Advances in Dynamical Systems and Applications.
ISSN 0973-5321, Volume 16, Number 2, (2021) pp. 1499-1514

https://www.ripublication.com/adsa.htm

Solving Fuzzy Multi-objective Linear Sum Assignment Problem with Modified Partial Primal Solution of ω - type 2 - Diamond Fuzzy Numbers by Using Linguistic Variables

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Abstract

In this paper, we introduce ω-type 1 and ω-type 2-diamond fuzzy numbers and solving a fuzzy multi-objective linear sum assignment problem (FMOLSAP) with linguistic variables whose λ_d −cut are expressed as ω-type 2-diamond fuzzy numbers (ω_{t2}-DFN). To solve the FMOLSAP, we can apply arithmetic operations of λ_d −cut of ω-type 2-diamond fuzzy numbers (ω_{t2}-DFN) to compute complete optimal matching.

Keywords: ω -type 1 and ω -type 2-diamond fuzzy numbers, Fuzzy assignment problem,

AMS Mathematics Subject Classification (2010): 90B80, 90C29, 03E72

1. Introduction

In 2010, Kagade and Bajaj[6] discussed for solving fuzzy multi-objective assignment problem with interval cost. Isabel et.al [5] an application of linguistic variables in fuzzy assignment problem, In 2014, Gupta and Mehlawat[3] For treating the fuzzy multi-objective assignment problems, a novel possibility programming technique was provided, Kayvan Salehi[7] proposed an approach for solving MOAP with interval parameters. In 2015, Pathinathan and Ponnivalavan [11] discussed diamond fuzzy number. In 2017, Nagoor Gani and Shiek Pareeth [9] are discussed dual variables and partial solution for solving FLSAP. In recently, many of the researchers work in this area of fuzzy multi-objective assignment problems like [1-4,8,10,12, 13-15].

We introduced ω-type 1-diamond fuzzy numbers and ω-type2-diamond fuzzy numbers are discussed in this paper. The upper and lower membership functions of diamond fuzzy numbers are described as ω-type 1 and ω-type 2-diamond fuzzy numbers. In λ_d- cut form, express the ω-type2 -diamond fuzzy numbers. Single fuzzy linear sum assignment problems are converted from fuzzy multi-objective linear sum assignment problems by using ranking method. obtain partial feasible solution and complete optimal solution by using λ_d −cut of ω-type 2-diamond fuzzy numbers (ω_{t2}-DFN) . Arithmetic operations of λ_d −cut of ω-type 2-diamond fuzzy numbers (ω_{t2}-DFN) to obtain complete optimal matching

2. Preliminaries

2.1. Definition: [11]A fuzzy set F is defined as $\widetilde{F_d} = \{d', d^*, d'' \ (\alpha_d, \beta_d)\}$is called diamond fuzzy number and it's the following membership function is given by

$$\mu_{\widetilde{F_d}} = \begin{cases} 0 & for\ x \leq d' \\ \frac{(x-d')}{(d^*-d')} & for\ d' \leq x \leq d^* \\ \frac{(d''-x)}{(d''-d^*)} & for\ d^* \leq x \leq d'' \\ & \alpha_d \\ \frac{(d'-\text{x})}{(d'-d^*)} & for\ d' \leq x \leq d^* \\ \frac{(x-d'')}{(d^*-d'')} & for\ d^* \leq x \leq d'' \\ 1 & x = \beta_d \\ 0 & otherwise \end{cases}$$

2.2.Definition: A ω-type1- diamond fuzzy number is upper and lower membership function of the diamond fuzzy number is defined as $[\underline{\omega_{t1}\widetilde{F_d}}, \overline{\omega_{t1}\widetilde{F_d}}]$ where $\underline{\omega_{t1}\widetilde{F_d}}$ $=\{\underline{d'}, d^*, \underline{d''}, (\underline{\alpha_d}, \underline{\beta_d}), \overline{\omega_{t1}\widetilde{F_d}} = \{\overline{d'}, d^*, \overline{d''}, (\overline{\alpha_d}, \overline{\beta_d})\}$ and it's the following membership function is given by

$$\mu_{\omega_{t1}\widetilde{F_d}} = \begin{cases} 0 & otherwise \\ \underline{\omega}\left(\frac{(x-\underline{d'})}{(d^*-\underline{d'})}\right) & for\ \underline{d'} \le x \le d^* \\ \underline{\omega}\left(\frac{(\underline{d''}-x)}{(\underline{d''}-d^*)}\right) & for\ d^* \le x \le \underline{d''} \\ \overline{\omega}\left(\frac{(x-\overline{d'})}{(d^*-\overline{d'})}\right) & for\ \overline{d'} \le x \le d^* \\ \overline{\omega}\left(\frac{(\overline{d''}-x)}{(\overline{d''}-d^*)}\right) & for\ d^* \le x \le \overline{d''} \\ \alpha_d \quad - \quad base & \\ \underline{\omega}\left(\frac{(\underline{d'}-x)}{(\underline{d'}-d^*)}\right) & for\ \underline{d'} \le x \le d^* \\ \underline{\omega}\left(\frac{(x-\underline{d''})}{(d^*-\underline{d''})}\right) & for\ d^* \le x \le \underline{d''} \\ \overline{\omega}\left(\frac{(\overline{d'}-x)}{(\overline{d'}-d^*)}\right) & for\ \overline{d'} \le x \le d^* \\ \overline{\omega}\left(\frac{(x-\overline{d''})}{(d^*-\overline{d''})}\right) & for\ d^* \le x \le \overline{d''} \\ \omega = 1 & x = \beta_d \\ 0 & otherwise \end{cases}$$

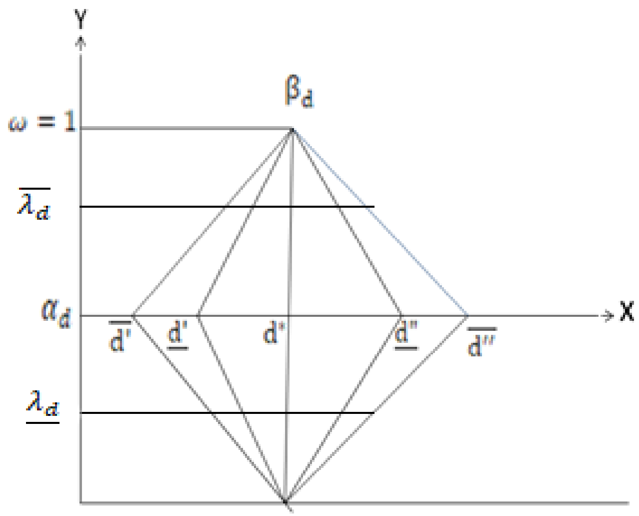

Figure 1: ω-type 1- diamond fuzzy number

2.3. Definition: A ω-type2 diamond fuzzy number is upper and lower membership function of the diamond fuzzy number is defined as $[\underline{\omega_{t2}\widetilde{F_d}}, \overline{\omega_{t2}\widetilde{F_d}}]$ where $\underline{\omega_{t2}\widetilde{F_d}}$

