An Improved Simulated Annealing Techniques (ISAT) for two variables Global Minimum Optimization Value Problems

Paiwan Wongsasinchai\textsuperscript{1} and Chuanpit Mungkala\textsuperscript{2}

\textsuperscript{1,2}Department of Mathematics, Faculty of Science and Technology, Rambhai Barni Rajabhat University, Chanthaburi, Thailand.

Abstract

This research studied an Improved Simulated Annealing Techniques (ISAT) for two variables Global Minimum Optimization Value Problems for finding global minimum solution which consist of main elements like search direction, temperature and energy value. These methods have been expected to increase, the speed and efficiency of Simulated Annealing, find the optimal solution for the global minimum increase at faster rate. This method was performed by using 9 selected test functions. Numerical performance tests were carried out using Random Search Method, Simulated Annealing Method and an Improved Simulated Annealing Techniques. From the numerical experiments with the test function, it was deduced that the ISAT method gave better results than the SA and RS method. The numerical experiments shows that ISAT yields the better solutions than SA and RS in term of function evolutions as an index.

Keywords: Optimization; Random Search Method; Simulated Annealing Method.

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1. INTRODUCTION

Using mathematical methods to analyze and solve problems most times proves difficult due to the complexity at the problem such as one function variable. On many occasions finding solutions becomes almost impossible. Instead of devoting resources in finding the solution, we consider the widely applied optimization process. This process is known to be the best way of finding the optimal solution of the function under the constraint conditions. For instance, resources for productive of transportation cost management has been a problem to address this problem. Heuristic method has been developed to select random sample from all possible answers. In many cases, the purpose function is complex. Simulated Annealing (SA) is a heuristic method developed in 1983 to solve the most optimal nonlinear problem. Consider the problem of imitating the toughening process of heating metal to achieve high toughness and hardness. When the metal is heated to a molten state, the atom has a high energy. Atoms in the hot metal can move freely. Since there is enough energy opportunity to get the solution out of the local optimum when the metal slowly slows down by lowering the system temperature, atomic energy is reduced and the motion of the atom is limited. Atoms in the metal begin to gradually settle in order, eventually causing the crust to change. The crystalline structure has the lowest internal energy. It is comparable to finding the solution gradually. This makes the metal stronger and more durable, which means getting the right result. Based on this concept, SA has applied the best value solution to many problems and widely studied today. SA has high efficiency in solving complicated problem. Especially the ability to fall out of a specific value. Recent research has focused on improving an animalized simulated annealing (ISA) to improve the efficiency of SA. So to speed up the calculation. And increase the chance of falling out of a specific value. As well as focusing on efficiency in finding solution, SA has proposed a technique to improve the simulated cohesion to help solve the optimal solution problem. In the study of techniques to improve the simulated drying, the method is not used to find the solution. And as a way to reproduce the solution randomly the best solution is then compared and then used as the starting point for the solution. The efficiency of this new methodology will be compared with the SA method.

The object function $f : D \subseteq R^n \rightarrow R$ where $x = (x_1, x_2, x_3, ..., x_n)$ is called a variable function. It has a minimum local or maximum local at multiple points such as schwefel function: $f(x_1, x_2) = 418.9829(2) + [(-x_1 \sin(\sqrt{|x_1|}) + (-x_2 \sin(\sqrt{|x_2|})))$ by $x_i \in [-500, 500]$ The lowest point is $x^* = (0, 0, 0, ..., 0)$ This causes the lowest function value $f(x_1, x_2) = (0, 0)$. 
Fig.1. Show $f(x_1, x_2)$ of schwefel function

