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Fermatean Fuzzy Normed Linear Space

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Abstract

The initiative of the presented work is to include the notion of fermatean fuzzy norm on fuzzy linear space which is called fermatean fuzzy n- normed linear space(f-f-n-NLS). The thought of fermatean fuzzy n- normed linear space is discussed here painstakingly. The definition of convergence of sequence is defined in this space. Then the result of convergence sequence is explained. The definition of Cauchy sequence is also introduced in this space. Some consequences of Cauchy sequence are derived here elaborately. Then we constructed the contraction mapping and cubic control function in this normed linear space and fixed point theorem. The application of this normed linear space is established in medical diagnosis process using contraction mapping and fixed point. The comparison between our proposed normed linear space, intuitionistic fuzzy normed linear space and neutrosophic fuzzy normed linear space is also derived here taking an example.
Keywords: Fuzzy normed linear space, Fermatean fuzzy normed linear space, Contraction mapping, Fixed point theorems.
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1. INTRODUCTION

After the introduction of fuzzy set by Zadeh [1], the theory fuzzy metric space was introduced by Kramosil and Michálek [2] in 1975. George and Veeramani [3] modified the theory of fuzzy metric space. Then many mathematicians developed the concept of fuzzy metric space in different way. In, 1984 Kataras [4] introduced the concept of fuzzy norm. Then M. Sangeetha and M. Sentamilselvi [5] established the theory of fuzzy normed and Banach space in fuzzy topological spaces.
The concept of 2-norm and n-norm was introduced by S.Gähler [6] on a linear space. S.S.Kim and Y.J.Cho [7], R. Malceski [8], M. Mashadi and Hendra Gunawan [9] developed the theory of an n-normed linear space systematically. Bag and Samanta [10] gave a

definition of fuzzy norm and proved the decomposition theorem of fuzzy norm. S.Vijaybalaji, N.Thillaigovindan and Y.B.Jun [11] introduced the intuitionistic fuzzy n-normed linear space and defined the Cauchy sequence and the convergence of the sequence. Kirişci and Şimşek [12] developed the concept of neutrosohic normed linear space and explained the statistical convergence in this space. Recently, V. Kumar, A. Sharma, S. Murtaza [13] introduce the neutrosophic n-normed linear spaces and discussed about convergence of sequence and Cauchy sequence in these spaces.

Senapati and Yager [14] introduced the fermatean fuzzy set. The aim of this paper is to introduce the concept of fermatean fuzzy n- normed linear space(f-f-n-NLS). The contribution of our work is described below.
In section 1, we included the introduction part. Some preliminary results are described in section 2. In section 3, we derived the concept of f-f-n-NLS, open ball, closed ball, Cauchy sequence, the condition of convergence of sequence. The contraction mapping, cubic control function and fixed point theorem are obtained in section 4. In section 5, we set up the application of the fixed point theorem in f-f-n-NLS in medical diagnosis. We established the comparison between three types of normed linear space with an example in section 6. In section 7, we ended our work with conclusion part.

2. PRELIMINARIES

In this part we state some basic definitions which will be useful to obtain the main results.

Definition 1. *([15])* $*$ is a binary operation from $[0,1] \times [0,1] \longrightarrow [0,1]$ is called continuous t-norm (CTN), for each $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ in $[0,1]$ if $*$ holds the following conditions:

(i) $\varepsilon_1 * 1 = \varepsilon_1$; for all $\varepsilon_1 \in [0,1]$

(ii) If $\varepsilon_1 \leq \varepsilon_3$ and $\varepsilon_2 \leq \varepsilon_4$ then $\varepsilon_1 * \varepsilon_2 \leq \varepsilon_3 * \varepsilon_4$ for all $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4 \in [0,1]$.

(iii) $*$ is continuous, commutative and associative.

EXAMPLE **1.** Some examples of t-norm are $\varepsilon_1 * \varepsilon_2 = \varepsilon_1\varepsilon_2, \varepsilon_1 * \varepsilon_2 = [\min\{\varepsilon_1^3 + \varepsilon_2^3 - 1, 1\}]^{\frac{1}{3}}, \varepsilon_1 * \varepsilon_2 = \Big[\min\{\varepsilon_1^3, \varepsilon_2^3\}\Big]^{\frac{1}{3}}$ where all $\varepsilon_1, \varepsilon_2 \in [0,1]$.

Definition 2. *([15])* $\diamondsuit$ is a binary operation from $[0,1] \times [0,1] \longrightarrow [0,1]$ is called continuous t-conorm (CTC) since $\diamondsuit$ satisfies the conditions described below:

(i) $\diamondsuit$ is continuous, commutative and associative.

(ii) $\varepsilon_1 \diamondsuit 0 = \varepsilon_1$; for all $\varepsilon_1 \in [0,1]$

(iii) $\varepsilon_1 \diamondsuit \varepsilon_3 \leq \varepsilon_2 \diamondsuit \varepsilon_4$ whenever $\varepsilon_1 \leq \varepsilon_3$ and $\varepsilon_2 \leq \varepsilon_4$, for all $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4 \in [0, 1]$

EXAMPLE **2.** Some examples of t-conorm are $\varepsilon_1 \diamondsuit \varepsilon_2 = [\max\{\varepsilon_1^3 + \varepsilon_2^3, 0\}]^{\frac{1}{3}}, \varepsilon_1 \diamondsuit \varepsilon_2 = [\max\{\varepsilon_1^3, \varepsilon_2^3\}]^{\frac{1}{3}}, \varepsilon_1 \diamondsuit \varepsilon_2 = (\varepsilon_1^3 + \varepsilon_2^3 - \varepsilon_1^3 \varepsilon_2^3)^{\frac{1}{3}}, \varepsilon_1 \diamondsuit \varepsilon_2 = [\max\{\varepsilon_1^3 + \varepsilon_2^3 - \varepsilon_1^3 \varepsilon_2^3\}, 0]^{\frac{1}{3}}$, where all $\varepsilon_1, \varepsilon_2 \in [0, 1]$.

