

Effects of Ohmic heating and viscous dissipation on MHD fluid flow past a vertical plate embedded with porous medium

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Abstract

An appraisal is made of a steady, two-dimensional, electroconductive fluid flow past a vertical porous plate embedded with porous medium accompanied by magnetic field in transverse direction. The ascendancy of Ohmic heating is contemplated with attribute of heat transfer in addition to mass transfer. Furthermore, chemical reaction is escorted by viscous dissipation with other physical variables intricated in the problem. The continuity, momentum, energy and species concentration equations with relevant boundary conditions are resolved using numerical technique and graphical illustrations are assembled. Related non-dimensional coefficients are solved, discussed for numerous physical parameters and dispensed graphically in possible cases. This type of flow has conspicuous applications in innumerable applied technological and chemical industries.

AMS Subject Classifications: 76Dxx, 76S05.

Keywords: MHD, Ohmic heating, viscous dissipation, shearing stress, Nusselt number, Sherwood number.

1 INTRODUCTION

The analysis of electrically-conducting fluid in view of its engineering and industrial implementation is a salient expanse of exploration for the analyst. In such procedures, the flow can be synchronized by an applied magnetic meadow. The foremost intension of present endeavour is to reconnoitre the MHD viscous fluid in existence of Ohmic heating, chemical reaction with dissipation of energy and mass transfer characteristics. The passage of an electricity through a conductor generates heat by the mechanism of Joule heating and has applicability in fermentation, dehydration, extraction etc. In

recent years, MHD flow coupled with above features have attracted many investigators and their contribution in this area is praiseworthy.

Shercliff [1], Ferraro and Plumpton [2], Cramer and Pai [3] have studied the magnetohydrodynamic flows in an elaborate way. In engineering and electronics fields, the application of MHD free-convection flows are noteworthy. Chien-Hsin-Chen [4], Chaudhary *et al.* [5], Palanimani [6], Sharma and Singh [7], Sibanda and Makinde [8], Babu and Reddy [9], Rashidi and Erfani [10], Loganathan and Sivapoornapriya [11], Rajakumar and Balamurugan *et al.* [12], Balamurugan and Gopikrishnan [13], Mishra *et al.* [14], Sharma and Mishra [15], Zigta [16], Swain *et al.* [17] have contributed outstandingly in this domain. Additionally, we can incorporate the endowment of the great researchers viz. Mishra and Mohanty [18] and Javed *et al.* [19] etc.

The endeavour of present study is to look into the effects of Ohmic heating of MHD fluid flow accompanying with chemical reaction and dissipation of energy past a vertical plate embedded with porous medium in presence of heat and mass flux surroundings. The problem is solved numerically using MATLAB's built in solver bvp4c and the solutions are represented graphically.

2. MATHEMATICAL FORMULATION:

The steady two-dimensional, free-convective, incompressible, electro-conductive fluid over a plate which is infinite, porous and vertically upward is contemplated. The $\bar{x}$ -axis is delineated alongside the plate when $\bar{y}$ -axis is upright to it. A magnetic field of strength B_0 is taken along $\bar{y}$ -direction where induced magnetic-flux is not encountered due to small magnetic Reynolds number. As the motion is two-dimensional and the plate is infinitely long, all the physical quantities are independent of $\bar{x}$. Let $\bar{u}$, $\bar{v}$ be the respective velocity components along $\bar{x}$ and $\bar{y}$ directions. Here, resistance heating, chemical reaction and dissipation of energy are accounted in appearance of heat transfer including mass transfer conditions also.

The boundary equations can be composed as follows:

$$\begin{aligned}\frac{\partial \bar{v}}{\partial \bar{y}} &= 0 \\ \Rightarrow \bar{v} &= -v_0\end{aligned}\quad (2.1)$$

where v_0 is constant, the negative symbol is introduced since the suction is in the direction of plate.

$$\bar{v}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right) = \nu\left(\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) + g\beta(\bar{T} - T_\infty) + g\beta(\bar{C} - C_\infty) - \left(\frac{\sigma B_0^2}{\rho}\right)\bar{u} - \frac{\nu \bar{u}}{K_p} \quad (2.2)$$

$$\bar{v}\left(\frac{\partial \bar{T}}{\partial \bar{y}}\right) = \frac{\kappa}{\rho c_p}\left(\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}\right) + \frac{\nu}{c_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 + \left(\frac{\sigma B_0^2}{\rho c_p}\right)\bar{u}^2 \quad (2.3)$$

$$\bar{v}\left(\frac{\partial \bar{C}}{\partial \bar{y}}\right) = D\left(\frac{\partial^2 \bar{C}}{\partial \bar{y}^2}\right) + D_1\left(\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}\right) - K_1(\bar{C} - C_\infty) \quad (2.4)$$

The boundary conditions are:

$$\begin{aligned} \bar{y} = 0: \quad \bar{u} = 0, \quad \bar{T} = T_w, \quad \bar{C} = C_w \\ \bar{y} \rightarrow \infty: \quad \bar{u} \rightarrow 0, \quad \bar{T} \rightarrow T_\infty, \quad \bar{C} \rightarrow C_\infty \end{aligned} \quad (2.5)$$

where ν is kinematic viscosity, κ is thermal conductivity, g is the acceleration due to gravity, $\bar{T}$ and $\bar{C}$ are fluid temperature and species concentration of the fluid, T_w and C_w are the temperature and concentration near the plate, T_∞ and C_∞ are the temperature and concentration away from the plate, β and $\bar{\beta}$ are coefficient of thermal expansion and mass expansion, ρ is density of the fluid, K_1 is chemical reaction parameter, σ is magnetic permeability, B_0 is magnetic parameter, D and D_1 are molecular diffusivity and coefficient of thermal diffusivity, C_p is specific heat at constant pressure.

