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Numerical Study of Thermoelastic Damping Effects on Diamond Based Beams with Plane Stress and Plane Strain Conditions Applying Nonclassical Elasticity Theory

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Abstract

Damping effects are very important in MEMS-based sensors and actuators. The factors of energy dissipation are either extrinsic or intrinsic losses. Thermoelastic damping (TED) is considered as an intrinsic loss due to its interaction within the material structure. Thermo elasticity occurs as a result of coupling between the temperature field and elastic field of the material. In this paper, analytical models and numerical simulations are used to quantify the energy losses due to thermo-elastic damping in diamond-based microbeams. In order to include the size effects, nonclassical elasticity theory like modified couple stress theory is used to accurately model thermoelastic damping. Both plane stress and strain conditions of diamond-based beams are numerically simulated using MATLAB 2015 and the thermoelastic damping limited quality factors (Q_{TED}) are analysed. Q_{TED} is maximized when the beams are under plane stress condition and the thickness of the beam comparable to the internal length scale parameter. The size effect on Q_{TED} is analysed by conducting numerical simulations based on a complex frequency approach and is found to be enhanced by nonclassical elasticity theory.

Keywords: Damping effects, Thermo elasticity, Microbeams, Plane stress and plane strain, Thermoelastic damping limited quality factor.

1. INTRODUCTION

Microbeams are one of the major elements widely used for sensing [1] and communication applications [2] due to its enormous merits. Quality Factor (QF) of microbeam resonators is an important performance index when used mainly as sensors and filters. QF gives a representation of energy dissipation due to its various energy losses which can be classified as extrinsic and intrinsic [3]. Extrinsic losses can be simply managed whereas to mitigate intrinsic losses proper design of the device should be done. Intrinsic losses like thermoelastic damping is a crucial energy dissipation mechanism at micro and nano-scales which limits the maximum achievable QF in the resonator denoted by Q_{TED} [4]. Presence of TED in various beam based resonating structures is identified in various research works.

Zener identified the existence of TED as a significant energy loss mechanism in micro-beams in 1937 [5][6][7]. Lifshitz and Roukes derived a closed-form expression for TED of beams with high degree of accuracy [8]. Prabhakar and Vengallatore analysed TED with heat transfer in 2D and presented an analytical relation for TED in the form of an infinite series for micro-resonators [[9]]. Parayil studied TED in Timoshenko microbeams and presented analytical and numerical solutions [10]. TED analysis and geometry optimizations in circular microplates is done by Resmi et al. [11]. Classical continuum theories are insufficient to predict the size-dependent mechanical behaviour in micro/nanobeams due to the lack of material length scale parameter [12][13]. The size effect in engineering materials is first studied in the works of Cauchy [14] and Voigt [15] which are later proved experimentally. Higher order continuum theories like modified couple stress theory (MCST) can be used to estimate the size dependencies at micron/submicron levels. Yang et al. developed modified couple stress theory in which only a single material length scale parameter is included to investigate the mechanical behaviour of microstructures [16]. Fathalilou used MCST to study the pull-in instability of a gold microbeam switch [17]. By applying MCST, Rezazadeh derived a size-dependent relation for TED in microbeams and studied the small-scale effects on the critical thickness [18]. Resmi R. et al applied MCST to study the figure of merit and impact of dimensionless length scale parameter in quality factor analysis of rectangular microplates [19][20].

In this paper, a comparative study of energy dissipation and quality factor evaluation in thin and thick beams pertaining to stress and strain conditions are done for diamond based microbeams. The Q_{TED} can be improved by properly selecting the plane stress/strain conditions of the beam as depicted in subsequent sections. In Section 2 expressions for plane stress and strain conditions of a microbeam are given. Section 3 presents the results and discussions- energy dissipation and Q_{TED} analysis are done for both plane stress and strain conditions applying CT and MCST. Section 4 deals with the conclusion regarding the work.

Expression for Plane Stress and Plane Strain Conditions

To derive the expression for thermoelastic damping limited quality factor applying Modified Couple Stress Theory (MCST), total strain energy in terms of its stress components are derived for both plane stress and plane stain conditions [18]. In the case of plane stress state, across a particular plane, the stress vector is zero which happens over the entire element of the structure resulting in a thin beam structure. In the case of a plane stress state, the stress state can be represented by a two dimensional tensor. Plane strain state corresponds to thick beams.

1.1. Stress and strain fields in a vibrating microbeam

According to the Bernoulli model of a beam, the displacements in the x and z directions are given by, Longitudinal displacement $u_x(x,z,t)$ in terms of transversal displacement is $u_z(x,t)$ as [18]

$$u_x = -zu_{z,x} \tag{1}$$

During vibrations of the beam, coupling of temperature and elastic fields occur and as a result deformations due to thermal and mechanical components arise.