Definition 3. *([16])* Let $\mathcal{X}$ be a linear space over a field $\mathbb{F}$. A fuzzy subset $\mu_{\mathcal{F}}$ of $\mathcal{X}^n \times \mathbb{R}$ is called fuzzy n-norm on $\mathcal{X}$ if the following conditions are satisfied:

(i) $\mu_{\mathcal{F}}(v_1, \cdots, v_n, \lambda) = 0$ for all $\lambda \in \mathbb{R}$ with $\lambda \leq 0$;

(ii) $\mu_{\mathcal{F}}(v_1, \cdots, v_n, \lambda) = 1$ iff $v_1, v_2, \cdots, v_n$ are linearly dependent for all $\lambda \geq 0$.

(iii) $\mu_{\mathcal{F}}(v_1, \cdots, v_n, \lambda)$ is invariant under any permutation of $v_1, \cdots, v_n$.

(iv) If $\alpha \neq 0$, $\alpha \in \mathbb{F}$ then $\mu_{\mathcal{F}}(v_1, \cdots, \alpha v_n, \lambda) = \mu_{\mathcal{F}}(v_1, \cdots, v_n, \frac{\lambda}{|\alpha|})$ for all $\lambda \in \mathbb{R}$ with $\lambda \geq 0$;

(v) $\mu_{\mathcal{F}}(v_1, \cdots, v_n + v_n, \lambda_1 + \lambda_2) \geq \min\{\mu_{\mathcal{F}}(v_1, \cdots, v_n, \lambda_1), \mu_{\mathcal{F}}(v_1, \cdots, v_n, \lambda_2)\}$ for all $\lambda_1, \lambda_2 \in \mathbb{R}$;

(vi) $\mu_{\mathcal{F}}(v_1, \cdots, v_n, \lambda)$ is a non-decreasing function of $\lambda \in \mathbb{R}$ and $\lim_{n \longrightarrow \infty} \mu_{\mathcal{F}}(v_1, \cdots, v_n, \lambda) = 1$.

Then $(\mathcal{X}, \mu_{\mathcal{F}})$ is a called fuzzy n-normed linear space(f-n-NLS).

3. FERMATEAN FUZZY n- NORMED LINEAR SPACE(F-F-N-NLS)

In this section, the definition of f-f-n-NLS is derived and some topological properties are discussed.

Definition 4. A f-f-n-NLS is of the form

$$\mathcal{A} = \Big\{ \big(\tilde{\mathcal{X}}^n, \mu_{\mathcal{F}}(\tilde{V}, \lambda), \nu_{\mathcal{F}}(\tilde{V}, \lambda) \big) : \ \tilde{V} = (v_1, \cdots, v_n) \in \tilde{\mathcal{X}}^n, \lambda \in (0, \infty) \Big\},$$

$\tilde{\mathcal{X}}^n$ is any linear space, satisfying the following condition:

(i) $\mu_{\mathcal{F}}(\tilde{V}, \lambda) > 0$ and $\nu_{\mathcal{F}}(\tilde{V}, \lambda) > 0$.

(ii) $\mu_{\mathcal{F}}^3(\tilde{V}, \lambda) + \nu_{\mathcal{F}}^3(V, \lambda) \leq 1$for all $\lambda > 0$.

(iii) $\mu_{\mathcal{F}}(\tilde{V}, \lambda) = 1$ iff $v_1, \cdots, v_n$ are linearly dependent.

(iv) $\mu_{\mathcal{F}}(\tilde{V}, \lambda)$ is not changed when the variables, $v_1, \cdots, v_n$ are rearranged.

(v) $\mu_{\mathcal{F}}(\alpha^3\tilde{V},\lambda) = \mu_{\mathcal{F}}\Big(\tilde{V}, \frac{\lambda}{|\alpha|}\Big)$ if $\alpha \neq 0, \alpha \in \mathbb{F}$;

(vi) $\mu^3_{\mathcal{F}}(v_1, \cdots, v_n, \lambda_1) * \mu^3_{\mathcal{F}}(v_1, \cdots, \tilde{v}_n, \lambda_2) \leq \mu^3_{\mathcal{F}}(v_1, \cdots, v_n + \tilde{v}_n, \lambda_1 + \lambda_2)$;

(vii) $\mu_{\mathcal{F}}(\tilde{V},\lambda) : (0,\infty) \longrightarrow [0,1]$ is continuous in λ;

(viii) $\nu_{\mathcal{F}}(\tilde{V},\lambda) = 0$ iff $v_1, \cdots, v_n$ are linearly dependent.

(ix) $\nu_{\mathcal{F}}(\tilde{V},\lambda)$ remains stable under any rearrangements of $\tilde{v}$;

(x) $\nu_{\mathcal{F}}(\alpha^3\tilde{V},\lambda) = \nu_{\mathcal{F}}(\tilde{V}, \frac{\lambda}{|\alpha|})$ if $\alpha \neq 0$, $\alpha \in \mathbb{F}$.

(xi) $\nu^3_{\mathcal{F}}(v_1, \cdots, v_n, \lambda_1) \diamondsuit \nu^3_{\mathcal{F}}(v_1, \cdots, \tilde{v}_n, \lambda_2) \geq \nu^3_{\mathcal{F}}(v_1, \cdots, v_n + \tilde{v}_n, \lambda_1 + \lambda_2)$;

(xii) $\nu_{\mathcal{F}}(\tilde{V},\lambda) : (0,\infty) \longrightarrow [0,1]$ is continuous in λ.