We introduce the following non-dimensional quantities:

$$\begin{aligned} y = \frac{\bar{y}v_0}{\nu} \quad u = \frac{\bar{u}}{v_0} \quad \theta = \frac{\bar{T} - T_\infty}{T_w - T_\infty} \quad \phi = \frac{\bar{C} - C_\infty}{C_w - C_\infty} \\ K_p = \frac{\bar{K}_p v_0^2}{\nu^2} \quad Ec = \frac{v_0^2}{C_p(T_w - T_\infty)} \quad Pr = \frac{\nu C_p \rho}{\kappa} \\ M = \frac{\sigma \nu B_0^2}{v_0^2 \rho} \quad So = \frac{D_1(T_w - T_\infty)}{\nu(C_w - C_\infty)} \quad K = \frac{K_1 \nu}{v_0^2} \quad Sc = \frac{\nu}{D} \\ Gr = \frac{\nu g \beta (T_w - T_\infty)}{v_0^3} \quad Gm = \frac{\nu g \bar{\beta} (C_w - C_\infty)}{v_0^3} \quad \nu = \frac{\mu}{\rho} \end{aligned} \quad (2.6)$$

where Gr is the Grashof number for heat transfer, Gm is the Grashof number for mass transfer, So is the Soret number, Ec is the Eckert number, K is the chemical reaction parameter, κ is the thermal conductivity, Sc is the Schmidt number, θ is the dimensionless temperature, ϕ is the dimensionless concentration.

On introducing the non-dimensional quantities in the equations (2.2) to (2.4), we get

$$\frac{d^2 u}{dy^2} + \frac{du}{dy} - u \left(M + \frac{1}{K_p} \right) = -Gr\theta - Gm\phi \quad (2.7)$$

$$\frac{d^2 \theta}{dy^2} + Pr \frac{d\theta}{dy} + Pr Ec \left(\frac{du}{dy} \right)^2 + Pr Ec M u^2 = 0 \quad (2.8)$$

$$\frac{d^2 \phi}{dy^2} + Sc \frac{d\phi}{dy} + Sc So \left(\frac{d^2 \theta}{dy^2} \right) - K Sc \phi = 0 \quad (2.9)$$

The metamorphosed boundary conditions are:

$$\begin{aligned} y = 0: \quad u = 0, \quad \theta = 1, \quad \phi = 1 \\ y \rightarrow \infty: \quad u \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \end{aligned} \quad (2.10)$$

3. METHOD OF SOLUTION

The non-linear coupled ordinary differential equations (2.7) to (2.9) are interpreted numerically by means of MATLAB's built in solver bvp4c by taking into consideration the boundary conditions (2.10).

4. RESULTS AND DISCUSSION

To acquire a physical perception of the considered problem, an emblematic set of numeric results are constituted graphically for various physical variables jumbled in the problem.

In figures 4.1 to 4.6, the velocity u against y has been delineated for assorted flow variables which are involved in the problem. It is revealed that the velocity increases to a optimal point near the plate and then reduces to the boundary value when we move far away from the plate in all the figures with fixed values $M=2$, $Gr=3.5$, $Gm=4.5$, $Sc=0.5$, $So=2.5$, $Pr=6.9$, $K=0.1$, $Kp=0.6$, $E=0.001$ unless otherwise stated.