Mechanical strain,

$$\varepsilon_{ij}\,(M) = \frac{1+v}{E}\sigma_{ij} - \frac{v}{E}\sigma_{kk}\delta_{ij}, \tag{2}$$

Thermal strain,

$$\varepsilon_{ij}\,(T) = \alpha\vartheta\delta_{ij}, \tag{3}$$

Considering Eqs. (13), (14) and (15), the total strain field can be written as

Total strain field,

$$\varepsilon_{ij} = \frac{1+v}{E}\sigma_{ij} + \left(\alpha\vartheta - \frac{v}{E}\sigma_{kk}\right)\delta_{ij} \tag{4}$$

in which α, σ_{ij}, δ_{ij},E and v are the thermal expansion coefficient, stress tensor, the Kronecker delta, Young's modulus and Poisson's ratio respectively. Here, $\vartheta = T - T_0$as the temperature change with respect to a reference temperature $T_0$and assuming shear strains created by temperature change are negligible.

1.1.1. Plane Stress Condition

In continuum mechanics, if the stress vector is zero across a particular plane, a material is said to be under plane stress and the stress analysis is considerably simplified. Under plane stress condition b $\leq$ 5h where b and h are the width and thickness of the beam respectively. The corresponding stress and strain fields were as obtained below [18].

The stress components are

$$\sigma_{xx} = E(\varepsilon_{xx} - \alpha\vartheta) \tag{5}$$

$$\sigma_{zz} = \sigma_{xz} = \sigma_{yz} = 0 \tag{6}$$

$$\sigma_{yy} = \sigma_{xy} = \sigma_{zy} = 0 \tag{7}$$

The strain components are

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} + \alpha\vartheta, \tag{8}$$

$$\varepsilon_{yy} = \varepsilon_{zz} = -v\varepsilon_{xx} + (1+v)\alpha\vartheta, \tag{9}$$

$$\varepsilon_{xy} = \varepsilon_{xz} = \varepsilon_{yz} = 0. \tag{10}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{11}$$

The displacement field of the beam according to Euler Bernoulli theory is given by

$$u_x = -zu_{z,x} \tag{12}$$

The non-zero strain, and stress tensors components in terms of the displacement field are:

$$\varepsilon_{xx} = u_{x,x} = -zu_{z,x\,x} \tag{13}$$

$$\varepsilon_{yy} = \varepsilon_{zz} = vzu_{z,x\,x} + \alpha(1+v)\vartheta \tag{14}$$

$$\sigma_{xx} = -E\,\left(zu_{z,x\,x} - \alpha\vartheta\right) \tag{15}$$

1.1.2. Plane Strain conditions

A material is considered to be under plane strain, if its width b ≥ 5h where h is the thickness of the beam i.e., applicable to very thick members [18].

Plane Strain Condition,

$$\varepsilon_{xx} = u_{x,x} = -zu_{z,x\,x} \tag{16}$$

$$\varepsilon_{zz} = \frac{v}{(1-v)} zu_{z,x\,x} + \frac{(1+v)}{1-v}\alpha\vartheta \tag{17}$$

$$\sigma_{xx} = \frac{-E}{(1-v^2)} zu_{z,x\,x} - \frac{E}{1-v}\alpha\vartheta \tag{18}$$

$$\sigma_{yy} = \frac{-Ev}{(1-v^2)} zu_{z,x\,x} - \frac{E}{1-v}\alpha\vartheta \tag{19}$$

The inverse of the quality factor according to complex frequency approach [18],

$$Q^{-1} = 2\left|\frac{\Re(\mathrm{s})}{\Im(\mathrm{s})}\right| = 2\left|\frac{\Re(\Omega)}{\Im(\Omega)}\right| \tag{20}$$

where $\Re(\Omega)$ is the real and $\Im(\Omega)$ is the imaginary parts considering the complex frequency approach.

2. RESULTS AND DISCUSSIONS

In diamond based microbeams, for evaluating the energy dissipation due to thermoelastic damping and quality factor limited by thermoelastic damping (Q_{TED}), the numerical simulations are conducted with MATLAB 2015. The thermoelastic damping limited quality factor, Q_{TED} of a resonator depends on the type of the structural material used since its thermal and mechanical properties greatly affect its energy dissipation related to thermoelastic damping.

Table 1. Mechanical and thermal properties of diamond material

Mechanical/Thermal properties		Diamond
Mechanical	E[GPa]	800
	v	0.069
	ρ	3515
Thermal	k	100
	C_p	510
	$\alpha[10^{-6}]$	1.2
	$\chi[cm^2/s]$	0.558
	C_v	1792650

The mechanical and thermal properties of diamond material are given in Table 1 [20].