The norm of any element A in $\mathcal{A}$ is defined by $||A||_f = \sqrt[3]{\mu^3_{\mathcal{F}}(\tilde{V},\lambda) + \nu^3_{\mathcal{F}}(\tilde{V},\lambda)}$.

EXAMPLE **3.** Let $\tilde{\mathcal{X}}$ be any non-empty set. Define $\varepsilon_1 * \varepsilon_2 = [\min\{{\varepsilon_1}^3, {\varepsilon_2}^3\}]^{\frac{1}{3}}$, $\varepsilon_1 \diamondsuit \varepsilon_2 = [\max\{{\varepsilon_1}^3, {\varepsilon_2}^3\}]^{\frac{1}{3}}$ for all $\varepsilon_1, \varepsilon_2 \in [0,1]$, $\mu_{\mathcal{F}}(\tilde{V},\lambda) = \frac{\lambda}{\{\lambda^3 + ||\tilde{V}||\}^{\frac{1}{3}}}$, $\nu_{\mathcal{F}}(\tilde{V},\lambda) = \Big\{\frac{||\tilde{V}||}{\lambda^3 + ||\tilde{V}||}\Big\}^{\frac{1}{3}}$, here $||\tilde{V}|| = (\sum_{i=1}^n |v_i|^3)^{\frac{1}{3}}$. Then $\mathcal{A} = (\tilde{\mathcal{X}}, \mu_{\mathcal{F}}, \nu_{\mathcal{F}})$ is a f-f-n-NLS.

Proof: In this example, we shall prove the conditions (ii), (v) and (x) from the definition 4. Others are obvious.

(ii) $\mu^3_{\mathcal{F}}(\tilde{V},\lambda) + \nu^3_{\mathcal{F}}(\tilde{V},\lambda) = 1$.

(v)

$$\begin{aligned}
\mu_{\mathcal{F}}(v_1, \cdots, v_n, \frac{\lambda}{|\alpha|}) &= \frac{\frac{\lambda}{|\alpha|}}{\{(\frac{\lambda}{|\alpha|})^3 + ||v_1, \cdots, v_n||\}^{\frac{1}{3}}} \\
&= \frac{\lambda}{\{\lambda^3 + |\alpha|^3||v_1, \cdots, v_n||\}^{\frac{1}{3}}} \\
&= \frac{\lambda}{\{\lambda^3 + ||v_1, \cdots, \alpha^3 v_n||\}^{\frac{1}{3}}} \\
&= \mu_{\mathcal{F}}(v_1, \cdots, \alpha^3 v_n, \lambda)
\end{aligned}$$

(x)

$$\begin{aligned}
\nu_{\mathcal{F}}(v_1, \cdots, \alpha^3 v_n, \lambda) &= \Big\{\frac{||v_1, \cdots, \alpha^3 v_n||}{\lambda^3 + ||v_1, \cdots, \alpha^3 v_n||}\Big\}^{\frac{1}{3}} \\
&= \Big\{\frac{|\alpha|^3||v_1, \cdots, v_n||}{\lambda^3 + |\alpha|^3||v_1, \cdots, v_n||}\Big\}^{\frac{1}{3}} \\
&= \nu_{\mathcal{F}}(v_1, \cdots, v_n, \frac{\lambda}{|\alpha|})
\end{aligned}$$

Therefore, $\mathcal{A}$ is a f-f-n-NLS. □

Definition 5. (I) An open ball in a f-f-n-NLS,$\mathcal{A} = \Big\{\big(\tilde{\mathcal{X}}, \mu_{\mathcal{F}}(\tilde{V}, \lambda), \nu_{\mathcal{F}}(\tilde{V}, \lambda)\big) : \tilde{v} = (v_1, v_2, \cdots, v_n) \in \tilde{\mathcal{X}}^n, \lambda \in (0, \infty)\Big\}$, is

$$\mathcal{B}(\tilde{\alpha}, r) = \{\tilde{\beta} \in \mathcal{A} : ||\tilde{\alpha} - \tilde{\beta}||_f < r\}.$$

(II) An open ball can be defined in the form of

$$\mathcal{B}(\tilde{V}, r, \lambda) = \{\tilde{W} \in \mathcal{A} : \mu^3_{\mathcal{F}}(\tilde{V}, \tilde{W}, \lambda) > 1 - r \; and \; \nu^3_{\mathcal{F}}(\tilde{V}, \tilde{W}, \lambda) < r\}.$$

Definition 6. (I) A sequence $\langle v_n \rangle$ in a f-f-n-NLS $\mathcal{A}$ is said to converge to $\tilde{v}$ in $\mathcal{A}$ if $||v_n - \tilde{v}||_f \longrightarrow 0$ as $n \longrightarrow \infty$.

(II) For each $0 < \epsilon < 1$ and $\lambda > 0$, $\exists n_0 \in \mathbb{N}$ s.t. $\mu_{\mathcal{F}}(v_1, \cdots, v_{n-1}, v_n - \tilde{v}, \lambda) > (1 - \epsilon)^{\frac{1}{3}}$ and $\nu_{\mathcal{F}}(v_1, \cdots, v_{n-1}, v_n - \tilde{v}, \lambda) < \epsilon^{\frac{1}{3}}$ for all $n \geq n_0$.

THEOREM **3.1.** If any sequence $\langle v_n \rangle$ converges to $\tilde{v}$ in f-f-n-NLS, then point of convergence is unique.

THEOREM **3.2.** Let $\mathcal{A} = (\tilde{\mathcal{X}}^n, \mu_{\mathcal{F}}, \nu_{\mathcal{F}})$ be a f-f-n-NLS. The sequence $\langle v_n \rangle$ in $\tilde{\mathcal{X}}^n$ converges $\tilde{v}$ iff $\{\mu_{\mathcal{F}}(v_1, \cdots, v_n - \tilde{v})\}^3 \longrightarrow 1$ and $\{\nu_{\mathcal{F}}(v_1, \cdots, v_n - \tilde{v})\}^3 \longrightarrow 0$.