$=\{\underline{d'}, \underline{d^*}, \underline{d''}\ (\underline{\alpha_d}, \underline{\beta_d}), \overline{\omega_{t2}\widetilde{F_d}} = (\overline{d'}, \overline{d^*}, \overline{d''}\ (\overline{\alpha_d}, \overline{\beta_d}))$ and it's the following membership function is given by

$$\mu_{\omega_{t1}\widetilde{F_d}} = \begin{cases} 0 & otherwise \\ \underline{\omega}\left(\frac{(x-\underline{d'})}{(d^*-\underline{d'})}\right) & for\ \underline{d'} \le x \le \underline{d^*} \\ \underline{\omega}\left(\frac{(\underline{d''}-x)}{(\underline{d''}-d^*)}\right) & for\ \underline{d^*} \le x \le \underline{d''} \\ \overline{\omega}\left(\frac{(x-\overline{d'})}{(d^*-\overline{d'})}\right) & for\ \overline{d'} \le x \le d^* \\ \overline{\omega}\left(\frac{(\overline{d''}-x)}{(\overline{d''}-d^*)}\right) & for\ d^* \le x \le \overline{d''} \\ \alpha_d \quad - \quad base & \\ \underline{\omega}\left(\frac{(\underline{d'}-x)}{(\underline{d'}-d^*)}\right) & for\ \underline{d'} \le x \le d^* \\ \underline{\omega}\left(\frac{(x-\underline{d''})}{(d^*-\underline{d''})}\right) & for\ d^* \le x \le \underline{d''} \\ \overline{\omega}\left(\frac{(\overline{d'}-x)}{(\overline{d'}-d^*)}\right) & for\ \overline{d'} \le x \le d^* \\ \overline{\omega}\left(\frac{(x-\overline{d''})}{(d^*-\overline{d''})}\right) & for\ d^* \le x \le \overline{d''} \\ \omega = 1 & x = \beta_d \\ 0 & otherwise \end{cases}$$

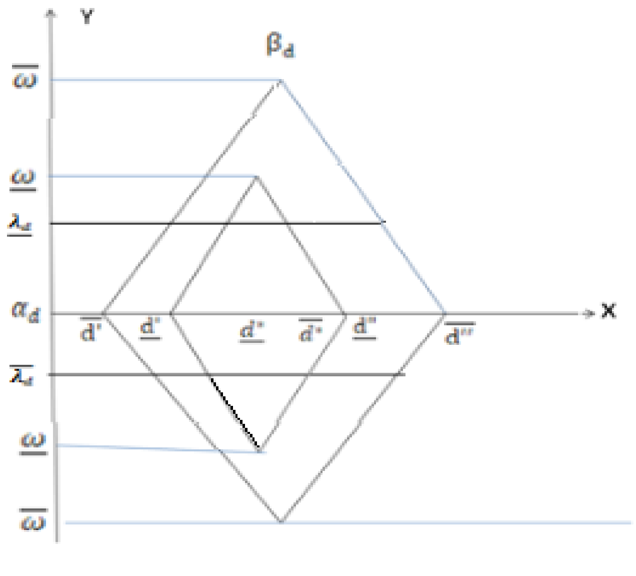

Figure 2: λ_d- cut of ω-type 2- diamond fuzzy number

3. Arithmetic Operations of ω-type2- Diamond Fuzzy Number ($\omega_{t2}\widetilde{F_d}$) :

Let us take two ω-type2 diamond fuzzy number are given below

$\omega_{t2}\widetilde{F_d}^1 = [\underline{\omega_{t2}\widetilde{F_d}^1}, \overline{\omega_{t2}\widetilde{F_d}^1}] = (\underline{d_1'}, \underline{d_2'}\underline{d_3'}\underline{d_4'}, \underline{d_5'}, \underline{d_6'}, \underline{\omega_{1F^1}}), (\overline{d_1'}, \overline{d_2'}, \overline{d_3'}, \overline{d_4'}, \overline{d_5'}, \overline{d_6'}, \overline{\omega_{1F^1}})$ and

$\omega_{t2}\widetilde{F_d}^2 = [\underline{\omega_{t2}\widetilde{F_d}^2}, \overline{\omega_{t2}\widetilde{F_d}^2}] = (\underline{d_1''}, \underline{d_2''}, \underline{d_3''}, \underline{d_4''}, \underline{d_5''}, \underline{d_6''}, \underline{\omega_{1F^2}}), (\overline{d_1''}, \overline{d_2''}, \overline{d_3''}, \overline{d_4''}, \overline{d_5''}, \overline{d_6''}, \overline{\omega_{1F^2}})$.

The following arithmetic operations of $\omega_{t2}\widetilde{F_d}^1$ and $\omega_{t2}\widetilde{F_d}^2$.

Addition

$\omega_{t2}\widetilde{F_d}^1 \oplus \omega_{t2}\widetilde{F_d}^2 =$

$((\underline{d_1'} \oplus \underline{d_1''}, \underline{d_2'} \oplus \underline{d_2''}, \underline{d_3'} \oplus \underline{d_3''}, \underline{d_4'} \oplus \underline{d_4''}, \underline{d_5'} \oplus \underline{d_5''}, \underline{d_6'} \oplus \underline{d_6''}), \min\{\underline{\omega_{1F^1}}, \underline{\omega_{1F^2}}\}),$

$((\overline{d_1'} \oplus \overline{d_1''}, \overline{d_2'} \oplus \overline{d_2''}, \overline{d_3'} \oplus \overline{d_3''}, \overline{d_4'} \oplus \overline{d_4''}, \overline{d_5'} \oplus \overline{d_5''}, \overline{d_6'} \oplus \overline{d_6''}), \min\{\overline{\omega_{1F^1}}, \overline{\omega_{1F^2}}\})$

Subtraction

$\omega_{t2}\widetilde{F_d}^1 \ominus \omega_{t2}\widetilde{F_d}^2 =$

$((\underline{d_1'} \ominus \underline{d_6''}, \underline{d_2'} \ominus \underline{d_5''}, \underline{d_3'} \ominus \underline{d_4''}, \underline{d_4'} \ominus \underline{d_3''}, \underline{d_5'} \ominus \underline{d_2''}, \underline{d_6'} \ominus \underline{d_6''}), \min\{\underline{\omega_{1F^1}}, \underline{\omega_{1F^2}}\}),$

