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Realization Problems in 1-movable Independent and 1-movable Doubly Connected Domination in Graphs

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Abstract

This paper shows that every pair of positive integers is realizable as independent domination number and 1-movable independent domination number and realizable as doubly connected domination number and 1-movable doubly connected domination number.

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Keywords: Domination, independent domination, doubly connected domination, 1-movable domination, 1-movable independent domination, 1-movable doubly connected domination.

1. Introduction

Let $G = (V(G), E(G))$ be a graph and $v \in V(G)$. The *open neighborhood* of v is the set $N_G(v) = N(v) = \{u \in V(G) : uv \in E(G)\}$ and the *closed neighborhood* of v is the set $N_G[v] = N[v] = N(v) \cup \{v\}$. If $S \subseteq V(G)$, then the *open neighborhood* of S is the set $N_G(S) = N(S) = \bigcup_{v \in S} N_G(v)$ and the *closed neighborhood* of S is the set $N_G[S] = N[S] = S \cup N(S)$.

A subset S of $V(G)$ is an *independent set* of G if for every two elements $x, y \in S$, $xy \notin E(G)$. The *independence number* of G, denoted by $\beta(G)$, is the largest cardinality of an independent set in G. An independent set S of G is called a *maximum independent set* if $|S| = \beta(G)$. A subset S of $V(G)$ is a *dominating set* of G if for every $v \in V(G)\backslash S$, there exists $u \in S$ such that $uv \in E(G)$, that is, $N_G[S] = V(G)$. It is an *independent*

dominating set of G if S is both an independent set and a dominating set. It is a *connected dominating set* of G if the subgraph $\langle S \rangle$ induced by S is connected and it is a *doubly connected dominating set* of G if S is a connected dominating set and the subgraph $\langle V(G) \setminus S \rangle$ induced by $V(G) \setminus S$ is connected. The *domination number* $\gamma(G)$ (resp. *independent domination number* $\gamma_i(G)$, *connected domination number* $\gamma_c(G)$ and *doubly connected domination number* $\gamma_{cc}(G)$) is the smallest cardinality of a dominating(resp. independent dominating, connected dominating and doubly connected dominating) set of G.

A nonempty set $S \subseteq V(G)$ is a 1-*movable dominating* (*resp.* 1-*movable independent dominating*) *set* of G if S is a dominating(resp. independent dominating) set of G and for every $v \in S$, $S \setminus \{v\}$ is a dominating set of G or there exists a vertex $u \in (V(G) \setminus S) \cap N_G(v)$ such that $(S \setminus \{v\}) \cup \{u\}$ is a dominating (resp. independent dominating) set of G. It is a 1-*movable doubly connected dominating set* of G if (i) $S = V(G)$ and for each $v \in S$, $S \setminus \{v\}$ is a doubly connected dominating set of G or (ii) S is a doubly connected dominating set of G and for each $v \in S$, $S \setminus \{v\}$ is a doubly connected dominating set of G or $(S \setminus \{v\}) \cup \{u\}$ is a doubly connected dominating set of G for some $u \in (V(G) \setminus S) \cap N_G(v)$. The 1-*movable domination number* $\gamma^1_m(G)$(*resp.* 1-*movable independent domination number* $\gamma^1_{mi}(G)$ *and* 1-*movable doubly connected domination number* $\gamma^1_{mcc}(G)$) is the smallest cardinality of a 1-movable dominating (resp. 1-movable independent dominating and 1-movable doubly connected dominating) set of G.

Domination and some of its variants were investigated in [1], [3] and [4]. Moreover, the 1-movable domination and its variants were investigated in [2], [5], [6], [7],[8], [9], [10], [11] and [12].

The next section presents some realization problems in 1-movable independent domination and 1-movable doubly connected domination in graphs and that the difference of two parameters can be made arbitrarily large.

2. Results

Theorem 2.1. For every pair of positive integers a and b, where $1 \leq a \leq b$, there exists a connected graph G such that $\gamma_i(G) = a$ and $\gamma^1_{mi}(G) = b$.

Proof. Consider the following cases:
Case 1: $1 = a = b$
Then take $G = K_n$, where $n \geq 2$ so that $\gamma_i(G) = 1 = \gamma^1_{mi}(G)$.

Case 2: $1 = a < b$
Then consider the graph $G = G_1$ as shown in Figure 1.

The set $S_1 = \{x\}$ is a γ_i-set of G. Hence, $\gamma_i(G) = |S_1| = 1$. Now, if S_2 is a γ^1_{mi}-set of G, then either $u_i \in S_2$ or $v_i \in S_2$ for each $i = 1, 2, 3, \ldots, b$. Suppose that $u_i \in S_2$. Then, $S_2 = \{u_1, u_2, \ldots, u_{b-1}, u_b\}$ is a γ^1_{mi}-set of G. Hence, $\gamma^1_{mi}(G) = |S_2| = b$.