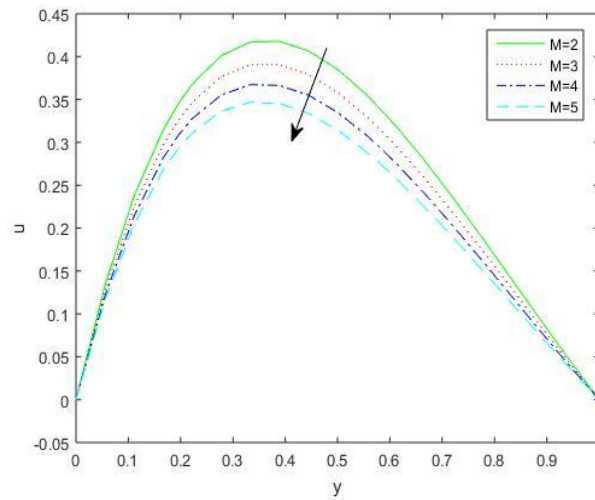


Figure 4.1: Velocity for M

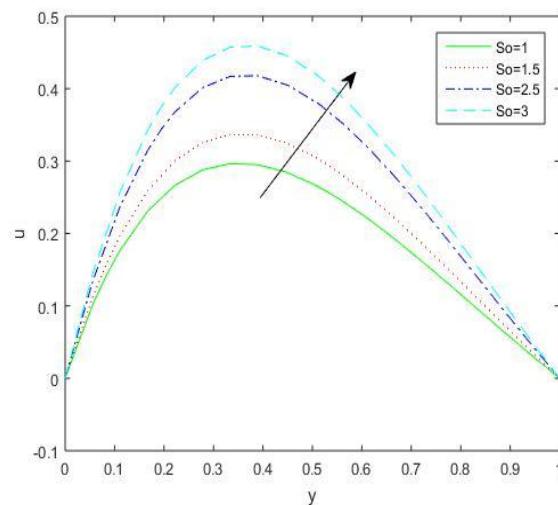


Figure 4.2: Velocity for So

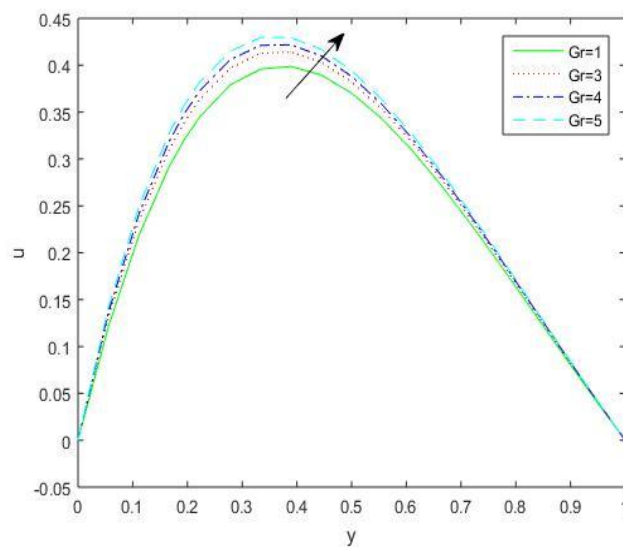


Figure 4.3: Velocity for Gr

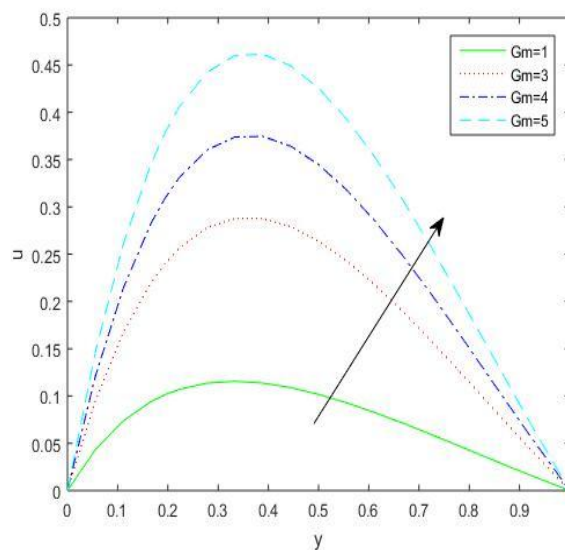


Figure 4.4: Velocity for Gm

Figure 4.1 displays the impact of magnetic parameter(M) on velocity component u . It is observed that growth in the intensity of magnetic field decelerates the fluid velocity because transverse magnetic field produces the Lorentz force which has a propensity to impede the motion.