$((\overline{d_1'} \ominus \overline{d_6''}, \overline{d_2'} \ominus \overline{d_5''}, \overline{d_3'} \ominus \overline{d_4''}, \overline{d_4'} \ominus \overline{d_3''}, \overline{d_5'} \ominus \overline{d_2''}, \overline{d_6'} \ominus \overline{d_1''}), \min\{\overline{\omega_{1F^1}}, \overline{\omega_{1F^2}}\}).$

Multiplication

$\omega_{t2}\widetilde{F_d}^1 \otimes \omega_{t2}\widetilde{F_d}^2 =$

$((\underline{d_1'} \otimes \underline{d_1''}, \underline{d_2'} \otimes \underline{d_2''}, \underline{d_3'} \otimes \underline{d_3''}, \underline{d_4'} \otimes \underline{d_4''}, \underline{d_5'} \otimes \underline{d_5''}, \underline{d_6'} \otimes \underline{d_6''}), \min\{\underline{\omega_{1F^1}}, \underline{\omega_{1F^2}}\}),$

$((\overline{d_1'} \otimes \overline{d_1''}, \overline{d_2'} \otimes \overline{d_2''}, \overline{d_3'} \otimes \overline{d_3''}, \overline{d_4'} \otimes \overline{d_4''}, \overline{d_5'} \otimes \overline{d_5''}, \overline{d_6'} \otimes \overline{d_6''}), \min\{\overline{\omega_{1F^1}}, \overline{\omega_{1F^2}}\}).$

Scalar multiplication

$\alpha_k (\omega_{t2}\widetilde{F_d}^1) =$

$((\alpha_k\underline{d_1'}, \alpha_k\underline{d_2'}, \alpha_k \underline{d_3'}, \alpha_k\overline{d_4'}, \alpha_k\underline{d_5'}, \alpha_k\underline{d_6'}, \underline{\omega_{1F^1}}),), (\alpha_k\overline{d_1'}, \alpha_k\overline{d_2'}, \alpha_k\overline{d_3'}, \alpha_k\overline{d_4'},$
$\alpha_k\overline{d_5'}, \alpha_k\overline{d_6'}, \overline{\omega_{1F^1}}))$ if $\alpha_k \geq 0$.

$\alpha_k (\omega_{t2}\widetilde{F_d}^1) =$

$((\alpha_k\underline{d_6'}, \alpha_k\underline{d_5'}, \alpha_k \underline{d_3'}, \alpha_k\underline{d_2'}, \underline{\alpha_k d_1'}, \underline{\omega_{1F^1}}), (\alpha_k\overline{d_6'}, \alpha_k\overline{d_5'}, \alpha_k\overline{d_4'}, \alpha_k\overline{d_3'}, \alpha_k\overline{d_2'}, \alpha_k\overline{d_1'}, \overline{\omega_{1F^1}}))$

if $k \leq 0$.

4. Mathematical Form of Multi-objective Fuzzy Linear Sum Assignment Problem is defined as:

Minimize $\tilde{z}^{k} = \sum_{i=1}^{n} \tilde{c}_{ij}^{k} \sum_{j=1}^{n} \tilde{x}_{ij}$

$$\text{Subject to } \sum_{i=1}^{n} \tilde{x}_{i} = 1, \text{ for } j = 1,2,3,4\ldots n$$

$$\sum_{i=1}^{n} \tilde{x}_{j} = 1, \text{ for } i = 1,2,3,4\ldots n$$

and $$\tilde{x}_{ij} = \begin{cases} 1, & if \quad job\ 'j' \ is\ matched \ to \ machine\ 'm' \\ 0, & otherwise \end{cases}$$

where $\tilde{z}^{k} = \{\tilde{z}^{1}, \tilde{z}^{2}, \tilde{z}^{3} \ldots\ldots \tilde{z}^{k}\}$ is vector of multi objectives

Table 1.

Job/ Machine	J_1	J_2	J_3	J_4
M_1	$[\underline{\tilde{c}_{11}},\overline{\tilde{c}_{11}}]$ $[\underline{\tilde{t}_{11}},\overline{\tilde{t}_{11}}]$ $[\underline{\tilde{q}_{11}},\overline{\tilde{q}_{11}}]$	$[\underline{\tilde{c}_{12}},\overline{\tilde{c}_{12}}]$ $[\underline{\tilde{t}_{12}},\overline{\tilde{t}_{12}}]$ $[\underline{\tilde{q}_{12}},\overline{\tilde{q}_{12}}]$	$[\underline{\tilde{c}_{13}},\overline{\tilde{c}_{13}}]$ $[\underline{\tilde{t}_{13}},\overline{\tilde{t}_{13}}]$ $[\underline{\tilde{q}_{13}},\overline{\tilde{q}_{13}}]$	$[\underline{\tilde{c}_{14}},\overline{\tilde{c}_{14}}]$ $[\underline{\tilde{t}_{14}},\overline{\tilde{t}_{14}}]$ $[\underline{\tilde{q}_{14}},\overline{\tilde{q}_{14}}]$
M_2	$[\underline{\tilde{c}_{21}},\overline{\tilde{c}_{21}}]$ $[\underline{\tilde{t}_{21}},\overline{\tilde{t}_{21}}]$ $[\underline{\tilde{q}_{21}},\overline{\tilde{q}_{21}}]$	$[\underline{\tilde{c}_{22}},\overline{\tilde{c}_{22}}]$ $[\underline{\tilde{t}_{22}},\overline{\tilde{t}_{22}}]$ $[\underline{\tilde{q}_{22}},\overline{\tilde{q}_{22}}]$	$[\underline{\tilde{c}_{23}},\overline{\tilde{c}_{23}}]$ $[\underline{\tilde{t}_{23}},\overline{\tilde{t}_{23}}]$ $[\underline{\tilde{q}_{23}},\overline{\tilde{q}_{23}}]$	$[\underline{\tilde{c}_{24}},\overline{\tilde{c}_{24}}]$ $[\underline{\tilde{t}_{24}},\overline{\tilde{t}_{24}}]$ $[\underline{\tilde{q}_{24}},\overline{\tilde{q}_{24}}]$
M_3	$[\underline{\tilde{c}_{31}},\overline{\tilde{c}_{31}}]$ $[\underline{\tilde{t}_{31}},\overline{\tilde{t}_{31}}]$ $[\underline{\tilde{q}_{31}},\overline{\tilde{q}_{11}}]$	$[\underline{\tilde{c}_{32}},\overline{\tilde{c}_{32}}]$ $[\underline{\tilde{t}_{32}},\overline{\tilde{t}_{32}}]$ $[\underline{\tilde{q}_{32}},\overline{\tilde{q}_{32}}]$	$[\underline{\tilde{c}_{33}},\overline{\tilde{c}_{33}}]$ $[\underline{\tilde{t}_{33}},\overline{\tilde{t}_{33}}]$ $[\underline{\tilde{q}_{33}},\overline{\tilde{q}_{33}}]$	$[\underline{\tilde{c}_{34}},\overline{\tilde{c}_{34}}]$ $[\underline{\tilde{t}_{34}},\overline{\tilde{t}_{34}}]$ $[\underline{\tilde{q}_{34}},\overline{\tilde{q}_{34}}]$
M_4	$[\underline{\tilde{c}_{41}},\overline{\tilde{c}_{41}}]$ $[\underline{\tilde{t}_{41}},\overline{\tilde{t}_{41}}]$ $[\underline{\tilde{q}_{41}},\overline{\tilde{q}_{41}}]$	$[\underline{\tilde{c}_{42}},\overline{\tilde{c}_{42}}]$ $[\underline{\tilde{t}_{42}},\overline{\tilde{t}_{42}}]$ $[\underline{\tilde{q}_{42}},\overline{\tilde{q}_{42}}]$	$[\underline{\tilde{c}_{43}},\overline{\tilde{c}_{43}}]$ $[\underline{\tilde{t}_{43}},\overline{\tilde{t}_{43}}]$ $[\underline{\tilde{q}_{43}},\overline{\tilde{q}_{43}}]$	$[\underline{\tilde{c}_{44}},\overline{\tilde{c}_{44}}]$ $[\underline{\tilde{t}_{44}},\overline{\tilde{t}_{44}}]$ $[\underline{\tilde{q}_{44}},\overline{\tilde{q}_{44}}]$