EXAMPLE **4.** Let $\tilde{\mathcal{X}}$ be an non empty set. Define $\varepsilon_1 * \varepsilon_2 = [\min\{{\varepsilon_1}^3, {\varepsilon_2}^3\}]^{\frac{1}{3}}$, $\varepsilon_1 \diamondsuit \varepsilon_2 = [\max\{{\varepsilon_1}^3, {\varepsilon_2}^3\}]^{\frac{1}{3}}$ for all $\varepsilon_1, \varepsilon_2 \in [0, 1]$, $\mu_{\mathcal{F}}(\tilde{V}, \lambda) = \frac{\lambda}{\{\lambda^3 + ||\tilde{V}||\}^{\frac{1}{3}}}$, $\nu_{\mathcal{F}}(\tilde{V}, \lambda) = \Big\{\frac{||\tilde{V}||}{\lambda^3 + ||\tilde{V}||}\Big\}^{\frac{1}{3}}$.
Then $\mathcal{A} = (\tilde{\mathcal{X}}, \mu_{\mathcal{F}}, \nu_{\mathcal{F}})$ is a f-f-n-NLS.
Let us take a sequence $\langle a_n \rangle = \frac{1}{n}$. For $\lambda > 0$, $\lim\limits_{n \longrightarrow \infty} ||v_1, \cdots, v_n - v|| = 0$.
Then $\lim\limits_{n \longrightarrow \infty} \mu_{\mathcal{F}}(v_1, \cdots, v_n - v, \lambda) = \lim\limits_{n \longrightarrow \infty} \frac{\lambda}{\{\lambda^3 + ||v_1, \cdots, v_n - v||\}^{\frac{1}{3}}} = 1$ and $\lim\limits_{n \longrightarrow \infty} \nu_{\mathcal{F}}(v_1, \cdots, v_n - v, \lambda) = \{\frac{||v_1, \cdots, v_n - v||}{\lambda^3 + ||v_1, \cdots, v_n - v||}\}^{\frac{1}{3}} = 0$. From theorem 3.2, we can say the sequence $\langle a_n \rangle$ is convergent.

Definition 7. A seuence $\langle v_n \rangle$ is said to be Cauchy sequence in a f-f-n-NLS $\mathcal{A}$, if for $0 < \epsilon < 1$ and $\lambda > 0$ $\exists$ a natural number n_0, such that $\mu_{\mathcal{F}}(v_1, \cdots, v_{n-1}, v_n - v_p, \lambda) > (1 - \epsilon)^{\frac{1}{3}}$ and $\nu_{\mathcal{F}}(v_1, \cdots, v_{n-1}, v_n - v_p, \lambda) < \epsilon^{\frac{1}{3}}$ for all $n, p \geq n_0$.

EXAMPLE **5.** Take a sequence $\langle v_n \rangle = \frac{1}{n}$ in $\mathcal{A}$, defined in Example 3. We get, $\mu^3_{\mathcal{F}}(v_1, \cdots, v_n - v_p) = 1$ and $\nu^3_{\mathcal{F}}(v_1, \cdots, v_n - v_p) = 0$ as $n, p \longrightarrow \infty$. Then $\mu_{\mathcal{F}}(v_1, \cdots, v_n - v_p) > (1 - \epsilon)^{\frac{1}{3}}$ and $\nu_{\mathcal{F}}(v_1, \cdots, v_n - v_p) < \epsilon$ for any $\epsilon \in (0, 1)$ and for all $n, p \geq n_0$ where $n_0 \in \mathbb{N}$.

THEOREM **3.3.** In a f-f-n-NLS $\mathcal{A}$, every convergent sequence is a Cauchy sequence.

Definition 8. A f-f-n-NLS , $\mathcal{A} = \Big\{\big(\tilde{\mathcal{X}}^n, \mu_{\mathcal{F}}(\tilde{V}, \lambda), \nu_{\mathcal{F}}(\tilde{V}, \lambda)\big) : \tilde{V} = (v_1, \cdots, v_n) \in \tilde{\mathcal{X}}^n, \lambda \in (0, \infty)\Big\}$ is said to be complete iff every Cauchy sequence in $\mathcal{A}$ is convergent in $\mathcal{A}$.

EXAMPLE **6.** Take a sequence $\langle a_n \rangle = \frac{1}{n}$ in $\mathcal{A}$, defined in Example 3. $\langle a_n \rangle$ is a Cauchy sequence and converges to 0. If we take $X = (0, 1]$, then $\langle a_n \rangle$ is not convergent in X. So the f-f-n-NLS is not complete with respect to the set X.

LEMMA **1.** In a f-f-n-NLS , $\mathcal{A} = \Big\{\big(\tilde{\mathcal{X}}^n, \mu_{\mathcal{F}}(\tilde{V}, \lambda), \nu_{\mathcal{F}}(\tilde{V}, \lambda)\big) : \tilde{V} = (v_1, \cdots, v_n) \in \tilde{\mathcal{X}}^n, \lambda \in (0, \infty)\Big\}$, a sequence $\langle v_n \rangle$ is Cauchy iff $\mu^3_{\mathcal{F}}(v_1, \cdots, v_n - v_p, \lambda) \longrightarrow 1$ and $\nu^3_{\mathcal{F}}(v_1, \cdots, v_n - v_p, \lambda) \longrightarrow 0$ as $n, p \longrightarrow \infty$.

4. CONTRACTION MAPPING AND FIXED POINT THEOREM IN F-F-N-NLS

Definition 9. The two mappings are such that $\tilde{\phi}, \tilde{\psi} : \tilde{\mathcal{X}}^n \times [0, \infty) \longrightarrow [0, 1]$ are defined here as a control function, having the following properties

(i) $\tilde{\phi}$ and $\tilde{\psi}$ are continuous.