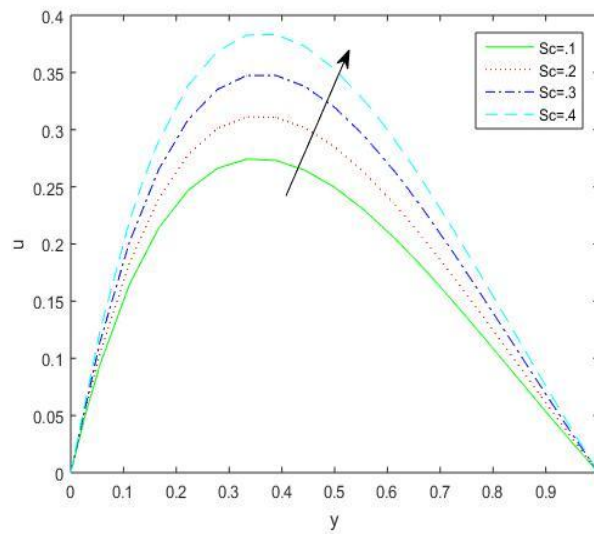


Figure 4.5: Velocity for Sc

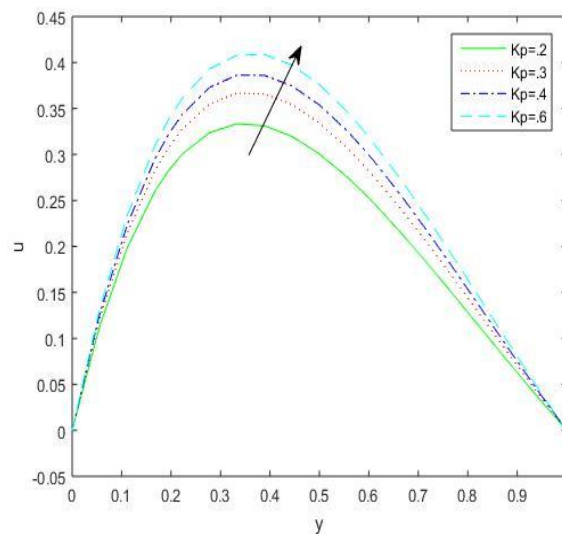


Figure 4.6: Velocity for K_p

Influence of Soret number (So), Grashof number for heat transfer (Gr), Grashof number for mass transfer (Gm), Schmidt number (Sc), permeability parameter (K_p) are represented by the figure 4.2, figure 4.3, figure 4.4, figure 4.5, figure 4.6 respectively. In all the cases, the fluid velocity experiences increasing trend during the rise of the flow parameters.

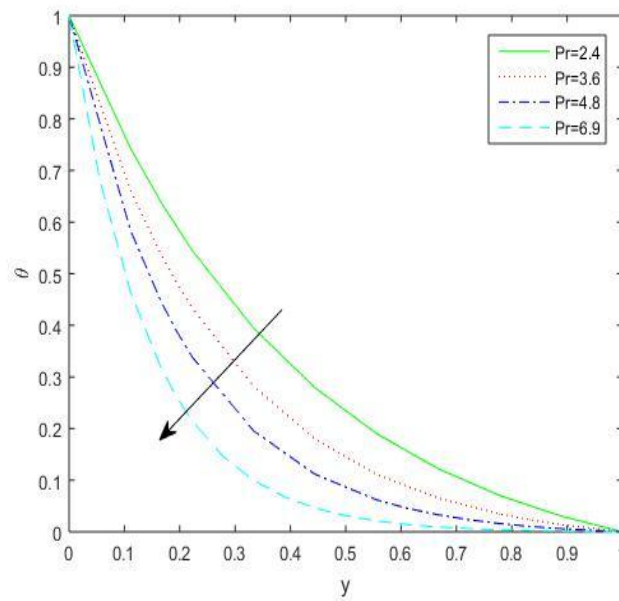


Figure 4.7: Temperature for Pr

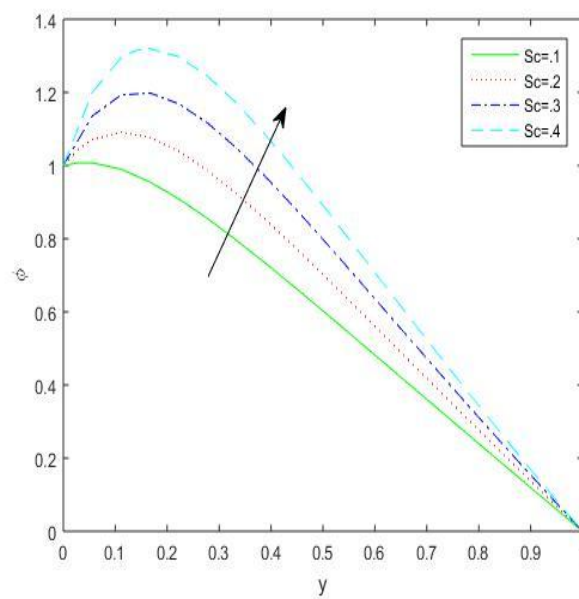


Figure 4.8: Concentration for Sc

Figure 4.7 depicts the temperature θ against y . The enhancement of Prandtl number decelerates the temperature.

Figures 4.8 to 4.12 embellish the etiquette of concentration in the concerned field. Concentration is accelerated in the neighbourhood of the plate and approaches boundary value when we move far away from the plate.