The diamond based microbeams are numerically simulated for analysing the energy dissipation due to thermoelastic damping applying classical theory for the first three vibrating modes. The quantification of energy dissipation for the diamond microbeams is again governed by applying non classical higher order theory like MCST for the first three vibrating modes. All the numerical simulations are done for plane stress (b $\leq$ 5h) and plane strain conditions (b $\geq$ 5h).

Table 2. Energy dissipation and Quality Factor of a diamond based microbeam under Clamped-Clamped Boundary condition applying Classical Theory and Modified Couple Stress Theory vibrating in first three modes.

Stress/ Strain	Energy dissipation/Quality factor	Mode Number	*l*=0 (CT)	*l*=1(MCST)
Stress	Energy dissipation	I	9.52E-05	3.96E-05
		II	9.52E-05	3.96E-05
		III	9.52E-05	3.96E-05
	Quality factor	I	10504.2	25200
		II	10504.2	25200
		III	10504.2	25200
Strain	Energy dissipation	I	1.02E-05	4.27E-05
		II	1.02E-05	4.27E-05
		III	1.02E-05	4.27E-05
	Quality factor	I	98039.2	23400
		II	98039.2	23400
		III	98039.2	23400

Figure. 1 shows the energy dissipation Vs thickness of a diamond beam under plane stress and plane strain conditions for first three vibrating modes.

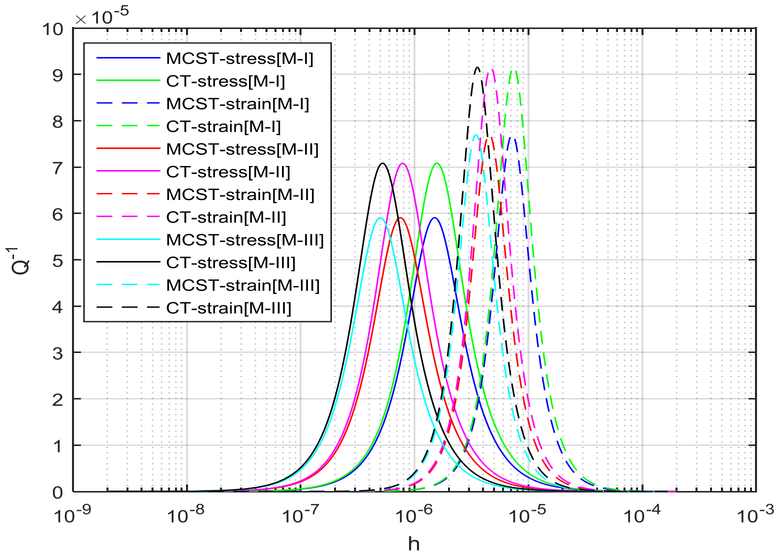

Figure. 1 Energy dissipation Vs thickness of a diamond beam under (a) plane stress condition (b) plane strain condition for first three modes applying classical theory and Modified Couple Stress Theory.

The numerical simulations are conducted based on classical theory and non classical theory. The non classical theory selected for analysis is Modified Couple Stress Theory.

When beams under plane stress and plane strain conditions are considered, beams with stress condition possess higher Q_{TED} than the beams with strain condition which is validated from Table 2. The inverse of energy dissipation due to thermoelastic damping directly gives the value of thermoelastic damping limited quality factor (Q_{TED}). The enhancement of Q_{TED} is realised by selecting plane stress state i.e. thin beams. The increase in Q_{TED} is found for both classical and non classical theories in plane stress state. The numerical simulations are done for different vibrating modes also. As the mode number increases, the Q_{TED} is constant and found to be applicable to both classical theory and MCST.

3. CONCLUSİON

Microbeam based resonators are widely used in MEMS-based systems. Thermoelastic damping is a prominent energy loss mechanism and the maximum attainable quality factor is limited due to the interaction within the material structure. When the systems are downsized, to accurately evaluate the energy dissipation and corresponding quality factors, instead of conventional elasticity theories, nonelasticity theory like Modified Couple Stress Theory (MCST) is applied. Impacts of plane stress and plane strain conditions on energy dissipation and Q_{TED} (the inverse of energy dissipation Q^{-1}) are evaluated by numerical methods for diamond based microbeams. Higher values of Q_{TED} are achieved for diamond microbeams under plane stress condition i.e. for thin beams. The quality factor is found to be enhanced while applying modified couple stress theory (MCST) for both plane stress and strain conditions compared to classical theory(CT). The results obtained help engineers to fabricate microbeam resonators with enhanced quality factors for different applications.

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