(ii) $\tilde{\phi}$ and $\tilde{\psi}$ are increasing

(iii) $\tilde{\phi}, \tilde{\psi} \geq 0$

(iv) $\tilde{\phi}(v_1, \cdots, v_n - v, \lambda) \longrightarrow 1$ and $\tilde{\psi}(v_1, \cdots, v_n - v, \lambda) \longrightarrow 0$ since $n \longrightarrow \infty$

and the both function satisfying the property,

$$\tilde{\phi}(\mu_{\mathcal{F}})^3 + \tilde{\psi}(\nu_{\mathcal{F}})^3 \leq 1$$

where $\mu_{\mathcal{F}}$ is membership function and $\nu_{\mathcal{F}}$ is non-membership function defined on the set $\tilde{\mathcal{X}}^n$ satisfying $\mu^3_{\mathcal{F}}(\tilde{V}, \lambda) + \nu^3_{\mathcal{F}}(\tilde{V}, \lambda) \leq 1$. Then the pair of functions $(\tilde{\phi}, \tilde{\psi})$ is said to be fermatean fuzzy cubic- control function.

Definition 10. A mapping $\tilde{\tau} : \tilde{\mathcal{X}}^n \longrightarrow \tilde{\mathcal{X}}^n$ is said to be contraction mapping in a f-f-n-NLS, $\mathcal{A} = (\tilde{\mathcal{X}}^n, \mu_{\mathcal{F}}, \nu_{\mathcal{F}})$ if

$$\mu^3_{\mathcal{F}}(\tilde{\tau}(v_1), \cdots, \tilde{\tau}(v_n), \lambda) \geq \frac{\mu^3_{\mathcal{F}}(v_1, \cdots, v_n, \lambda)}{1 + \xi\{1 - \mu^3_{\mathcal{F}}(v_1, \cdots, v_n, \lambda)\}} + \eta\tilde{\phi}(v_1, \cdots, v_n, \lambda) \tag{1}$$

$$\nu^3_{\mathcal{F}}(\tilde{\tau}(v_1), \cdots, \tilde{\tau}(v_n), \lambda) \leq \frac{\nu^3_{\mathcal{F}}(v_1, \cdots, v_n, \lambda)}{1 + \xi\nu^3_{\mathcal{F}}(v_1, \cdots, v_n, \lambda)} - \eta\tilde{\psi}(v_1, \cdots, v_n, \lambda) \tag{2}$$

Definition 11. In a f-f-n-NLS, $\mathcal{A} = (\tilde{\mathcal{X}}^n, \mu_{\mathcal{F}}, \nu_{\mathcal{F}})$, if $\tilde{\tau} : \tilde{\mathcal{X}}^n \longrightarrow \tilde{\mathcal{X}}^n$ is a fermatean fuzzy n-contraction mapping(FF-n-CM) then v is said to be fixed point in $\mathcal{A}$ if $\tilde{\tau}(v) = v$.

THEOREM **4.1.** Let $\mathcal{A} = (\tilde{\mathcal{X}}^n, \mu_{\mathcal{F}}, \nu_{\mathcal{F}})$, be a complete f-f-n-NLS. Then any FF-n-CM attains unique fixed point in $\mathcal{A}$.

Proof: Let $\langle v_n \rangle$ be any sequence and $\tilde{\mathcal{F}}$ be any FF-n-CM such that $\tilde{\tau}(v_n) = v_{n+1}$. From the definition of FF-n-CM

$$\begin{aligned}\mu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1}, v_{n+2}) &= \mu_{\mathcal{F}}^3(\tilde{\tau}(v_1), \cdots, \tilde{\tau}(v_n), \tilde{\tau}(v_{n+1}), \lambda) \\ &\geq \frac{\mu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1}, \lambda)}{1 + \xi\{1 - \mu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1}, \lambda)\}} + \eta\tilde{\phi}(v_1, \cdots, v_n, v_{n+1}, \lambda)\end{aligned}$$

Since $\eta\tilde{\phi}(v_1, \cdots, v_n, v_{n+1}, \lambda) \geq 0$ and $0 < \mu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1}, \lambda) < 1$ we get

$$\mu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1}, v_{n+2}, \lambda) \geq \frac{\mu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1}, \lambda)}{1 + \xi\{1 - \mu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1}, \lambda)\}}$$

We get, $\mu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1}, v_{n+2}, \lambda) > \mu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1}, \lambda)$ From this we can say that the sequence $\langle v_n \rangle$ is increasing and bounded above by 1 and sequence $\langle v_n \rangle$ is convergent. Let the sequence $\langle v_n \rangle$ converges to $\tilde{v}$.

$$\mu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1}, v_{n+2} - \tilde{v}, \lambda) \geq \frac{\mu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1} - \tilde{v}, \lambda)}{1 + \xi\{1 - \mu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1} - \tilde{v}, \lambda)\}}$$

$$\begin{aligned}&\mu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1}, v_{n+2} - \tilde{v}, \lambda) - \xi\mu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1}, v_{n+2} - \tilde{v}, \lambda) - \\ &\xi\mu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1} - \tilde{v}, \lambda)\{\mu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1}, v_{n+2} - \tilde{v}, \lambda) - 1\} \geq 0\end{aligned}$$

This is shown that $\mu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1}, v_{n+2} - \tilde{v}, \lambda)$ tends to 1 as $n \longrightarrow \infty$. From the property (vi) of the definition 4,

$$\mu_{\mathcal{F}}^3(v_1, \cdots, v_n - v_p, \lambda) \geq \mu_{\mathcal{F}}^3(v_1, \cdots, v_{n-1}, v_n - \tilde{v}, \lambda) * \mu_{\mathcal{F}}^3(v_1, \cdots, v_{n-1}, \tilde{v} - v_p, \lambda)$$