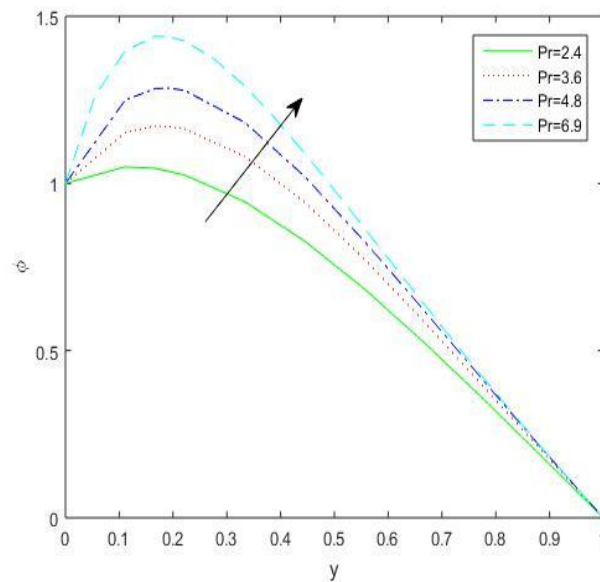


Figure 4.9: Concentration for Pr

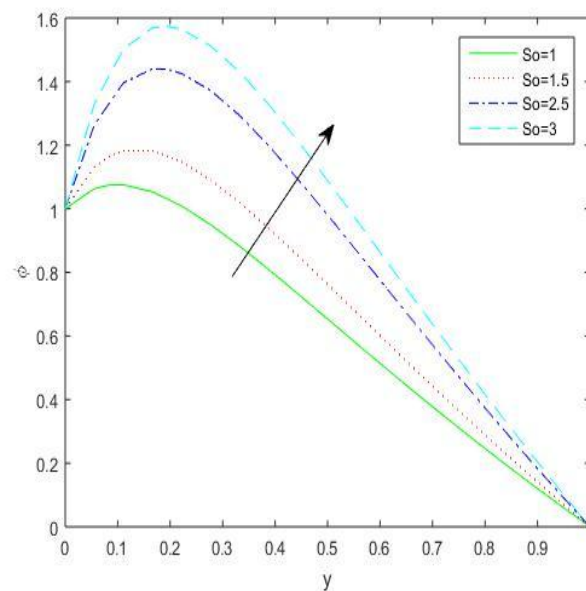


Figure 4.10: Concentration for So

Effect of Schmidt number, Prandtl number and Soret number are represented by figure 4.8, figure 4.9 and figure 4.10 respectively and in all the cases concentration increases with the increment in the parameters.

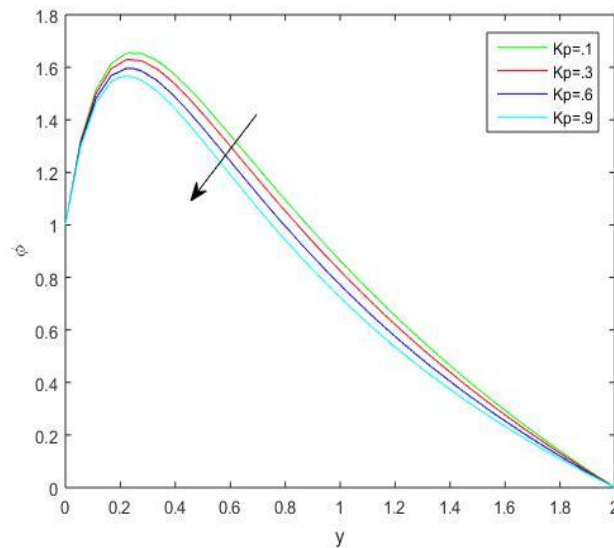


Figure 4.11: Concentration for K_p

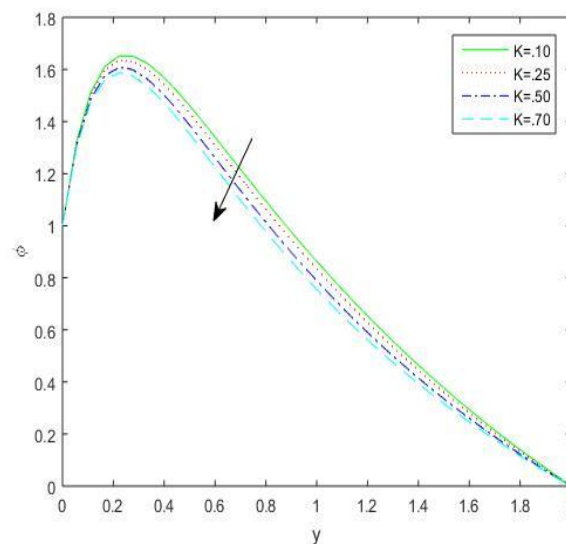


Figure 4.12: Concentration for K

In figures 4.11 and 4.12, diminishing trend of concentration is observed with variation of the physical parameters viz. K_p and K .

Figures 4.13 to 4.18 display the impact of Grashof number for mass transfer (G_m), Soret

number (So), Grashof number for heat transfer(Gr), magnetic parameter(M), permeability parameter(K_p), Schmidt number (Sc) on skin friction.