5. Ranking function of ω-type 2 diamond fuzzy numbers

Let $\omega_{t2}\widetilde{F_d}^1 = [\underline{\omega_{t2}\widetilde{F_d}^1}, \overline{\omega_{t2}\widetilde{F_d}^1}]$ and $\omega_{t2}\widetilde{F_d}^2 = [\underline{\omega_{t2}\widetilde{F_d}^2}, \overline{\omega_{t2}\widetilde{F_d}^2}]$ are two ω-type2 diamond fuzzy numbers. $\underline{\omega_{t2}\widetilde{F_d}^1}$, $\underline{\omega_{t2}\widetilde{F_d}^2}$ are lower ω-type 2 diamond fuzzy number and $\overline{\omega_{t2}\widetilde{F_d}^1}$, $\overline{\omega_{t2}\widetilde{F_d}^2}$ are upper ω-type 2 diamond fuzzy number. Then the following ranking function of ω-type2 diamond fuzzy number and defined as $R(\omega_{t2}\widetilde{F_d})$

$$R(\omega_{t2}\widetilde{F_d}) = \frac{\omega_{t2}\widetilde{F_d}^1+\omega_{t2}\widetilde{F_d}^2}{2} = \frac{(\underline{\omega_{t2}\widetilde{F_d}^1} + \underline{\omega_{t2}\widetilde{F_d}^2}) + (\overline{\omega_{t2}\widetilde{F_d}^1} + \overline{\omega_{t2}\widetilde{F_d}^2}])}{2}$$

6. Algorithm and Properties:

6.1: A New Algorithm for fuzzy multi-objective linear sum assignment:

ω-type 2 diamond fuzzy numbers are considered as linguistic variables. The fuzzy cost coefficient, fuzzy time, and fuzzy quality are expressed in λ_d- cut of ω-type 2 diamond fuzzy numbers to compute the partial feasible solution and complete optimal solution.

Step 1: First let us take the cost matrix $[\widetilde{c_{ij}}]$, whose elements are linguistic variables that have been substituted by fuzzy numbers, is presented. Examine whether or not the provided ω-type 2 diamond fuzzy multi-objective linear sum assignment table is balanced.

a) If the number of machines and the number of jobs are equal, go to step 3.

b) Proceed to step 2 if the number of machines does not equal the number of jobs.

Step 2: In the ω-type 2 diamond fuzzy multi objective linear sum assignment table, add a dummy row or column. Dummy row/column cost, time, and quantity entries are always zero.

Step 3: In λ_d- cut form, express the above ω-type 2 diamond fuzzy multi-objective linear sum assignment problems. The upper and lower ω-type 2 diamond fuzzy numbers of the multi-objective linear sum assignment problem are then merged into single λ_d- cut form of ω-type 2 diamond fuzzy number of the multi-objective linear sum assignment problem.

Step 4: By applying ranking method, convert a λ_d- cut of ω-type 2-diamond fuzzy multi objective linear sum assignment problem to λ_d- cut of ω-type 2-diamond fuzzy linear sum assignment problem..

Step 5: Find dual variables $(\tilde{u}_i, \tilde{v}_j)$,

If $M_i = M_1, M_2 \ldots M_n$ then find $\tilde{u}_i = \min \{\widetilde{c_{ij}}$; $j_i = J_1, J_2 \ldots J_n$

If $j_i = J_1, J_2 \ldots J_n$ then find $\tilde{v}_j = \min \{\widetilde{c_{ij}} - \tilde{u}_i$; $M_i = M_1, M_2 \ldots M_n\}$;

Step 6: Calculate ($\overline{\widetilde{c_{ij}}}$) and find a partial feasible solution

if j_i = J_1 , J_2 …..J_n then row (j) = 0;

if $M_i = M_1, M_2 \ldots M_n$ and j_i = J_1 , J_2 …..J_n then obtain $\overline{\widetilde{c_{ij}}} = \widetilde{c_{ij}} - \tilde{u}_i - \tilde{v}_j = 0$ and the solution is

row (j) =i , $\tilde{x}_{ij} = 1$ and

a) If there are less than 'n' rows of matching. go to the next step

b) An optimal solution is found if the number of matches is equal to n..

Step7: If the number of matching solution is less than (the order of the matrix) n matching solution by using the following alternative path method.

The matching vertex$|\boldsymbol{U}|$< n then increase the partial solution and let E be any vertex in U and select the elementary path from k whose edges are alternatively not assigned and assigned.

If E $\notin \overline{U}$ then sink = Alternate(k);

If sink > 0 then $\overline{U} = \overline{U} \cup \{E\}$; j = sink and obtain in new graph.

Step 8: update the dual variables and obtain complete optimal solution

Select the minimum value of an unassigned row δ = min{$\widetilde{c_{ij}} - \tilde{u}_i - \tilde{v}_j$= 0 and then updated dual variables are $\tilde{u}_i^* = \tilde{u}_i + \delta$: $\tilde{v}_j^* = \widetilde{c_{ij}} - \tilde{u}_i^*$ then obtain $\overline{\widetilde{c_{ij}}}^* = \widetilde{c_{ij}} - \tilde{u}_i^* - \tilde{v}_j^* = 0$ is the new bipartite graph of the current solution. Alternate (k) is then executed again for k = δ Producing the augmented tree. Finally, each machine (M_i) and job (j_i) has one and only matching edges, complete optimum solution is reached.