As $\mu_{\mathcal{F}}^3(v_1, \cdots, v_{n-1}, v_n - \tilde{v}, \lambda)$ tends to 1 as $n \longrightarrow \infty$ and $\mu_{\mathcal{F}}^3(v_1, \cdots, v_{n-1}, \tilde{v} - v_p, \lambda)$ tend to 1 as $p \longrightarrow \infty$. Therefore, $\mu_{\mathcal{F}}^3(v_1, \cdots, v_n - v_p, \lambda) \longrightarrow 1$ as $n, p \longrightarrow \infty$.

$$\begin{aligned}\nu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1}, v_{n+2}) &= \nu_{\mathcal{F}}^3(\tilde{\tau}(v_1), \cdots, \tilde{\tau}(v_n), \tilde{\tau}(v_{n+1}), \lambda) \\ &\leq \frac{\nu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1}, \lambda)}{1 + \xi\nu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1}, \lambda)} - \eta\tilde{\psi}(v_1, \cdots, v_n, v_{n+1}, \lambda) \\ &\leq \frac{\nu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1}, \lambda)}{1 + \xi\nu_{\mathcal{F}}^3(v_1, \cdots, v_n, v_{n+1}, \lambda)}\end{aligned}$$

We get, $\nu^3_{\mathcal{F}}(v_1, \cdots, v_n, v_{n+1}, v_{n+2}, \lambda) < \nu^3_{\mathcal{F}}(v_1, \cdots, v_n, v_{n+1}, \lambda)$ From this we can say that the sequence $\langle v_n \rangle$ is increasing and bounded above by 1 and sequence $\langle v_n \rangle$ is convergent. Let the sequence $\langle v_n \rangle$ converges to $\tilde{v}$.

$$\nu^3_{\mathcal{F}}(v_1, \cdots, v_n, v_{n+1}, v_{n+2} - \tilde{v}, \lambda) \leq \frac{\nu^3_{\mathcal{F}}(v_1, \cdots, v_n, v_{n+1} - \tilde{v}, \lambda)}{1 + \xi \nu^3_{\mathcal{F}}(v_1, \cdots, v_n, v_{n+1} - \tilde{v}, \lambda)\}}$$

$$\nu^3_{\mathcal{F}}(v_1, \cdots, v_n, v_{n+1}, v_{n+2} - \tilde{v}, \lambda) + \{\xi \nu^3_{\mathcal{F}}(v_1, \cdots, v_n, v_{n+1}, v_{n+2} - \tilde{v}, \lambda) - 1\} \\ \nu^3_{\mathcal{F}}(v_1, \cdots, v_n, v_{n+1} - \tilde{v}, \lambda) \leq 0$$

It implies that $\nu^3_{\mathcal{F}}(v_1, \cdots, v_n, v_{n+1}, v_{n+2} - \tilde{v}, \lambda)$ tends to 0 as $n \longrightarrow \infty$. From the property (xi) of the definition 4,

$$\nu^3_{\mathcal{F}}(v_1, \cdots, v_n - v_p, \lambda) \leq \nu^3_{\mathcal{F}}(v_1, \cdots, v_{n-1}, v_n - \tilde{v}, \lambda) \Diamond \nu^3_{\mathcal{F}}(v_1, \cdots, v_{n-1}, \tilde{v} - v_p, \lambda)$$

As $\nu^3_{\mathcal{F}}(v_1, \cdots, v_{n-1}, v_n - \tilde{v}, \lambda)$ tends to 0 as $n \longrightarrow \infty$ and $\nu^3_{\mathcal{F}}(v_1, \cdots, v_{n-1}, \tilde{v} - v_p, \lambda)$ tend to 0 as $p \longrightarrow \infty$. So $\nu^3_{\mathcal{F}}(v_1, \cdots, v_n - v_p, \lambda) \longrightarrow 0$ as $n, p \longrightarrow \infty$. The sequence $\langle v_n \rangle$ is Cauchy sequence from lemma 1. Since the space $\mathcal{A} = (\tilde{\mathcal{X}}, \mu_{\mathcal{F}}, \nu_{\mathcal{F}})$ is complete the sequence $\langle v_n \rangle$ converges to any point $\tilde{v} \in \tilde{\mathcal{X}}$. We get $\mu^3_{\mathcal{F}}(v_1, \cdots, v_n - \tilde{v}, \lambda) \longrightarrow 1$ and $\nu^3_{\mathcal{F}}(v_1, \cdots, v_n - \tilde{v}, \lambda) \longrightarrow 0$ as $n \longrightarrow \infty$. Now, we shall show that $\tilde{v}$ is a fixed of the contraction operator $\tilde{\tau}$,that is $\tilde{\tau}(\tilde{v}) = \tilde{v}$. From the property (vi) in the definition 4,

$$\begin{aligned} \mu^3_{\mathcal{F}}(v_1, \cdots, v_{n-1}, \tilde{\tau}(\tilde{v}) - \tilde{v}, \lambda) &\geq \mu^3_{\mathcal{F}}(v_1, \cdots, v_n, v_i - \tilde{v}, \lambda) * \mu^3_{\mathcal{F}}(v_1, \cdots, v_n, \tilde{\tau}(\tilde{v}) - v_i, \lambda) \\ &\geq \mu^3_{\mathcal{F}}(v_1, \cdots, v_n, v_i - \tilde{v}, \lambda) * \mu^3_{\mathcal{F}}(v_1, \cdots, v_n, \tilde{\tau}(\tilde{v}) - \tilde{\tau}(v_{i-1}), \lambda) \\ &\geq \mu^3_{\mathcal{F}}(v_1, \cdots, v_n, v_i - \tilde{v}, \lambda) * \frac{\mu^3_{\mathcal{F}}(v_1, \cdots, v_n, v_i - \tilde{v}, \lambda)}{1 + \xi\{1 - \mu^3_{\mathcal{F}}(v_1, \cdots, v_n, v_i - \tilde{v}, \lambda)\}} \\ &\quad + \eta \tilde{\phi}(v_1, \cdots, v_n, v_i - \tilde{v}, \lambda) \end{aligned} \tag{3}$$

