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Presentation of Finite Dimensions

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Abstract

We present subsets of Euclidean spaces $\mathbb{R}^n$ in the ordinary plane $\mathbb{R}^2$. Naturally some informations are lost. We provide examples.

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1. INTRODUCTION

It is trivial that one can picture objects of the space $\mathbb{R}^n$ only if n is less than four. The best presentation is a picture in $\mathbb{R}^2$. Mathematicians often deal with objects in higher dimensional spaces, but since we live in a three dimensional space we have no real imagination of these objects. Here we show methods to represent something of the $\mathbb{R}^n$ in $\mathbb{R}^2$. The way is by dividing a vector of $\mathbb{R}^n$ into small parts consisting of some components. After this we take barycenters. The resulting point can be presented in the two dimensonal space $\mathbb{R}^2$.

2. DEMONSTRATION

First we give names. We have methods $\mathsf{way}_2, \mathsf{way}_3$ and way_4. To use way_2 we need points in $\mathbb{R}^n$ for $n \in \{4, 8, 16\}$. To use way_3 we need points in $\mathbb{R}^n$ for $n = 9$ or $n = 27$. For way_4 we use a point in $\mathbb{R}^8$. We calculate barycenters of polygons. Further we define method_n, which requires a vector from $\mathbb{R}^n$, and which is suitable for all integers n and which does not need barycenters.

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Let us take a vector $\vec{a} := (a_1, a_2, \ldots, a_{n-1}, a_n)$ of $\mathbb{R}^n$. We define

$$\mathsf{method}_n(\vec{a}) := \begin{cases} \left(a_1 + \ldots + a_{\frac{n}{2}-1} + a_{\frac{n}{2}},\ a_{\frac{n}{2}+1} + a_{\frac{n}{2}+2} + \ldots + a_{n-1} + a_n\right) \\ \text{if } n \text{ is even, } n \text{ larger than 4} \\ \left(a_1 + \ldots + a_{\frac{n-1}{2}-1} + a_{\frac{n-1}{2}} + \frac{1}{2} \cdot a_{\frac{n-1}{2}+1},\ \frac{1}{2} \cdot a_{\frac{n-1}{2}+1} + a_{\frac{n-1}{2}+2} + \ldots + a_n\right) \\ \text{if } n \text{ is odd, } n \text{ larger than 5} \end{cases}$$

We define $\mathsf{method}_1(a) := (a, a)$, $\mathsf{method}_2(a, b) := (a, b)$, $\mathsf{method}_3(a, b, c) := \left(a + \frac{1}{2} \cdot b, \frac{1}{2} \cdot b + c\right)$, $\mathsf{method}_4(a, b, c, d) := (a + b, c + d)$, $\mathsf{method}_5(a, b, c, d, e) := (a + b + \frac{1}{2} \cdot c, \frac{1}{2} \cdot c + d + e)$.

Let us demonstrate way_2. If we have an element $(a, b, c, d) \in \mathbb{R}^4$ we take two vectors $(a, b), (c, d) \in \mathbb{R}^2$. Then we compute the barycenter and we get the image point

$$\mathsf{way}_2(a, b, c, d) := \left(\frac{1}{2} \cdot (a, b) + \frac{1}{2} \cdot (c, d)\right) = \left(\frac{1}{2} \cdot (a + c),\ \frac{1}{2} \cdot (b + d)\right), \tag{2.1}$$

which can be drawn in $\mathbb{R}^2$. In the case of a vector $\vec{y} := (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8) \in \mathbb{R}^8$ we divide it in four parts $(a_1, a_2), (a_3, a_4), (a_5, a_6), (a_7, a_8) \in \mathbb{R}^2$. First we calculate two barycenters of the pairs $(a_1, a_2), (a_3, a_4)$ and $(a_5, a_6), (a_7, a_8)$, respectively. After this we take the barycenter of the two barycenters. We get

$$\mathsf{way}_2(\vec{y}) := \left(\frac{1}{2} \cdot \left[\frac{1}{2} \cdot (a_1 + a_3),\ \frac{1}{2} \cdot (a_2 + a_4)\right] + \frac{1}{2} \cdot \left[\frac{1}{2} \cdot (a_5 + a_7),\ \frac{1}{2} \cdot (a_6 + a_8)\right]\right)$$

hence

$$\mathsf{way}_2(\vec{y}) = \left(\frac{1}{4} \cdot (a_1 + a_3 + a_5 + a_7),\ \frac{1}{4} \cdot (a_2 + a_4 + a_6 + a_8)\right) \tag{2.2}$$

which is a point in $\mathbb{R}^2$.

If we have a vector $\vec{u} := (a_1, a_2, a_3, \ldots, a_{14}, a_{15}, a_{16}) \in \mathbb{R}^{16}$ we can use also way_2. We compute the barycenter of two barycenters of four barycenters of 8 points $(a_1, a_2), (a_3, a_4), (a_5, a_6), (a_7, a_8), (a_9, a_{10}), (a_{11}, a_{12}), (a_{13}, a_{14})$ and (a_{15}, a_{16}). We get $\mathsf{way}_2(\vec{u}) =$

$$\left(\frac{1}{8} \cdot (a_1 + a_3 + a_5 + a_7 + a_9 + a_{11} + a_{13} + a_{15}),\ \frac{1}{8} \cdot (a_2 + a_4 + a_6 + a_8 + a_{10} + a_{12} + a_{14} + a_{16})\right). \tag{2.3}$$