Case 3: $1 < a = b$

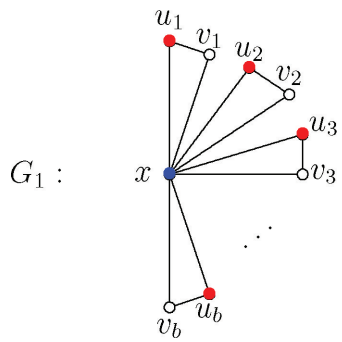

Figure 1: A graph with $\gamma_(G) = 1 < b = \gamma^1_{mi}(G)$

Then consider the graph $G = G_2$ as shown in Figure 2.

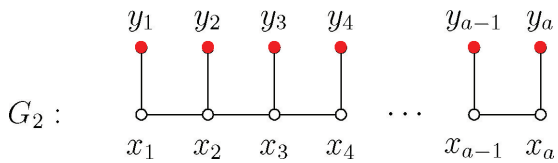

Figure 2: A graph with $1<\gamma_i(G) = a = \gamma^1_{mi}(G)$

The set $S_3 = \{y_1, y_2, \ldots, y_{a-1}, y_a\}$ is a γ_i-set of G. Hence, $\gamma_i(G) = |S_3| = a$. Moreover, for each $i = 1, 2, \ldots, a$, the set $(S_3 \setminus \{y_i\}) \cup \{x_i\}$ is an independent dominating set of G. Hence, S_3 is a γ^1_{mi}-set of G. Thus, $\gamma^1_{mi}(G) = |S_3| = a$.

Case 4: $1 < a < b$
Then consider the graph $G = G_3$ as shown in Figure 3.

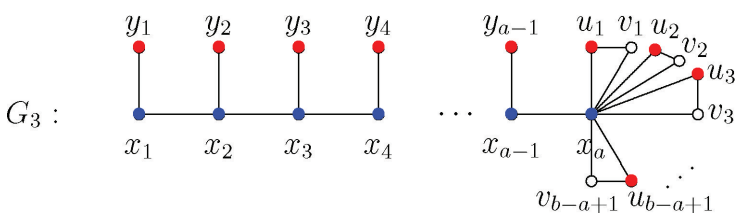

Figure 3: A graph with $1<\gamma_i(G) = a < b = \gamma^1_{mi}(G)$

If S_4 is a γ_i-set of G, then $x_a \in S_4$ and $y_i \in S_4$ for each $i = i, 2, \ldots, a-1$. Hence, $S_4 = \{y_1, y_2, \ldots, y_{a-1}\} \cup \{x_a\}$ and $\gamma_i(G) = |S_4| = a$. Now, if S_5 is a γ^1_{mi}-set of G, then $y_i \in S_5$ for each $i = 1, 2, 3, \ldots, a-1$ and either $u_j \in S_5$ or $v_j \in S_5$ for each $j = 1, 2, \ldots, b-a+1$. Suppose that $u_j \in S_5$ for each $j = 1, 2, \ldots, b-a+1$. Then, $S_5 = \{y_1, y_2, \ldots, y_{a-1}\} \cup \{u_1, u_2, \ldots, u_{b-a+1}\}$ is a γ^1_{mi}-set of G. Hence, $\gamma^1_{mi}(G) = |S_5| = (a-1) + (b-a+1) = b$. ■

Corollary 2.2. The difference $\gamma^1_{mi} - \gamma_i$ can be made arbitrarily large.

Proof. Let $n \in \mathbb{N}$. By Theorem 2.1, there exists a connected graph G such that $\gamma_i(G) = 1$ and $\gamma^1_{mi}(G) = n + 1$. Thus, $\gamma^1_{mi}(G) - \gamma_i(G) = (n + 1) - 1 = n$. ■

Note that for every connected nontrivial graph G, $\gamma_{cc}(G) \leq \gamma^1_{mcc}(G)$ and $\gamma^1_{mcc}(K_n) = 1$ for all $n \geq 2$. Now, we show that every pair of positive integers is realizable as doubly connected domination number and 1-movable doubly connected domination number.

Theorem 2.3. For every pair of positive integers a and b, where $1 \leq a \leq b$ and $b > 3a$, there exists a connected graph G without cut-vertices such that $\gamma_{cc}(G) = a$ and $\gamma^1_{mcc}(G) = b$.

Proof. Consider the following cases:

Case 1: $1 = a = b$
Then take $G = K_n$, where $n \geq 2$ so that $\gamma_{cc}(G) = 1 = \gamma^1_{mcc}(G)$.

Case 2: $1 = a < b$
Then consider the graph $G = G_4$ as shown in Figure 4.

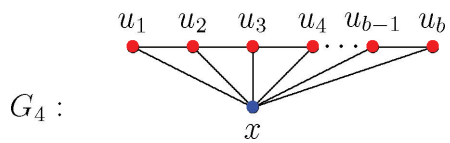

Figure 4: A graph with $\gamma_{cc}(G) = 1 < b = \gamma^1_{mcc}(G)$

The set $S_1 = \{x\}$ is a γ_{cc}-set of G. Hence, $\gamma_{cc}(G) = |S_1| = 1$. Note that $G_4 \cong K_1 + P_b$. Hence, it follows that $\gamma^1_{mcc}(G) = \gamma^1_{mcc}(K_+P_b) = |P_b| = b$.