As i tends to ∞, v_i tends to $\tilde{v}$ and $\mu^3_{\mathcal{F}}(v_1, \cdots, v_n, v_i - \tilde{v}, \lambda)$ tends to 1 as i tends to ∞ the right hand side of the equation 3 tends to 1 as i tends to ∞. This implies that $\mu^3_{\mathcal{F}}(v_1, \cdots, v_{n-1}, \tilde{\tau}(\tilde{v}) - \tilde{v}, \lambda)$ also tends to 1 as i tends to ∞.

From the property (xi) in the definition 4,

$$\begin{aligned} \nu^3_{\mathcal{F}}(v_1, \cdots, v_{n-1}, \tilde{\tau}(\tilde{v}) - \tilde{v}, \lambda) &\leq \nu^3_{\mathcal{F}}(v_1, \cdots, v_n, v_i - \tilde{v}, \lambda) \Diamond \nu^3_{\mathcal{F}}(v_1, \cdots, v_n, \tilde{\tau}(\tilde{v}) - v_i, \lambda) \\ &\leq \nu^3_{\mathcal{F}}(v_1, \cdots, v_n, v_i - \tilde{v}, \lambda) \Diamond \nu^3_{\mathcal{F}}(v_1, \cdots, v_n, \tilde{\tau}(\tilde{v}) - \tilde{\tau}(v_{i-1}), \lambda) \\ &\leq \nu^3_{\mathcal{F}}(v_1, \cdots, v_n, v_i - \tilde{v}, \lambda) \Diamond \frac{\nu^3_{\mathcal{F}}(v_1, \cdots, v_n, v_i - \tilde{v}, \lambda)}{1 + \xi \nu^3_{\mathcal{F}}(v_1, \cdots, v_n, v_i - \tilde{v}, \lambda)} \\ &\quad - \eta \tilde{\psi}(v_1, \cdots, v_n, v_i - \tilde{v}, \lambda) \end{aligned} \tag{4}$$

When i tends to ∞, v_i tends to $\tilde{v}$ and $\nu^3_{\mathcal{F}}(v_1, \cdots, v_n, v_i - \tilde{v}, \lambda)$ tends to 0 as i tends to ∞ the right hand side of the equation 4 tends to 0 as i tends to ∞, i.e. $\nu^3_{\mathcal{F}}(v_1, \cdots, v_{n-1}, \tilde{\tau}(\tilde{v}) -$

$\tilde{v}, \lambda)$ also tends to 0 as i tends to ∞. So $\tilde{v}$ is a fixed point. To prove the unicity of the fixed point $\tilde{v}$, let v' is the other fixed point in $\mathcal{A}$,i.e. $\tilde{\tau}(\tilde{v}) = \tilde{v}$ and $\tilde{\tau}(v') = v'$. Then using the equation (1), we can have

$$\begin{aligned}\mu^3_{\mathcal{F}}(v_1, \cdots, v_n, v' - \tilde{v}, \lambda) &= \mu^3_{\mathcal{F}}(v_1, \cdots, v_n, \tilde{\tau}(v') - \tilde{\tau}(\tilde{v}), \lambda)\\ &\geq \frac{\mu^3_{\mathcal{F}}(v_1, \cdots, v_n, v' - \tilde{v}, \lambda)}{1 + \xi\{1 - \mu^3_{\mathcal{F}}(v_1, \cdots, v_n, v' - \tilde{v}, \lambda)\}} + \eta\tilde{\phi}(v_1, \cdots, v_n, v' - \tilde{v}, \lambda)\\ &\geq \frac{\mu^3_{\mathcal{F}}(v_1, \cdots, v_n, v' - \tilde{v}, \lambda)}{1 + \xi\{1 - \mu^3_{\mathcal{F}}(v_1, \cdots, v_n, v' - \tilde{v}, \lambda)\}} \\ &\quad \text{since } \tilde{\phi}(v_1, \cdots, v_n, v' - \tilde{v}, \lambda) \geq 0\end{aligned} \tag{5}$$

The inequality (3) holds if $\mu^3_{\mathcal{F}}(v_1, \cdots, v_n, v' - \tilde{v}, \lambda) = 1$. This indicates that $v' = \tilde{v}$. Using (2),

$$\begin{aligned}\nu^3_{\mathcal{F}}(v_1, \cdots, v_n, v' - \tilde{v}, \lambda) &= \nu^3_{\mathcal{F}}(v_1, \cdots, v_n, \tilde{\tau}(v') - \tilde{\tau}(\tilde{v}), \lambda)\\ &\leq \frac{\nu^3_{\mathcal{F}}(v_1, \cdots, v_n, v' - \tilde{v}, \lambda)}{1 + \xi\nu^3_{\mathcal{F}}(v_1, \cdots, v_n, v' - \tilde{v}, \lambda)} - \eta\tilde{\psi}(v_1, \cdots, v_n, v' - \tilde{v}, \lambda)\\ &\leq \frac{\nu^3_{\mathcal{F}}(v_1, \cdots, v_n, v' - \tilde{v}, \lambda)}{1 + \xi\nu^3_{\mathcal{F}}(v_1, \cdots, v_n, v' - \tilde{v}, \lambda)} \\ &\quad \text{since } \tilde{\psi}(v_1, \cdots, v_n, v' - \tilde{v}, \lambda) \geq 0\end{aligned} \tag{6}$$