Step 9: Stop.

6.2. Properties on fuzzy matching:

6.2.1. Theorem: If partial feasible matching is the minimum matching edges in any bipartite graph.

Proof: Consider the fuzzy cost matrix (nxn) is $\widetilde{c_{ij}}$ and define the fuzzy dual variables are $\tilde{u}_i$ = min {$\widetilde{c_{ij}}$} and $\tilde{v}_j$ = min {$\widetilde{c_{ij}} - \tilde{u}_i$ }. Then, we have by applying complementary slackness conditions for transform cost matrix $\widetilde{c_{ij}}$ to reduced cost matrix $\overline{\widetilde{c_{ij}}}$ (ie), $\overline{\widetilde{c_{ij}}} = \widetilde{c_{ij}} - \tilde{u}_i - \tilde{v}_j = 0, \forall\, 0 \le i, j \le n$. Therefore, assign only one matching edge to each rows and columns but both rows and columns are less than n. Then we have a solution is partial if there are a minimum number of matching edges in any bipartite graph.

6.2.2. Theorem: If an optimal complete matching is the number of matching is equal to the order of the matrix (nxn).

Proof: From Theorem (5.1). Let us take the partial feasible matching edges in bipartite graph. The matching vertex is less then n and increase the partial solution and let 'E' be any vertex in U and choose the elementary path from 'E' whose edges are alternatively not matched and matched. In a bipartite graph, an alternating tree rooted in a vertex 'r' is a tree in which all paths emanating from 'r' alternate. adding new matching vertex is $\overline{U} = \overline{U} \cup \{E\}$. Choose the minimum value of an unassigned row δ = min{$\widetilde{c_{ij}}$ - $\tilde{u}_i$ - $\tilde{v}_j$= 0 and then updated the dual variables are $\tilde{u}_i^* = \tilde{u}_i + \delta$: $\tilde{v}_j^* = \widetilde{c_{ij}} - \tilde{u}_i^*$ then compute $\overline{\widetilde{c_{ij}}}^* = \widetilde{c_{ij}} - \tilde{u}_i^* - \tilde{v}_j^* = 0$ is the new bipartite graph of the current solution. The alternate method executed again for k = δ Producing the augmented tree. Then we have a solution is complete optimal if there are a maximum number of matching is equal to the order of the matrix (nxn).

7. Example:

Let us considered the four machines given below. M_1, M_2, M_3, M_4, and four jobs J_1, J_2, J_3, J_4 respectively. To optimize the fuzzy cost, fuzzy time, and fuzzy quality are each considered as a ω-type 2 diamond fuzzy numbers. The fuzzy cost, the fuzzy time and the fuzzy quality for solving λ_d- cut of ω-type 2 diamond fuzzy numbers of multi-objective linear sum assignment problem.

Table 2: ω-type 2 diamond fuzzy numbers are representing to the linguistic variables.

Job/Machine	J_1	J_2	J_3	J_4
M_1	Fairly high	Very high	High	Very high
M_2	Very low	High	low	Fairly high
M_3	Extremely low	Medium	Medium	Very high
M_4	Fairly low	Medium	Very low	Very low

Table 3.

Job/Machine	J_1	J_2	J_3	J_4
M_1	(23,30,37) (22,31,38) (31,38,45) (30,39,46) (36,43,50) (35,44,51)	(25,33,41) (24,34,42) (31,39,47) (30,40,48) (36,44,52) (35,45,53)	(21,27,33) (20,28,34) (18,24,30) (17,25,31) (27,33,39) (26,34,40)	(25,33,41) (24,34,42) (31,39,47) (30,40,48) (36,44,52) (35,45,53)
M_2	(1,2,4) (0,3,7) (3,4,7) (2,5,9) (4,6,8) (3,7,10)	(21,27,33) (20,28,34) (18,24,30) (17,25,31) (27,33,39) (26,34,40)	(8,13,18) (7,15,21) (14,18,24) (13,23,28) (11,13,21) (10,17,22)	(23,30,37) (22,31,38) (31,38,45) (30,39,46) (36,43,50) (35,44,51)
M_3	(1,2,4) (0,3,5) (2,3,5) (1,4,6) (4,5,7) (3,6,8)	(7,12,15) (6,13,16) (8,13,16) (7,14,17) (13,18,21) (12,19,22)	(7,12,15) (6,13,16) (8,13,16) (7,14,17) (13,18,21) (12,19,22)	(25,33,41) (24,34,42) (31,39,47) (30,40,48) (36,44,52) (35,45,53)
M_4	(3,5,9) (2,6,10) (3,4,7) (2,5,10) (4,6,8) (3,7,11)	(7,12,15) (6,13,16) (8,13,16) (7,14,17) (13,18,21) (12,19,22)	(1,2,4) (0,3,7) (3,4,7) (2,5,9) (4,6,8) (3,7,10)	(1,2,4) (0,3,7) (3,4,7) (2,5,9) (4,6,8) (3,7,10)

Table 4: ω-type 2 diamond fuzzy numbers are converted into λ_d-cut of fuzzy numbers