Simulated Annealing becomes the incumbent. This migration through local minima in search of a global minimum continues until the global minimum is found or some termination criteria are reached. Belisle [14] presents a special simulated annealing algorithm for global optimization, which uses a heuristically motivated cooling schedule. This algorithm is easy to implement and provides a reasonable alternative to existing methods. Belisle et al. [15] discuss convergence properties of simulated annealing algorithms applied to continuous functions and apply these results to hit-and-run algorithms used in global optimization. The presented convergence properties are consistent with those presented in [16] and provide a good contrast between convergence in probability and (the stronger) almost sure convergence. This work is further extended in [17] to an improved hit-and-run algorithm used for global optimization. Fleischer and Jacobson [18] propose cybernetic optimization by simulated annealing as a method of parallel processing that accelerates the convergence of simulated annealing to the global optima. This theory is extended by Fleischer [19] into the continuous domain by applying probabilistic feedback control to the generation of candidate solutions. The probabilistic feedback control method of generating candidate solutions effectively accelerates convergence to a global optimum using parallel simulated annealing on continuous variable problems. Locatelli [26] presents convergence properties for a class of simulated annealing algorithms for continuous global optimization by removing the restriction that the next candidate point must be generated according to a probability distribution whose support is the whole feasible set. A study on simulated annealing algorithms for globally minimizing functions of multiple continuous variables is conducted by Siarry et al. [21]. The study focuses on how high dimensionality can be addressed using variables discretization and addresses the design and implementation issues for several complementary stopping criteria. Convergence results and criteria for simulated annealing applied to continuous global optimization problems are also provided in [22] and [26]. A general-purpose simulated annealing algorithm that solves mixed integer linear programs is introduced by Kiatsupaibul and Smith [24]. The simulated annealing algorithm is constructed using a Markov chain sampling algorithm to generate uniformly distributed points on an arbitrary bounded region of a high-dimensional integer lattice. They show that the algorithm converges in probability to a global optimum. Romeijn et al.
[15] that uses a reflection generator for mixed integer/continuous global optimization problems. Locatelli [26] establishes the convergence of simulated annealing algorithms for continuous global optimization and an upper bound for the expected first hitting time, i.e., the expected number of iterations before reaching the global optimum value within accuracy $\epsilon$.

1.1 Random Search Method

Random Search is a non-derivative random method for finding the answer, which has the following steps:

Step 1: Define objective function

Step 2: Random start And set the start point to the lowest value. And the starting point function is the lowest value at the moment.

Step 3: Random direction vector

Step 4: Compare function values If no point is found for a better function, return to step 2.

Step 5: Stopping the operation will get the lowest value given the minimum function value.

1.2 simulated Annealing

Analysis of current problems with mathematical methods is very important and necessary. In particular, to solve the problem of optimal value, which must solve the problem in a proper manner. Simulated Annealing, SA is an alternative solution. It is a technique to find random solutions to find the optimal value. SA is the concept of the hollowing of metal, which is the melting of metal until melted. By controlling the temperature of the metal. While the molten metal is highly energetic, it gradually decreases to the temperature at which the metal solidifies, which lowers the final state. These features make the metal strong and durable. If the temperature is dropped improperly, it will cause the metal structure to defect or defect. From the concept, SA has applied the most appropriate solution to solve the problem. SA has high efficiency in solving complicated problems, especially the ability to fall off. The specific value of the SA method at the start of the search process would allow the search to go in the wrong direction, with a probability value that helps determine the A cursory survey of the general first and then gradually, over time, make a thorough search was expected that the final results will not be based on the initial state too much.

When considering the minimum value of a target function at a certain point It is in the domain of the studied domain. The value of the objective function is determined in terms of energy $E^k = f(\vec{x}^k)$ repeat action $k$ Probability $P$ to use to decide to accept the next point or repeat action $k + 1$ here are the starting points $\vec{x}^{k+1}$ the opportunity
to accept the following points depends on the energy difference at both points $\Delta E = E^{k+1} - E^k$ and the Boltzmann distribution $P = \min\{1, e^{\frac{-\Delta E}{kT}}\}$. If $\Delta E$ a negative value is the value of the decreasing function, and thus the probability value $P$ of one point. therefore $\vec{x}^{k+1}$ will be accepted if the value $\Delta E$ is positive or means point $\vec{x}^{k+1}$ the value of the lower-order function is usually not recognized by other methods. But according to this methodology, this point is acceptable with the possibility $P$ which depends on $\Delta E$ still depends $T$ if that’s the value of $T$ high $P$ very valuable $\vec{x}^{k+1}$ more with That is, we allow the search to go in the wrong direction easily in the beginning, but if the value of $T$ low in the end of the search will be worth less $P$ acceptance point $\vec{x}^{k+1}$ With less to follow. That is, leapfrogging from good to bad is a big step, more difficult than a small step, which is a satisfactory feature. Because we believe that the status of the answer is often deep. To get out of the deep hole must be a big step, which should not happen easily. The best particular state is usually a shallow hole. This makes it easier to escape. And simulated annealing can be explained by the flow chart (Fig.2).