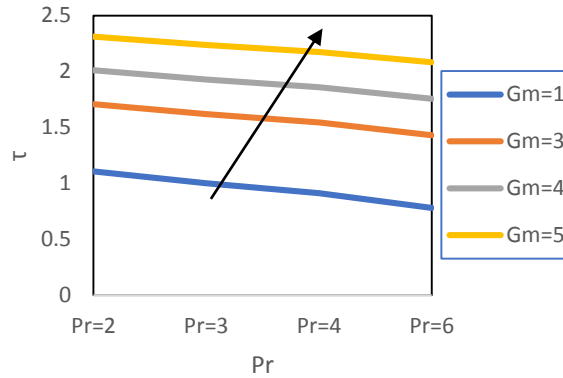


Figure 4.13: Skin friction for Gm

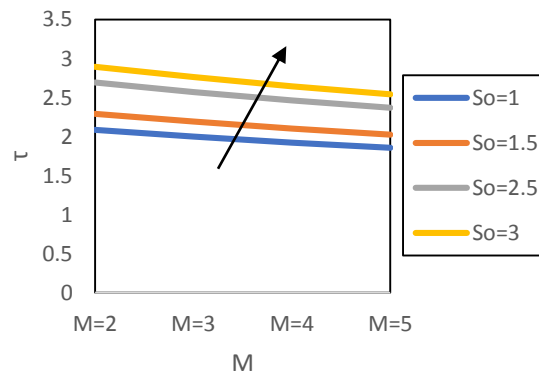


Figure 4.14: Skin friction for So

Figure 4.13 elucidates, rise in numerical values of Gm , skin friction gets raised. From figure 4.14, we notice that with the elevation of Soret number, skin friction increases. Influence of Gr number on skin friction is shown in figure 4.15, which indicates that with shootup of Gr number, skin friction gets elevated. Figure 4.16 evinces that the skin friction lessen with the enlargement of magnetic parameter(M). Figure 4.17 and 4.18 illustrate that the skin friction enhances with the raise of K_p and Sc .

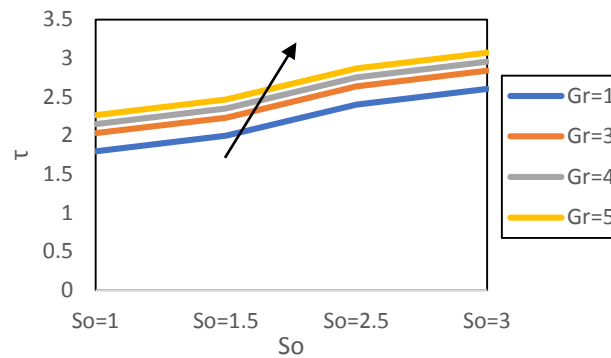


Figure 4.15: Skin friction for Gr

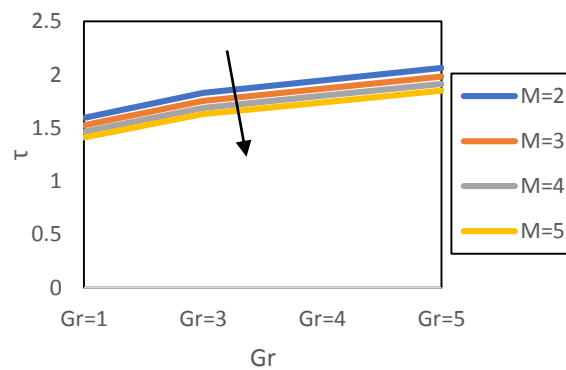


Figure 4.16: Skin friction for M

Figures 4.19 to 4.21 exhibit the impact of chemical reaction parameter(K), Soret number(So) and permeability parameter(K_p) on Nusselt number.

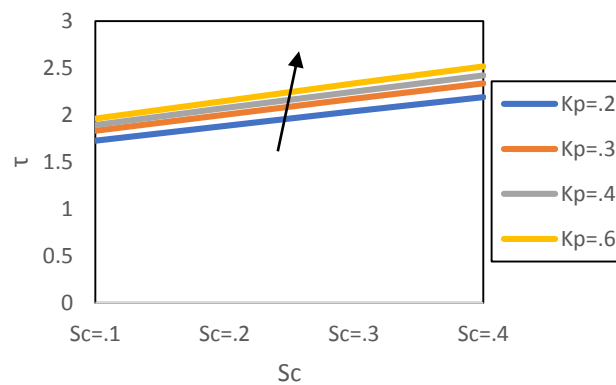


Figure 4.17: Skin friction for K_p

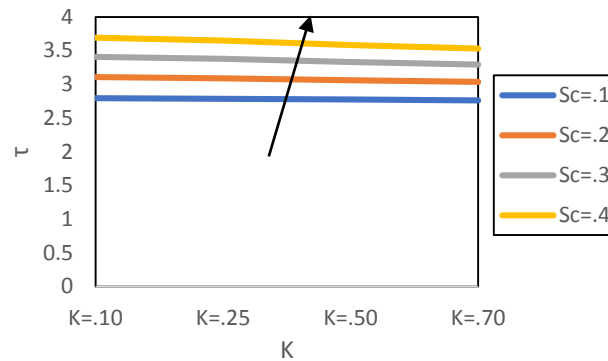


Figure 4.18: Skin friction for Sc

In figure 4.19, a reduction in Nusselt number is noticed due to expansion of chemical reaction parameter and a reverse trend is acquired in case of Soret number and permeability parameter(K_p) which can be seen from the figures 4.20 and 4.21