Job/ Machine	J_1	J_2	J_3	J_4
M_1	[6$\underline{\lambda_d}$ +21, 33-6$\underline{\lambda_d}$] [8$\overline{\lambda_d}$ +20, 34-6$\overline{\lambda_d}$] [6$\underline{\lambda_d}$+18,30- 6$\underline{\lambda_d}$] [8$\overline{\lambda_d}$ +17, 31-6$\overline{\lambda_d}$] [6$\underline{\lambda_d}$+27,39-6$\underline{\lambda_d}$][8$\overline{\lambda_d}$ +26, 40-6$\overline{\lambda_d}$]	[8λ_d +25, 41-8$\underline{\lambda_d}$] [10$\overline{\lambda_d}$+24, 42-8$\overline{\lambda_d}$] [8$\underline{\lambda_d}$+31,47- 8$\underline{\lambda_d}$] [10$\overline{\lambda_d}$+30, 48- [8$\underline{\lambda_d}$+36,52-8$\underline{\lambda_d}$][10$\overline{\lambda_d}$ +35, 53-8$\overline{\lambda_d}$]	[7$\underline{\lambda_d}$ +23, 37-7$\underline{\lambda_d}$] [9$\overline{\lambda_d}$ +22, 38-7$\overline{\lambda_d}$] [7$\underline{\lambda_d}$+31,45- 7$\underline{\lambda_d}$] [9$\overline{\lambda_d}$ +30, 46-7$\overline{\lambda_d}$] [7$\underline{\lambda_d}$+36,50-7$\underline{\lambda_d}$][9$\overline{\lambda_d}$ +35, 51-7$\overline{\lambda_d}$]	[8λ_d +25, 41-8$\underline{\lambda_d}$] [10$\overline{\lambda_d}$ +24, 42-8$\overline{\lambda_d}$] [8$\underline{\lambda_d}$+31,47- 8$\underline{\lambda_d}$] [10$\overline{\lambda_d}$ +30, 48-8$\overline{\lambda_d}$] [8$\underline{\lambda_d}$+36,52- 8$\underline{\lambda_d}$][10$\overline{\lambda_d}$ +35, 53-8$\overline{\lambda_d}$]
M_2	[$\underline{\lambda_d}$+1, 4-2$\underline{\lambda_d}$] [3$\overline{\lambda_d}$+0, 7-4$\overline{\lambda_d}$] [$\underline{\lambda_d}$+3,7- 3$\underline{\lambda_d}$] [3$\overline{\lambda_d}$ +2, 9-4$\overline{\lambda_d}$] [2$\underline{\lambda_d}$+4,8-2$\underline{\lambda_d}$][4$\overline{\lambda_d}$ +3, 10-3$\overline{\lambda_d}$]	[7$\underline{\lambda_d}$ +23, 37-7$\underline{\lambda_d}$] [9$\overline{\lambda_d}$ +22, 38-7$\overline{\lambda_d}$] [7$\underline{\lambda_d}$+31,45- 7$\underline{\lambda_d}$] [9$\overline{\lambda_d}$ +30, 46-7$\overline{\lambda_d}$] [7$\underline{\lambda_d}$+36,50-7$\underline{\lambda_d}$][9$\overline{\lambda_d}$ +35, 51-7$\overline{\lambda_d}$]	[5$\underline{\lambda_d}$ +8, 18-5$\underline{\lambda_d}$] [8$\overline{\lambda_d}$ +7, 21-6$\overline{\lambda_d}$] [4$\underline{\lambda_d}$+14,24- 4$\underline{\lambda_d}$] [10$\overline{\lambda_d}$+13, 28-5$\overline{\lambda_d}$] [2$\underline{\lambda_d}$+11,21-5$\underline{\lambda_d}$][7$\overline{\lambda_d}$ +10, 22-5$\overline{\lambda_d}$]	[6$\underline{\lambda_d}$ +21, 33-6$\underline{\lambda_d}$] [8$\overline{\lambda_d}$ +20, 34-6$\overline{\lambda_d}$] [6$\underline{\lambda_d}$+18,30- 6$\underline{\lambda_d}$] [8$\overline{\lambda_d}$ +17, 31-6$\overline{\lambda_d}$] [6$\underline{\lambda_d}$+27,39- 6$\underline{\lambda_d}$][8$\overline{\lambda_d}$ +26, 40-6$\overline{\lambda_d}$]
M_3	[$\underline{\lambda_d}$ +1, 4-2$\underline{\lambda_d}$] [3$\overline{\lambda_d}$], 5-2$\overline{\lambda_d}$]] [$\underline{\lambda_d}$+2,5- 2$\underline{\lambda_d}$] [3$\overline{\lambda_d}$] +1, 6-2$\overline{\lambda_d}$]] [$\underline{\lambda_d}$+4,7-2$\underline{\lambda_d}$][3$\overline{\lambda_d}$] +3, 8-2$\overline{\lambda_d}$]]	[5$\underline{\lambda_d}$ +7, 15-3$\underline{\lambda_d}$] [7$\overline{\lambda_d}$ +6, 16-3$\overline{\lambda_d}$] [5$\underline{\lambda_d}$λ+8,16- 3$\underline{\lambda_d}$] [7$\overline{\lambda_d}$ +7, 17-3$\overline{\lambda_d}$] [5$\underline{\lambda_d}$+13,21-3$\underline{\lambda_d}$][7$\overline{\lambda_d}$ +12, 22-3$\overline{\lambda_d}$]	[5$\underline{\lambda_d}$ +7, 15-3$\underline{\lambda_d}$] [7$\overline{\lambda_d}$ +6, 16-3$\overline{\lambda_d}$] [5$\underline{\lambda_d}$λ+8,16- 3$\underline{\lambda_d}$] [7$\overline{\lambda_d}$ +7, 17-3$\overline{\lambda_d}$] [5$\underline{\lambda_d}$+13,21-3$\underline{\lambda_d}$][7$\overline{\lambda_d}$ +12, 22-3$\overline{\lambda_d}$]	[8$\underline{\lambda_d}$+ +25, 41-8$\underline{\lambda_d}$] [10$\overline{\lambda_d}$ +24, 42-8$\overline{\lambda_d}$] [8$\underline{\lambda_d}$+31,47- 8$\underline{\lambda_d}$] [10$\overline{\lambda_d}$ +30, 48-8$\overline{\lambda_d}$] [8$\underline{\lambda_d}$+36,52- 8$\underline{\lambda_d}$][10$\overline{\lambda_d}$ +35, 53-8$\overline{\lambda_d}$]
M_4	[2$\underline{\lambda_d}$ +3, 9-4$\underline{\lambda_d}$] [4$\overline{\lambda_d}$, +2, 10-4$\overline{\lambda_d}$] [$\underline{\lambda_d}$+3,7- 3$\underline{\lambda_d}$] [3$\overline{\lambda_d}$+2, 9-5$\overline{\lambda_d}$] [2$\underline{\lambda_d}$+4,8-2$\underline{\lambda_d}$][4$\overline{\lambda_d}$+3 , 11-4$\overline{\lambda_d}$]	[5$\underline{\lambda_d}$ +7, 15-3$\underline{\lambda_d}$] [7$\overline{\lambda_d}$ +6, 16-3$\overline{\lambda_d}$] [5$\underline{\lambda_d}$λ+8,16- 3$\underline{\lambda_d}$] [7$\overline{\lambda_d}$ +7, 17-3$\overline{\lambda_d}$] [5$\underline{\lambda_d}$+13,21-3$\underline{\lambda_d}$][7$\overline{\lambda_d}$ +12, 22-3$\overline{\lambda_d}$]	[$\underline{\lambda_d}$+1, 4-2$\underline{\lambda_d}$] [3$\overline{\lambda_d}$+0, 7-4$\overline{\lambda_d}$] [$\underline{\lambda_d}$+3,7- 3$\underline{\lambda_d}$] [3$\overline{\lambda_d}$ +2, 9-4$\overline{\lambda_d}$] [2$\underline{\lambda_d}$+4,8- 2$\underline{\lambda_d}$][4$\overline{\lambda_d}$ +3, 10-3$\overline{\lambda_d}$]	[$\underline{\lambda_d}$+1, 4-2$\underline{\lambda_d}$] [3$\overline{\lambda_d}$+0, 7-4$\overline{\lambda_d}$] [$\underline{\lambda_d}$+3,7- 3$\underline{\lambda_d}$] [3$\overline{\lambda_d}$ +2, 9-4$\overline{\lambda_d}$] [2$\underline{\lambda_d}$+4,8- 2$\underline{\lambda_d}$][4$\overline{\lambda_d}$ +3, 10-3$\overline{\lambda_d}$]