To demonstrate way_3 for $\mathbb{R}^9$ we use a point $\vec{v} := (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9) \in \mathbb{R}^9$. First we take the barycenter of the triangle of three points $(a_1, a_2, a_3), (a_4, a_5, a_6), (a_7, a_8, a_9)$ of $\mathbb{R}^3$, then we use method_3. We define

$$\mathsf{way}_3(\vec{v}) := \left(\frac{1}{3} \cdot (a_1 + a_4 + a_7) + \frac{1}{6} \cdot (a_2 + a_5 + a_8),\ \frac{1}{6} \cdot (a_2 + a_5 + a_8) + \frac{1}{3} \cdot (a_3 + a_6 + a_9)\right). \tag{2.4}$$

In the case of a vector $\vec{w} := (a_1, a_2, a_3, \ldots, a_{26}, a_{27})$ from $\mathbb{R}^{27}$ we use method_3 for the barycenter of three barycenters of three triangles, which are generated by nine points $(a_1, a_2, a_3), (a_4, a_5, a_6), \ldots, (a_{25}, a_{26}, a_{27})$ of $\mathbb{R}^3$. This means

$$\mathsf{way}_3(\vec{w}) := \left(\frac{1}{9} \cdot a + \frac{1}{18} \cdot b,\ \frac{1}{18} \cdot b + \frac{1}{9} \cdot c \right) \tag{2.5}$$

where

$$a := a_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16} + a_{19} + a_{22} + a_{25}\ , \tag{2.6}$$

$$b := a_2 + a_5 + a_8 + a_{11} + a_{14} + a_{17} + a_{20} + a_{23} + a_{26}\ , \tag{2.7}$$

$$c := a_3 + a_6 + a_9 + a_{12} + a_{15} + a_{18} + a_{21} + a_{24} + a_{27}\ . \tag{2.8}$$

For way_4 we define the same formula as for way_2. Please see line (2.2). Note that in way_4 we calculate the barycenter of a 4-gon. We define

$$\mathsf{way}_4(\vec{y}) := \left(\frac{1}{4} \cdot (a_1 + a_3 + a_5 + a_7),\ \frac{1}{4} \cdot (a_2 + a_4 + a_6 + a_8) \right) \tag{2.9}$$

Remark 2.1. If $n < k$ it holds $\mathbb{R}^n \subset \mathbb{R}^k$. In place of the vector $(a_1, a_2, a_3, \ldots, a_{n-1}, a_n)$ of $\mathbb{R}^n$ we can use $(a_1, a_2, a_3, \ldots, a_{n-1}, a_n, 0, 0, 0, \ldots, 0, 0) \in \mathbb{R}^k$.

Remark 2.2. To avoid fractions we may multiply $\mathsf{method}_k(\vec{x})$ or $\mathsf{way}_k(\vec{x})$ with a suitable factor.

3. EXAMPLE

As an example we take the four dimensional cube, which is the convex hull of four dimensional vectors $(a, b, c, d) \in \mathbb{R}^4$, where the variables a, b, c, d either are 0 or 1. Each point is called a *vertex* of the cube. Hence a four dimensional cube has 16 vertices. By method_4 we get 9 vertices (x, y), where x and y is 0 or 1 or 2.
We get the same result if we use way_2 and Remark 2.2. We multiply all values with 2 by Remark 2.2. By line (2.1) the presentation in $\mathbb{R}^2$ has 9 vertices

$$(0,0), (0,1), (0,2), (1,0), (1,1), (1,2), (2,0), (2,1), (2,2).$$

We repeat the presentation by Remark 2.1 since $\mathbb{R}^4 \subset \mathbb{R}^5$. Instead of points (a, b, c, d) we use points $(a, b, c, d, 0)$ with variables a, b, c, d either 0 or 1. It holds $(s, t) := \mathsf{method}_5\,(a, b, c, d, 0) = \left(a + b + \frac{1}{2} \cdot c, \frac{1}{2} \cdot c + d\right)$.

By Remark 2.2 we multiply all points with 2 to avoid fractions. We get with method_5 12 points

$$(0,0), (2,0), (4,0), (0,2), (2,2), (4,2), (1,1), (1,3), (3,1), (3,3), (5,1), (5,3).$$

Note that c occurs both in s and t.

With way_3 we repeat the presentation by Remark 2.1, since $\mathbb{R}^4 \subset \mathbb{R}^9$. Instead of points $(a, b, c, d) \in \mathbb{R}^4$ we take points $(a, b, c, d, 0, 0, 0, 0, 0) \in \mathbb{R}^9$. By Remark 2.2 we multiply the 12 resulting points with 6. We get by line (2.4): $(0, 0), (2, 0)$, $(4, 0)$,$(0, 2)$, $(2, 2)$, $(4, 2)$, $(1, 1)$, $(1, 3)$, $(3, 1)$, $(3, 3)$, $(5, 1)$, $(5, 3)$.
With way_4 we repeat the presentation by Remark 2.1, since $\mathbb{R}^4 \subset \mathbb{R}^8$. Instead of points $(a, b, c, d) \in \mathbb{R}^4$ we take points $(a, b, c, d, 0, 0, 0, 0) \in \mathbb{R}^8$. Since in $\mathbb{R}^8$ we have $\mathsf{way}_4 = \mathsf{way}_2$ we get by line (2.2) the same points as above $(0, 0)$,$(0, 1)$,$(0, 2)$,$(1, 0)$,$(1, 1)$, $(1, 2)$,$(2, 0)$,$(2, 1)$,$(2, 2)$.

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