Equation (6) holds if $\nu^3_{\mathcal{F}}(v_1, \cdots, v_n, v' - \tilde{v}, \lambda) = 0$. This shows that $v' = \tilde{v}$. Therefore $\tilde{\tau}$ has a unique fixed point. □

5. APPLICATION IN MEDICAL DIAGNOSIS

Consider a patient is examined by many doctors. Some examinations highly imply existence of disease when other examinations also highly imply nonexistence of disease. Here $v_1, \cdots, v_n$ represent the value of diagnosis and λ represents time of evaluation. When time increases, number of evaluation also increases and diagnosis is closed to the correct value. Here we considered diagnosis by iteration method. $\mu_{\mathcal{F}}(v_1, \cdots, v_n, \lambda)$ indicates the similarity of diagnosis and $\nu_{\mathcal{F}}(v_1, \cdots, v_n, \lambda)$ indicates the dissimilarity of diagnosis. If we take x=0.8746 and y=0.5, in example 3 $\mu_{\mathcal{F}}(v_1, \cdots, v_n, \lambda) = 0.8$ and $\nu_{\mathcal{F}}(v_1, \cdots, v_n, \lambda) = 0.7$ when $\lambda = 1$.
In this case we cannot use intuitionistic fuzzy normed linear space because $\mu_{\mathcal{F}}(v_1, \cdots, v_n, \lambda)^2 + \nu_{\mathcal{F}}(v_1, \cdots, v_n, \lambda)^2 = 0.8^2 + 0.7^2 > 1$.
In this situation, we can use fermatean fuzzy normed linear spaceto handle uncertainty because $\mu_{\mathcal{F}}(v_1, \cdots, v_n, \lambda)^3 + \nu_{\mathcal{F}}(v_1, \cdots, v_n, \lambda)^3 = 0.8^3 + 0.7^3 < 1$. Now, construct a contraction mapping or diagnosis operator,

$$\tilde{\tau} : \tilde{\mathcal{X}} \longrightarrow \tilde{\mathcal{X}} = (\rho\zeta + (1 - \rho)\zeta', \rho\eta + (1 - \rho)\eta') \tag{7}$$

where $\mathcal{X} = \{(\mu_{\mathcal{F}}, \nu_{\mathcal{F}}) : \mu_{\mathcal{F}}^3 + \nu_{\mathcal{F}}^3 \leq 1\}$, $0 < \rho < 1$ and $(\zeta, \eta) = (0.8, 0.7)$ represent the initial state of diagnosis and $(\zeta', \eta') = (0.9, 0.51)$ represent the final or desired state of diagnosis.
For $\rho = 0.5$
$\xi_1 = (0.5 \times 0.8 + 0.5 \times 0.9, 0.5 \times 0.7 + 0.5 \times 0.51) = (0.85, 0.605)$
$\xi_2 = (0.5 \times 0.85 + 0.5 \times 0.9, 0.5 \times 0.605 + 0.5 \times 0.51) = (0.875, 0.5575)$
$\xi_3 = (0.5 \times 0.875 + 0.5 \times 0.9, 0.5 \times 0.5575 + 0.5 \times 0.51) = (0.8875, 0.53375)$
$\xi_4 = (0.5 \times 0.8875 + 0.5 \times 0.9, 0.5 \times 0.53375 + 0.5 \times 0.51) = (0.89375, 0.521875)$
$\xi_5 = (0.5 \times 0.89375 + 0.5 \times 0.9, 0.5 \times 0.521875 + 0.5 \times 0.51) = (0.896875, 0.5159375)$
$\xi_6 = (0.5 \times 0.896875 + 0.5 \times 0.9, 0.5 \times 0.5159375 + 0.5 \times 0.51) = (0.8984375, 0.51296875)$
After sixth iteration, we got our desired state of diagnosis which is similar to $(0.9, 0.51)$ which is a fixed point of the contraction mapping, $\tilde{\tau}$.

HIGHLIGHTS

(I) Larger uncertainty region

(II) Supports stronger contraction mappings

(III) Ensures convergence to stable solution

6. DISCUSSION

Table 1: Comparison Table

Normed Spaces	Examples	Remarks
i-f-n-NLS [11]	Example 3	Conditions (i),(v),(xi) in the Definition 3.1 , are not satisfied
N-n-NLS [13]	In Example 3 the membership and non-membership function are same as truth-membership and indeterminacy-membership function. Take falsity membership function as $F_B(v_1, \cdots, v_n, \lambda_1) = \left\{ \frac{\|v_1, \cdots, v_n\|}{\lambda_1^3} \right\}^{\frac{1}{3}}$	Conditions (i), (v), (xii), (xix) in the Definition 3.1, are not satisfied
f-f-n-NLS	Example 3	No violating conditions in the Definition 4

In this section we consider some examples which are not intuitionistic fuzzy n-normed linear space (i-f-n-NLS),introduced by S.Vijaybalaji et al. in their paper [11]. These examples also can not form neutrosophic n-normed linear space(N-n-NLS), introduced by V. Kumar et al. in their paper [13]. But these examples form f-f-n-NLS, introduced in this paper. There are such many examples of normed space which are cannot form i-f-n-NLS and N-n-NLS but can form f-f-n-NLS. For this purpose we derive f-f-n-NLS, that is more generalization of another normed space.

7. CONCLUSION

This paper introduced the concept of f-f-n-NLS as a generalization of fuzzy normed linear space. This normed linear space handles more uncertainty because of its larger uncertainty region. The obtained contraction mapping is more stronger. The convergence in this space ensures the more stable solution.This work showed here the advantage of f-f-n-NLS taking a specific case in medical diagnosis where other normed linear space is not working. For future, this paper may open a new era on f-f-n-NLS. This concept can be used to solve the problems in Optimization and Game theory.

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