Table 5: ω-type 2 diamond multi-objective fuzzy numbers are converted into single λ-cut fuzzy number

Job/ Machine	J_1	J_2	J_3	J_4
M_1	[18$\underline{\lambda_d}$+66,102- 18$\underline{\lambda_d}$] [24$\overline{\lambda_d}$ +63, 105-18$\overline{\lambda_d}$]	[24$\underline{\lambda_d}$+92,140-24$\underline{\lambda_d}$] [30$\overline{\lambda_d}$ +89, 143-24$\overline{\lambda_d}$]	[21$\underline{\lambda_d}$+90,132-21$\underline{\lambda_d}$] [27$\overline{\lambda_d}$ +87, 135-21$\overline{\lambda_d}$]	[24$\underline{\lambda_d}$+92,140-24$\underline{\lambda_d}$] [30$\overline{\lambda_d}$ +89, 143-24$\overline{\lambda_d}$]
M_2	[4$\underline{\lambda_d}$+8,19-7$\underline{\lambda_d}$] [10$\overline{\lambda_d}$ +5, 26-11$\overline{\lambda_d}$]	[21$\underline{\lambda_d}$+90,132-21$\underline{\lambda_d}$] [27$\overline{\lambda_d}$ +87, 135-21$\overline{\lambda_d}$]	[11$\underline{\lambda_d}$++33,63-14$\underline{\lambda_d}$] [25$\overline{\lambda_d}$ +30, 71-16$\overline{\lambda_d}$]	[18$\underline{\lambda_d}$ +66,102- 18$\underline{\lambda_d}$] [24$\overline{\lambda_d}$ +63, 105-18$\overline{\lambda_d}$]
M_3	[3$\underline{\lambda_d}$+7,16-6$\underline{\lambda_d}$] [9$\overline{\lambda_d}$ +4, 19-6$\overline{\lambda_d}$]	[15$\underline{\lambda_d}$+28,52- 9$\underline{\lambda_d}$] [21$\overline{\lambda_d}$ +25, 55-9$\overline{\lambda_d}$]	[15$\underline{\lambda_d}$+28,52- 9$\underline{\lambda_d}$] [21$\overline{\lambda_d}$ +25, 55-9$\overline{\lambda_d}$]	[24$\underline{\lambda_d}$+92,140-24$\underline{\lambda_d}$] [30$\overline{\lambda_d}$ +89, 143-24$\overline{\lambda_d}$]
M_4	[5$\underline{\lambda_d}$+10,24-9$\underline{\lambda_d}$] [11$\overline{\lambda_d}$ +7, 30-13$\overline{\lambda_d}$]	[15$\underline{\lambda_d}$+28,52- 9$\underline{\lambda_d}$] [21$\overline{\lambda_d}$ +25, 55-9$\overline{\lambda_d}$]	[4$\underline{\lambda_d}$ +8,19-7$\underline{\lambda_d}$] [10$\overline{\lambda_d}$ +5, 26-11$\overline{\lambda_d}$]	[4$\underline{\lambda_d}$ +8,19-7$\underline{\lambda_d}$] [10$\overline{\lambda_d}$ +5, 26-11$\overline{\lambda_d}$]

Table 6: Upper and lower ω-type 2 diamond multi-objective fuzzy numbers are converted into single objective λ_d-cut fuzzy number

Job/ Machine	J_1	J_2	J_3	J_4
M_1	[21$\underline{\lambda_d}$+64.5,103-18$\overline{\lambda_d}$]	[27$\underline{\lambda_d}$+90.5,141.5-24$\overline{\lambda_d}$]	[24$\underline{\lambda_d}$+88.5,133.5-21λ]	[27$\underline{\lambda_d}$+90.5,141.5-24$\overline{\lambda_d}$]
M_2	[7$\underline{\lambda_d}$+6.5,22.5 -9$\overline{\lambda_d}$]	[24$\underline{\lambda_d}$+88.5,133.5-21$\overline{\lambda_d}$]	[18$\underline{\lambda_d}$+31.5,67-15$\overline{\lambda_d}$]	[21$\underline{\lambda_d}$+64.5,103.5-18$\overline{\lambda_d}$]
M_3	[6$\underline{\lambda_d}$+5.5,17.5-6$\overline{\lambda_d}$]	[18$\underline{\lambda_d}$+26.5,53.5-9$\overline{\lambda_d}$]	[18$\underline{\lambda_d}$+26.5,53.5-9$\overline{\lambda_d}$]	[27$\underline{\lambda_d}$+90.5,141.5-24$\overline{\lambda_d}$]
M_4	[8$\underline{\lambda_d}$+8.5,27-11$\overline{\lambda_d}$]	[18$\underline{\lambda_d}$+26.5,53.5-9$\overline{\lambda_d}$]	[7$\underline{\lambda_d}$+6.5,22.5 -9$\overline{\lambda_d}$]	[7$\underline{\lambda_d}$+6.5,22.5 -9$\overline{\lambda_d}$]

Obtain dual variables

$\tilde{\boldsymbol{u}}$ ={[21$\underline{\lambda_d}$+64.5,103-18$\overline{\lambda_d}$], [7$\underline{\lambda_d}$+6.5,25.5-9$\overline{\lambda_d}$], [6$\underline{\lambda_d}$+5.5,17.5-6$\overline{\lambda_d}$], [7$\underline{\lambda_d}$+6.5,25.5-9$\overline{\lambda_d}$]

$\tilde{\boldsymbol{v}}$ = {0, [6$\underline{\lambda_d}$+26,38.5- 6$\overline{\lambda_d}$],0 , 0}

Table 7.