Case 3: $1 < a = b$
Then consider the graph $G = G_5$ as shown in Figure 5.

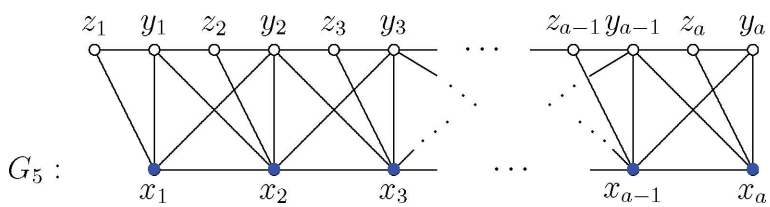

Figure 5: A graph with $1<\gamma_{cc}(G) = a = \gamma^1_{mcc}(G)$

The set $S_3 = \{x_1, x_2, \ldots, x_{a-1}, x_a\}$ is a γ_{cc}-set of G. Hence, $\gamma_{cc}(G) = |S_3| = a$. The vertex y_i dominates z_i for each $i = 1, 2, \ldots, a$. Moreover, $y_1x_2 \in E(G)$, $x_{i-1}y_i \in E(G)$ and $y_ix_{i+1} \in E(G)$ for each $i = 2, 3, \ldots, a - 1$. Hence, it follows that

$(S_3 \setminus \{x_i\}) \cup \{y_i\}$ is a dominating set in G and $\langle V(G) \setminus [(S_3 \setminus \{x_i\}) \cup \{y_i\}]\rangle$ is connected for each $i = 1, 2, \ldots, a$. Hence, S_3 is a γ^1_{mcc}-set of G. Therefore, $\gamma^1_{mcc}(G) = |S_3| = a$.

Case 4: $1 < a < b$
Then consider the graph $G = G_6$ as shown in Figure 6.

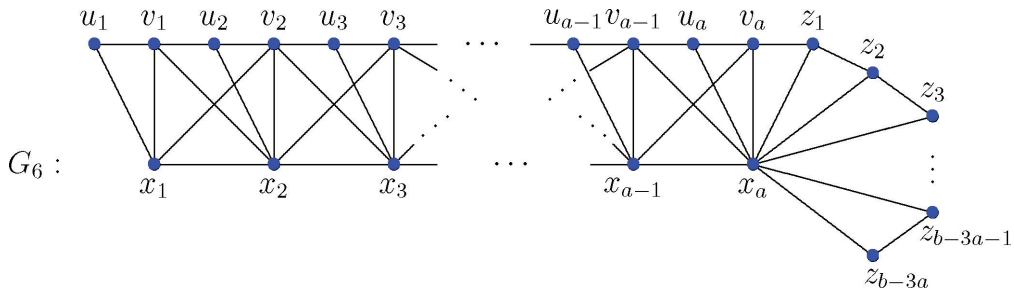

Figure 6: A graph with $1<\gamma_{cc}(G) = a < b = \gamma^1_{mcc}(G)$

The set $S_4 = \{x_1, x_2, \ldots, x_{a-1}, x_a\}$ is a γ_{cc}-set of G. Hence, $\gamma_{cc}(G) = |S_4| = a$. Suppose that S_5 is a γ^1_{mcc}-set of G. Since the graph G_6 has a subgraph isomorphic to the graph G_4, it follows that $u_a \in S_5$, $v_a \in S_5$ and $z_i \in S_5$ for each $i = 1, 2, \ldots, b-3a$. Also, since the graph G_6 has a subgraph isomorphic to the graph G_5, it follows that $x_i \in S_5$ for each $i = 1, 2, \ldots, a-1$. Suppose further that $u_i \notin S_5$ for each $i = 1, 2, \ldots, a-1$. Then $u_i z_1 \notin E(\langle V(G) \setminus (S_5 \setminus \{z_1\})\rangle)$. Thus, $\langle V(G) \setminus (S_5 \setminus \{z_1\})\rangle$ is disconnected. Hence, $S_5 \setminus \{z_1\}$ is not a doubly connected dominating set of G. Moreover, the same conclusion is obtained whenever $v_i \notin S_5$. A contradiction to the assumption. Thus, $u_i \in S_5$ and $v_i \in S_5$ for each $i = 1, 2, \ldots, a-1$. Therefore, $S_5 = \{u_1, u_2, \ldots, u_a\} \cup \{v_1, v_2, \ldots, v_a\} \cup \{x_1, x_2, \ldots, x_a\} \cup \{z_1, z_2, \ldots, z_{b-3a}\}$ is a γ^1_{mcc}-set of G and $\gamma^1_{mcc}(G) = |S_5| = 3a + (b - 3a) = b$. ■

Corollary 2.4. The difference $\gamma^1_{mcc} - \gamma_{cc}$ can be made arbitrarily large.

Proof. Let $n \in \mathbb{N}$. By Theorem 2.3, there exists a connected graph G such that $\gamma_{cc}(G) = 1$ and $\gamma^1_{mcc}(G) = n + 1$. Thus, $\gamma^1_{mcc}(G) - \gamma_{cc}(G) = (n + 1) - 1 = n$. ■

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