Step 1: set start point $\vec{x}^{k=0}$, Maximum temperature $T_{max}$, Minimum temperature $T_{min}$, set $T = T_{min}$, Random number $n$, find value $E^k = f(\vec{x}^k)$

Step 2: random point selection $\vec{x}^{k+1}$ around the point $\vec{x}^{k=0}$, find value $E^{k+1} = f(\vec{x}^{k+1})$

Step 3: consider value $E^k = f(\vec{x}^k)$ when $\Delta E = E^{k+1} - E^k$

- if $\Delta E < 0$ then point $\vec{x}^{k+1}$ accepted
- but if $\Delta E \geq 0$ then point $\vec{x}^{k+1}$ to be accepted or not depends on the probability of the simulated annealing drying $e^{\frac{-\Delta E}{kT}}$ compare with random numbers $\alpha \in (0, 1)$ when
  - if $\alpha \leq e^{\frac{-\Delta E}{kT}}$ then point $\vec{x}^{k+1}$ accepted
  - but if $\alpha \geq e^{\frac{-\Delta E}{kT}}$ then point $\vec{x}^{k+1}$ not accepted back to Step 2 and new point random

Step 4: When the value $T < T_{min}$ then stop repeating

If the value $T \geq T_{min}$ then when the random number is $n$ Will reduce the value of $T$ the set $k = k + 1$ and back to Step 2If the random number is less than $n$ set $k = k + 1$ and back to Step 2
2. AN ADAPTIVE SIMULATED ANNEALING TECHNIQUES

An adaptive simulated annealing technique, ASAT is an improved SA method. Can be applied to the solution of the optimal value. The technique of improving the simulated Annealing process will focus on improving the efficiency of finding the SA solution.

\[ P\{\min f(x), \text{s.t.} x \in \Omega\} \]

when \( P \) is optimization problem and \( \Omega \subset \mathbb{R}^n \)

\[ \Omega = \{x \in \mathbb{R}^n | x(i) \in [a_i, b_i], \ i = 1, 2, 3, ..., n\} \] and \( f \) is a real value function
ASAT is a way of imitating the cohesion of metal, a process of loop work, to find solutions in space, and then compare all of these solutions to the best possible point. Solution There are steps and techniques to improve the simplicity of the coil simulator, as described in the following diagram

Step 1: randomly answer the initial answer \(x_0\) and set the maximum temperature \(T_{max}\), minimum temperature \(T_{min}\), number of loops \(L_{max}\), set \(T = T_{max}\) when \(k = 0\) and set \(x^* = x_0\) is the best answer and \(f^* = f(x^*)\) is the best answer of the function.

Step 2: while \(T > T_{min}\)

2.1 when \(k \leq L_{max}\)

2.1.1 Set as \(z\) the new answer
when \(z(l_k) = x_k(l_k) + \alpha \times (b_{l_k} - a_{l_k}) \times N_k(0, 1)\)
when \(l_k\) is the value obtained from random \(\{1, 2, 3, ..., n\}\)
\(N_k(0, 1)\) is randomly distributed with a mean value of 0 and a variance of 1
\(a_{l_k}, b_{l_k}\) is the domain of the variable \(x_{l_k}\)
\(\alpha\) is a variable whose value decreases indefinitely when \(\alpha = \alpha \times e^{-\beta}\) and
\(\beta = 1.01\) and default of \(\alpha = 1\) if \(\alpha < 10^{-4}\) then \(\alpha = 1\)

2.1.2 Adjust the value \(z_{l_k}\) given in the domain answer domain \([a_{l_k}, b_{l_k}]\) when
\[
  z_{l_k} = \begin{cases} 
  a_{l_k} + [z_{l_k} - b_{l_k}] & \text{if } z_{l_k} > b_{l_k}, \\
  b_{l_k} + [a_{l_k} - z_{l_k}] & \text{if } z_{l_k} < a_{l_k}, \\
  z_{l_k} & \text{if } a_{l_k} \leq z_{l_k} \leq b_{l_k}
  \end{cases}
\]