$\overline{\widetilde{c_{ij}}}$ =

<u>0</u>	-[6$\underline{\lambda_d}$+26,38.5- 6$\overline{\lambda_d}$]	0	0
0	0	[3$\underline{\lambda_d}$+24,30.5-3$\overline{\lambda_d}$]	[6$\underline{\lambda_d}$+26,38.5- 6$\overline{\lambda_d}$]
0	[11$\underline{\lambda_d}$+56,72.5-6$\overline{\lambda_d}$]	[11$\underline{\lambda_d}$+25,44.5-6$\overline{\lambda_d}$]	[14$\underline{\lambda_d}$+58,81-9$\overline{\lambda_d}$]
0	[6$\underline{\lambda_d}$-5, -2.5+3$\overline{\lambda_d}$]	[12$\underline{\lambda_d}$+21,36-3$\overline{\lambda_d}$]	[21$\underline{\lambda_d}$+85,124- 18$\overline{\lambda_d}$]
[$\underline{\lambda_d}$+2.5,4.5-2$\overline{\lambda_d}$]	[5$\underline{\lambda_d}$-6,-7.5+6$\overline{\lambda_d}$]	**<u>0</u>**	0

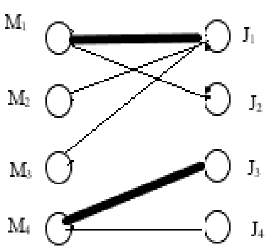

Figure :1

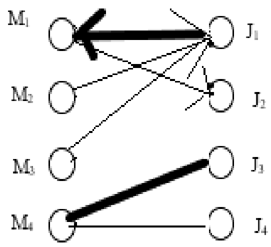

Figure : 2

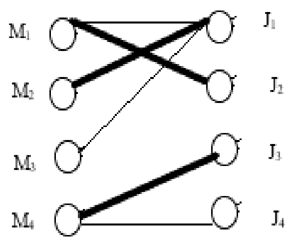

Figure : 3

Table 8.

Job/ Machine	J_1	J_2	J_3	J_4
M_1	[21$\underline{\lambda_d}$+64.5,103-18$\overline{\lambda_d}$]	[27$\underline{\lambda_d}$+90.5,141.5-24$\overline{\lambda_d}$]	[24$\underline{\lambda_d}$+88.5,133.5-21λ]	[27$\underline{\lambda_d}$+90.5,141.5-24$\overline{\lambda_d}$]
M_2	[7$\underline{\lambda_d}$+6.5,22.5 -9$\overline{\lambda_d}$]	[24$\underline{\lambda_d}$+88.5,133.5-21$\overline{\lambda_d}$]	[18$\underline{\lambda_d}$+31.5,67-15$\overline{\lambda_d}$]	[21$\underline{\lambda_d}$+64.5,103.5-18$\overline{\lambda_d}$]
M_3	[6$\underline{\lambda_d}$+5.5,17.5-6$\overline{\lambda_d}$]	[18$\underline{\lambda_d}$+26.5,53.5-9$\overline{\lambda_d}$]	[18$\underline{\lambda_d}$+26.5,53.5-9$\overline{\lambda_d}$]	[27$\underline{\lambda_d}$+90.5,141.5-24$\overline{\lambda_d}$]
M_4	[8$\underline{\lambda_d}$+8.5,27-11$\overline{\lambda_d}$]	[18$\underline{\lambda_d}$+26.5,53.5-9$\overline{\lambda_d}$]	[7$\underline{\lambda_d}$+6.5,22.5 -9$\overline{\lambda_d}$]	[7$\underline{\lambda_d}$+6.5,22.5 -9$\overline{\lambda_d}$]

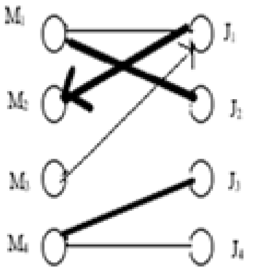

Figure :4

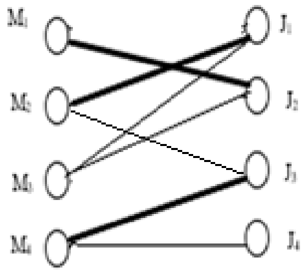

Figure : 5

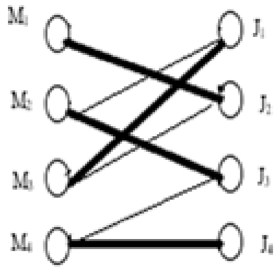

Figure : 6

Updated dual variables

$\tilde{u}_i^* = \{[21\underline{\lambda_d}+64.5, 103-18\overline{\lambda_d}], [18\underline{\lambda_d}+31.5, 67-15\overline{\lambda_d}], [12\underline{\lambda_d}, 15-9\overline{\lambda_d}], [7\underline{\lambda_d}+6.5, 25.5-9\overline{\lambda_d}]\};$

$\tilde{v}_j^* = \{[-11\underline{\lambda_d}-25, -44.5+6\overline{\lambda_d}], [6\underline{\lambda_d}+26, 38.5-6\overline{\lambda_d}], 0, 0\}$

Table 9.

$\overline{\widetilde{c_{ij}}}^* =$

Job/ Machine	J_1	J_2	J_3	J_4
M_1	0	**0**	$[3\underline{\lambda_d}+24, 30.5-3\overline{\lambda_d}]$	$[6\underline{\lambda_d}+26, 38.5-6\overline{\lambda_d}]$
M_2	0	$[0\underline{\lambda_d}-1.5, 28-0\overline{\lambda_d}]$	**0**	$[3\underline{\lambda_d}+33, 36.5+3\overline{\lambda_d}]$
M_3	**0**	0	$[6\underline{\lambda_d}+26.5, 38.5-0\overline{\lambda_d}]$	$[15\underline{\lambda_d}+90.5, 126.5-15\overline{\lambda_d}]$
M_4	$[12\underline{\lambda_d}+27, 49-4\overline{\lambda_d}]$	$[0\underline{\lambda_d}-6, -7.5-4\overline{\lambda_d}]$	0	**0**

Fuzzy optimal schedule $\mathbf{M_1 \rightarrow J_2,\ M_2 \rightarrow J_3,\ M_3 \rightarrow J_1,\ M_4 \rightarrow J_4}$

Fuzzy optimal cost = **{(25,33,41) + (8,13,18) + (1,2,4) + (1,2,4)}{(24,34,42) + (7,15,21) + (0,3,5) + (0,3,7)}**

Fuzzy optimal cost = (35, 50, 67) (31,55,75)

Fuzzy optimal time= **{(31,39,47) + (14,18,24) + (2,3,5) + (3,4,7)}{(30,40,48 + (13,23,28) + (1,4,6) + (2,5,9)}**

Fuzzy optimal time = (50, 64, 83) (46,72,91)

Fuzzy optimal quality = **{(36,44,52) + (4,6,8) + (4,5,7) + (4,6,8)} {(35,45,53) + (3,7,10) + (3,6,8) + (3,7,10)}**

Fuzzy optimal quality = (48,61,75) (44,65,81)

CONCLUSION

We discussed ω-type 1 and ω-type 2-diamond fuzzy numbers. We proposed a new method for solving λ_d- cut of ω-type 2-diamond fuzzy multi-objective linear sum assignment problem and involving linguistic variables and by using alternate method and augmented method of bipartite graph to compute partial feasible solution and complete optimal solution. To modified partial primal solution and obtain complete optimal solution using the alternate path method producing augment path method of the bipartite graph.

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