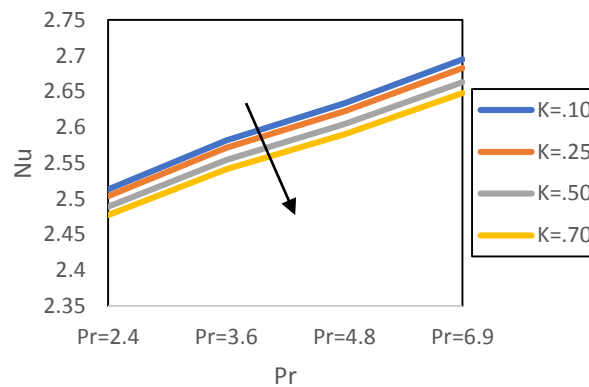


Figure 4.19: Nusselt number for K

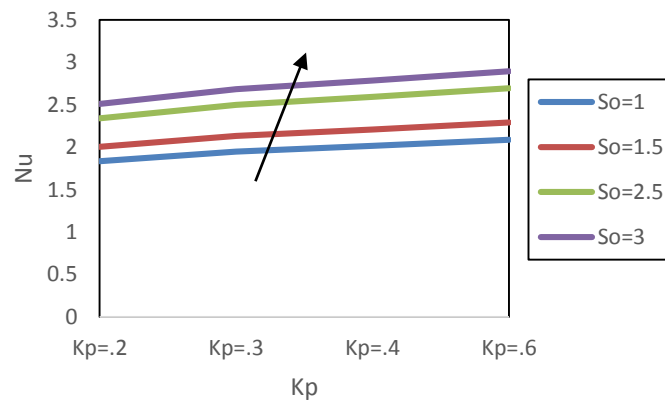


Figure 4.20: Nusselt number for So

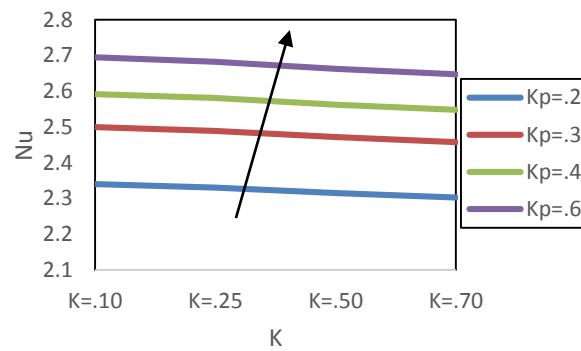


Figure 4.21: Nusselt number for K_p

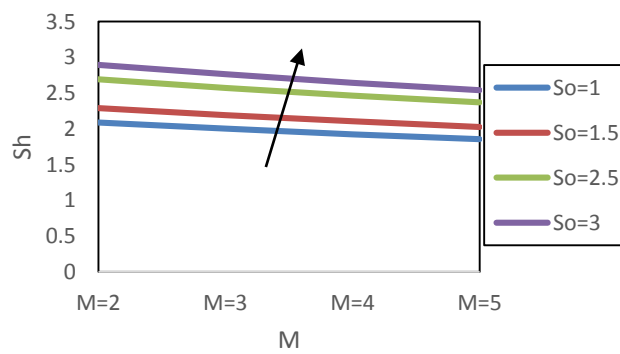


Figure 4.22: Sherwood number for S_o

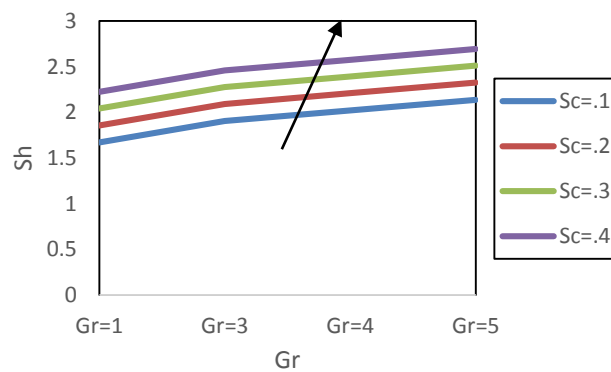


Figure 4.23: Sherwood number for Sc

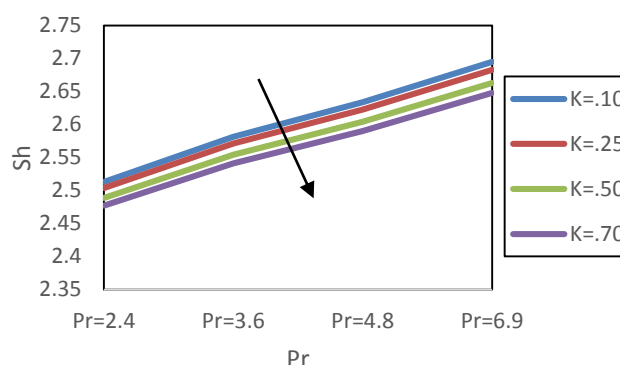


Figure 4.24: Sherwood number for K

Figures 4.22 to 4.24 illustrate the effect of pertinent flow parameters on Sherwood number. In figures 4.22 and 4.23, it is detected that with the enhancement of Soret number and Schmidt number, Sherwood number expands. From figure 4.24 it is seen that the Sherwood number decelerates with the acceleration of chemical reaction parameter.

5 CONCLUSION

In this paper, the upshots of diverse relevant parameters on MHD free-convection fluid flow along with Joule-heating and chemical reaction in companionship of viscous dissipation is explored. We have listed below some notable points regarding the present study:

- “The velocity field” is considerably affected at every point of the fluid flow region by the variation of the several flow parameters.
- The magnetic parameter hits the velocity in the zone of flow prominently. It is perceived that the enhancement of magnetic parameter diminishes the fluid velocity and this is matched with the physical behaviour of the magnetic parameter.
- The disparity of “Prandtl number” affects significantly the temperature field in comparison with other flow parameters.
- The concentration field is also affected by the flow parameters.
- The skin friction cavorts an indispensable part in the flow characteristics.
- The Nusselt number fluctuates prominently by the variation of various fluid flow parameters.
- The Sherwood number oscillate notably by the variation of pertinent flow parameters.