2.1.3 Determine the level of energy change \(\Delta E^* = f(z) - f^*\) and
\(\Delta E = f(z) - f(x_k)\)

2.1.4 If \(\Delta E^* \leq 0\) make adjustments to the value of the variable \(x^* = z\) and
the best value function is \(f^* = f(x^*)\)

2.1.5 If \(\Delta E \leq 0\) update the value of the variable \(x_{k+1} = z\)

2.1.6 If \(\Delta E > 0\) accept the value of the modified variable with probability
\(e^{(-\frac{\Delta E}{kT})}\)

2.1.7 \(k = k + 1\)

2.2 \(L_{max} = L_{max} + L, \ k = 0\)

2.3 decrease temperature \(T\) when \(T = \delta \times T\)

Step 3: Show results \(x^*\) is the best answer and \(f^*\) is the value of the best function.
In this study, differences in methodological techniques for improving the An adaptive simulated Annealing Techniques, ASAT and simulated Annealing, SA is a new random answer $\vec{x}^{k+1}$ round the spot $\vec{x}^k$ in a way ASAT will use normal distribution $N_k(0, 1)$ by $\vec{x}^k$ the center of gravity is the center of gravity $\vec{x}^{k+1}$ there will be opportunities near the value $\vec{x}^k$ more and more, including the adjustment process $\vec{x}^{k+1}$ new in the area.

**Fig 2.** An adaptive simulated Annealing Techniques, ASAT
3. RESEARCH SCOPE

The scope of this study is the development of An Improved Simulated Annealing Techniques, ISAT for two variables Global Minimum Optimization Value Problems with two variable number. And the following 9 function tests.

1. \( f_1 : B_2 \) function
   \[ f(x_1, x_2) = x_1^2 + 2x_1^2 - 0.3\cos(3\pi x_1) - 0.4\cos(4\pi x_2 + 0.7) \]
   Input Domain: \( x_1 \in [-50, 100], x_2 \in [-50, 100] \)
   Global Minimum: \( f(x_1, x_2) = 0, (x_1, x_2) = (0, 0) \)

2. \( f_2 : \) De Joung function
   \[ f(x_1, x_2) = x_1^2 + x_2^2 \]
   Input Domain: \( x_1 \in [-2.56, 5.12], x_2 \in [-2.56, 5.12] \)
   Global Minimum: \( f(x_1, x_2) = 0, (x_1, x_2) = (0, 0) \)

3. \( f_3 : \) Sum Squares function
   \[ f_3(x) = \sum_{i=1}^{n} ix_i^2 \]
   Input Domain: \( x_i \in [-5, 10] \) for \( i = 1, 2, 3, \ldots, n \)
   Global Minimum: \( f_3^*(x) = 0, (x^*) = 0 \)

4. \( f_4 : \) Schwefel function
   \[ f_4(x) = 418.9829n + \sum_{i=1}^{n} \left[ -x_i\sin(\sqrt{|x_i|}) \right] \]
   Input Domain: \( x_1 \in [-500, 500], x_2 \in [-500, 500] \) for \( i = 1, 2, 3, \ldots, n \)
   Global Minimum: \( f_4^*(x) = 0, (x^*) = 0 \)

5. \( f_5 : \) Griwank function
   \[ f_5(x) = \sum_{i=1}^{n} \frac{x_i^2}{4000} - \prod_{i=1}^{n} \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1 \]
   Input Domain: \( x_i \in [-300, 300] \) for \( i = 1, 2, 3, \ldots, n \)
   Global Minimum: \( f_5^*(x) = 0, (x^*) = 0 \)

6. \( f_6 : \) Rastrigin function
   \[ f_6(x) = 10n + \sum_{i=1}^{n} \left[ x_i^2 - 10\cos(2\pi x_i) \right] \]
   Input Domain: \( x_i \in [-2.56, 5.12] \) for \( i = 1, 2, 3, \ldots, n \)
   Global Minimum: \( f_6^*(x) = 0, (x^*) = 0 \)
7. $f_7$: Ackley function

$$f_7(x) = -ae^{-\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}} - e^{\sqrt{\frac{1}{n}\sum_{i=1}^{n} \cos(c x_i)}} + a + \exp(1)$$

when $a = 20$, $b = 0.2$, $c = 2\pi$

Input Domain: $x_i \in [-32.768, 32.768]$ for $i = 1, 2, 3, ..., n$

Global Minimum: $f_7^*(x) = 0$, $(x^*) = 0$

8. $f_8$: Easom function

$$f_8(x_1, x_2) = -\cos(x_1)\cos(x_2)e^{-[(x_1-\pi)^2-(x_2-\pi)^2]}$$

Input Domain: $x_1 \in [-100, 100]$, $x_2 \in [-100, 100]$ for $i = 1, 2, 3, ..., n$

Global Minimum: $f(x_1, x_2) = -1$, $(x_1, x_2) = (\pi, \pi)$

9. $f_9$: Drop wave function

$$f_9(x_1, x_2) = -\frac{1 + \cos(12\sqrt{x_1^2 + x_2^2})}{0.5(x_1^2 + x_2^2) + 2}$$

Input Domain: $x_1 \in [-5.12, 5.12]$, $x_2 \in [-5.12, 5.12]$ for $i = 1, 2, 3, ..., n$

Global Minimum: $f(x_1, x_2) = -1$, $(x_1, x_2) = (0, 0)$

4. RESEARCH RESULTS

Calculations to determine the minimum value of each test function $f_1 - f_9$ with an Improved Simulated Annealing Techniques, ISAT

<table>
<thead>
<tr>
<th>Optimization method</th>
<th>Global Minimum (best)</th>
<th>Mean(best)</th>
<th>Standard deviation (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
<td>1.8967212</td>
<td>1.9868216</td>
<td>0.1097356</td>
</tr>
<tr>
<td>SA</td>
<td>0.0063094</td>
<td>0.0058904</td>
<td>0.0126835</td>
</tr>
<tr>
<td>ISAT</td>
<td><strong>0.0009596</strong></td>
<td><strong>0.0008597</strong></td>
<td><strong>0.0000392</strong></td>
</tr>
</tbody>
</table>

Table 1. Show Mean and SD values for the test function $f_1$ when $N = 2$

<table>
<thead>
<tr>
<th>Optimization method</th>
<th>Global Minimum (best)</th>
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<th>Standard deviation (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
<td>1.0000007</td>
<td>1.0000091</td>
<td>0.0000151</td>
</tr>
<tr>
<td>SA</td>
<td>0.0000024</td>
<td>0.0038179</td>
<td>0.0031183</td>
</tr>
<tr>
<td>ISAT</td>
<td><strong>0.0000006</strong></td>
<td><strong>0.0000007</strong></td>
<td><strong>3.627D-08</strong></td>
</tr>
</tbody>
</table>

Table 2. Show Mean and SD values for the test function $f_2$ when $N = 2$
Table 4.3: $f_3(x_1, x_2)$: Sum Squares function

<table>
<thead>
<tr>
<th>Optimization method</th>
<th>Global Minimum (best)</th>
<th>Mean(best)</th>
<th>Standard deviation (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
<td>1.3333335</td>
<td>1.3333803</td>
<td>0.0000558</td>
</tr>
<tr>
<td>SA</td>
<td>0.0007086</td>
<td>0.0001275</td>
<td>0.0052614</td>
</tr>
<tr>
<td><strong>ISAT</strong></td>
<td><strong>0.0000025</strong></td>
<td><strong>0.0000028</strong></td>
<td><strong>6.338D-11</strong></td>
</tr>
</tbody>
</table>

Table 3. Show Mean and SD values for the test function $f_3$ when $N = 2$

Table 4.4: $f_4(x_1, x_2)$: Schwefel function

<table>
<thead>
<tr>
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<th>Global Minimum (best)</th>
<th>Mean(best)</th>
<th>Standard deviation (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
<td>0.1266607</td>
<td>0.1666243</td>
<td>0.0716124</td>
</tr>
<tr>
<td>SA</td>
<td>0.0034356</td>
<td>0.0045082</td>
<td>0.0976315</td>
</tr>
<tr>
<td><strong>ISAT</strong></td>
<td><strong>0.0007186</strong></td>
<td><strong>0.0005495</strong></td>
<td><strong>0.0000352</strong></td>
</tr>
</tbody>
</table>