REFERENCES

- [1] Shercliff, J.A., 1965, "A Text Book of Magnetohydrodynamics", Pergamon Press, London.
- [2] Ferraro, V.C.A., and Plumpton, C., 1966, "An Introduction to Magneto Fluid Mechanics", Clarendon Press, Oxford.
- [3] Cramer, K.P., and Pai, S.I., 1973, "Magneto Fluid Dynamics for Engineers and Applied Physics", McGraw-Hill Book Co., New York.
- [4] Chien-Hsin-Chen, 2004, "Combined heat and mass transfer in MHD free convection from a vertical surface with Ohmic heating and viscous dissipation", *Int. J. Eng. Sci.*, 42(7), pp. 699-713.
- [5] Chaudhary, R.C., Sharma, B.K., and Jha, A.K., 2006, "Radiation effect with simultaneous thermal and mass diffusion in MHD mixed convection flow from a vertical surface with Ohmic heating", *Rom. J. Phys.*, 51(7-8), pp. 715-727.
- [6] Palanimani, P.G., 2007, "Effects of chemical reactions, heat and mass transfer on non-linear Magnetohydrodynamic boundary layer flow over a wedge with a porous medium in the presence of Ohmic heating and viscous dissipation", *J. Porous Media*, 10(5), pp. 489-502.
- [7] Sharma, P. R., and Singh, G., 2009. "Effects of Ohmic heating and viscous dissipation on steady MHD flow near a stagnation point on an isothermal stretching sheet, *Thermal Science*, 13(1), pp. 5-12.
- [8] Sibanda, P., and Mankinde, O.D., 2010, "On steady MHD flow and heat transfer past a rotating disk in a porous medium with Ohmic heating and viscous dissipation", *Int. J. Num. methods for heat and fluid flow*, 20(3), pp. 269-285.
- [9] Babu, VSH., and Reddy, GV R., 2011, "Mass transfer effects on MHD mixed convective flow from a vertical surface with Ohmic heating and viscous dissipation", *Adv. in Appl. Sci. Res.*, 2(4), pp. 138-146.
- [10] Rashidi, M.M., and Erfani, E., 2012, "Analytical method for solving steady MHD convective and slip flow due to a rotating disk with viscous dissipation and Ohmic heating", *Eng. Computations*, 29(6), pp. 562-579.
- [11] Loganathan, P., and Sivapoornapriya, C., 2016, "Ohmic heating and viscous dissipation effects over a vertical plate in the presence of porous medium", *J. Appl. Fluid Mech.*, 9(1), pp. 225-232.
- [12] Rajakumar, K.V.B., Balamurugan, K.S., Murthy, Ch.V.R., and Reddy, M.U., 2017, "Chemical reaction and viscous dissipation on MHD free convective flow past a semi-infinite moving vertical porous plate with radiation absorption", *Global J. Pure and Appl. Math.*, 13(12), pp. 8297-8322.
- [13] Balamurugan, K., and Gopikrishnan, V., 2017, "Viscous dissipation and Ohmic heating effects on heat and mass transfer of MHD mixed convection flow over an inclined porous plate in presence of Soret effect and temperature gradient

- dependent heat source”, *Int. J. Math. Trends and Tech.*, 49(2), pp. 93-99.
- [14] Mishra, A., Pandey, A., and Kumar, M., 2018, “Ohmic-viscous dissipation and slip effects on nanofluid flow over a stretching cylinder with suction/injection”, *Nanoscience and Technology: An Int. J.*, 9(2), pp. 99-115.
- [15] Sharma, R.P., and Mishra, S.R., 2019, “Combined effects of free convection and chemical reaction with heat-mass flux conditions: A semianalytical technique”, *Pramana-Journal of Physics*, 93(6), pp. 93-99.
- [16] Zigta, B., 2019, “Thermal radiation, chemical reaction, viscous and Joule dissipation effects on MHD flow embedded in a porous medium”, *Int. J. Appl. Mech. and Eng.*, 24(3), pp. 725-737.
- [17] Swain, B.K., Parida, B.C., Kar, S., and Senapati, N., 2020, “Viscous dissipation and Joule heating effect on MHD flow and heat transfer past a stretching sheet embedded in a porous medium”, *Heliyon*, 6(10), e05338.
- [18] Mishra, S.R., and Mohanty, B., 2020, “Heat and mass transfer flow over a stretching surface with Ohmic heating and chemical reaction”, *Heat Transfer*, 49(4).
- [19] Javed, T., Faisal, M., and Ahmed, I., 2020, “Action of viscous dissipation and Ohmic heating on bidirectional flow of a magneto Prandtl nanofluid with prescribed heat and mass fluxes”, *Heat Transfer*, 49(8), pp. 4801-4819.