Table 4. Show Mean and SD values for the test function $f_4$ when $N = 2$

Table 4.5: $f_5(x_1, x_2)$: Griwank function

<table>
<thead>
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<th>Standard deviation (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
<td>0.0446315</td>
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<td>0.0140352</td>
</tr>
<tr>
<td>SA</td>
<td>0.0028806</td>
<td>0.0072414</td>
<td>0.1186954</td>
</tr>
<tr>
<td><strong>ISAT</strong></td>
<td><strong>0</strong></td>
<td><strong>0</strong></td>
<td><strong>0</strong></td>
</tr>
</tbody>
</table>

Table 5. Show Mean and SD values for the test function $f_5$ when $N = 2$

Table 4.6: $f_6(x_1, x_2)$: Rastrigin function

<table>
<thead>
<tr>
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<th>Mean(best)</th>
<th>Standard deviation (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
<td>1.994999</td>
<td>1.9959378</td>
<td>0.0012024</td>
</tr>
<tr>
<td>SA</td>
<td>0.0005708</td>
<td>0.0009134</td>
<td>0.0001179</td>
</tr>
<tr>
<td><strong>ISAT</strong></td>
<td><strong>0.0000618</strong></td>
<td><strong>0.0000802</strong></td>
<td><strong>0.0000013</strong></td>
</tr>
</tbody>
</table>

Table 6. Show Mean and SD values for the test function $f_6$ when $N = 2$

Table 4.7: $f_7(x_1, x_2)$: Ackley function

<table>
<thead>
<tr>
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<th>Global Minimum (best)</th>
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<th>Standard deviation (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
<td>11.683868</td>
<td>11.686457</td>
<td>0.0032835</td>
</tr>
<tr>
<td>SA</td>
<td>0.0009921</td>
<td>0.0056661</td>
<td>0.0112112</td>
</tr>
<tr>
<td><strong>ISAT</strong></td>
<td><strong>4.320D-18</strong></td>
<td><strong>0</strong></td>
<td><strong>7.550D-15</strong></td>
</tr>
</tbody>
</table>

Table 7. Show Mean and SD values for the test function $f_7$ when $N = 2$
Table 4.8: $f_8(x_1, x_2)$: Easom function

<table>
<thead>
<tr>
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<th>Global Minimum (best)</th>
<th>Mean(best)</th>
<th>Standard deviation (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
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<td>-0.9231956</td>
<td>0.0089786</td>
</tr>
<tr>
<td>SA</td>
<td>-0.994013</td>
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<td>0.0463695</td>
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<td>ISAT</td>
<td><strong>-0.999699</strong></td>
<td><strong>-0.9997865</strong></td>
<td><strong>0.000908</strong></td>
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</table>

Table 8. Show Mean and SD values for the test function $f_8$ when $N = 2$

Table 4.9: $f_9(x_1, x_2)$: Drop Wave function

<table>
<thead>
<tr>
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<th>Global Minimum (best)</th>
<th>Mean(best)</th>
<th>Standard deviation (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
<td>-1.5715027</td>
<td>-1.5097163</td>
<td>0.194972</td>
</tr>
<tr>
<td>SA</td>
<td>-0.9988814</td>
<td>-0.917791</td>
<td>0.0010912</td>
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<tr>
<td>ISAT</td>
<td><strong>-1</strong></td>
<td><strong>0</strong></td>
<td><strong>0</strong></td>
</tr>
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</table>

Table 9. Show Mean and SD values for the test function $f_9$ when $N = 2$

5. CONCLUSION

Based on numerical experiments with 9-function test function with An Improved Simulated Annealing Techniques (ISAT) for two variables Global Minimum Optimization Value Problems, we found that numerical results were better than those of Random Search and Simulated Annealing. We used ISAT technique in finding the initial solution of the problem. It has high efficiency in finding the solution more precisely.

REFERENCES


