Soft Sets - Motivation and Overview

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Abstract

In this paper we present some of the main developments in the soft set theory as well as in the theory of algebraic structures and soft topology as a review of literature motivated by Molodsov.

Keywords: Soft set, Fuzzy Soft set, Soft operations, Soft algebraic structures and Soft Topological Spaces.

1. INTRODUCTION

The problem of uncertainty or vagueness has been tackled for several years by philosophers, logicians, mathematicians and computer scientists. Many practical problems in engineering, economics, social science, environmental science, and medical science etc., cannot be resolved by classical methods due to these methods inherent difficulties such as inadequacy of the theories of parameterization tools.

To understand and manipulate imperfect knowledge there are many ways and the most successful approach to tackle this problem is the Fuzzy set theory. In 1965 L. A. Zadeh[102] introduced Fuzzy set theory and he described the control problems of complex systems and dealing with fuzzy system. The idea and the concept of Fuzzy set were introduced by Zadeh and he used the unit interval [0,1] to describe

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In 1982, P.Pawlak[81] proposed the rough set theory as an important mathematical tool to deal with imprecise, inconsistent, incomplete information and knowledge.

Molodtsov[56] introduced the new concept of Soft Set as a completely generic mathematical tool for modeling uncertainties. There is no limited condition to the description of objects, so researchers can choose the form of parameters they need, which greatly simplifies the decision-making process and make the process more efficient in the absence of partial information.

Ever since the soft sets came into existence, some mathematicians started imposing and studying algebraic structures on soft sets. Since any soft set involves two component sets, namely, a universal set and a parameter set, interestingly, some researchers algebrized universal set and others algebrized parameter set.

2. A BRIEF REVIEW ON SOFT SETS

In 1999 D. Molodtsov[56] introduced the Soft set theory as a general mathematical tool for dealing with uncertainty or vagueness. In his pioneering paper Soft Set Theory- First Results which was published in the journal Computers and Mathematics with Applications.

2.1 Algebraic operations on soft sets

In 2003 Maji[49,51] presented some new definitions on Soft sets and discussed the application of Soft set theory in decision making problems. They also defined the concepts of equality of two soft sets, absolute soft set and null soft set.

In 2009 Irfan-Fu[6,7,8] presented some new notions on soft set theory. In 2010 Babitha-Sunil[15] presented the concept of soft set relations and introduced as a sub soft set of the Cartesian product of the soft sets. Majumdar-Samanta[52] introduced and studied various properties of soft mappings, images and inverse images of soft mappings. Xiao[99] proposed the notion of exclusive disjunctive soft sets and studied some of its operations Gong[32] proposed the concept of bijective soft set and some of its operations and also discussed an application of bijective soft set in decision-making problem.


In 2012 Min [55] studied the concept of similarity between soft sets which is an extension of the equality for soft set theory. Singh-Onyeozili [75, 93, 94] presented the main objective and clarified some conceptual misunderstandings of the fundamentals of soft set theory and investigated some distributive and absorption properties of operations on soft sets. Ali discussed the idea of reduction of parameters in case of soft sets and studied approximation space of Pawlak associated with a soft set. Park [78] studied the equivalence soft set relations and obtained soft analogues of many results concerning ordinary equivalence relations and partitions. In 2013 Feng-Ali-Shabir [26] introduced the concepts of soft binary relations and some related properties are investigated on Soft semigroups. In 2014 Xin-Li [100] initiated the study of soft congruence relations by using the soft set theory.

2.2 Some Algebraic Structures on Soft sets

In 2007 the algebraic nature of set theories dealing with uncertainties has been studied by some authors like Aktas-Cagman. Aktas-Cagman [5] compared Soft sets to the related concepts of Fuzzy sets and Rough sets. And also defined the notion of soft groups and derived some properties. In 2009 Ali [8] studied some important properties associated with these new operations. A collection of all soft sets with respect to new operations give rise to four idempotent monoids. Sezgin-Atagun [91] introduced Soft groups and normalistic soft groups.

In 2014 Aktas-Ozlu [4] introduced order of the soft groups and some of the properties are investigated. Ali-Shabir [9] and Murthy-Maheshwari [66, 68, 69, 71] considered various soft substructures over a given semigroup and obtained some interesting properties.

In 2008 Feng [27] introduced the notions of soft semirings. In 2010 Acar [1] introduced initial concepts of soft rings. Ozturk-Inna [76] presented soft-rings and idealistic soft rings. In 2011 Celik [18] studied soft rings and ideals. Qiu-Sun [97] introduced the definition of soft modules and constructed some basic properties. Atagun-Sezgin [11] introduced soft subfields of a field and soft submodule of a left R-module and proposed their related properties. Xin-Li [100] introduced the notions of soft quotient rings, generalized soft ideals and generalized soft quotient rings and several related properties are investigated. Also, they obtained a one-to-one correspondence between soft congruence relations and idealistic soft rings and a one-to-one correspondence between soft congruence relations and soft ideals. In
particular, the first, second and third soft isomorphism theorems are established. In 2011 Sezgin-Atagun- Ayuğn[90] investigated the properties of idealistic soft near-rings with respect to the near-ring mappings and they have shown that the structure is preserved under the near-ring epimorphisms.


In 2017, Murthy-Maheswari[64] generalized the notion of soft set to that of a generalized soft set, deriving its theory from f-Set Theory of Murthy[59] in 1997, generalizing the soft (inverse) image of a soft subset under a soft map to that of a generalized soft (inverse) image of a generalized soft subset under a generalized soft map which now exist between soft sets with different universal sets.

In 2019 Murthy-Maheswari[70] proved that for any soft set over a universal set, there is a crisp set in such a way that the complete lattice of all soft subsets of the former is complete epimorphic to a complete lattice of certain subsets of the later crisp set. Further, in Murthy-Maheswari[63] and Murthy-Gouthami[72] similar results were proved for both Semigroups and Groups.

In 2001 Maji-Biswa-Roy[50] introduced the notion of fuzzy soft set by fuzzyfying the soft set and defined some basic operations on these sets. They also studied some
algebraic properties of these operations and discussed in detail an application of fuzzy soft set theory in a decision making problem. In 1991 Murali[57] introduced the notion of fuzzy subalgebras of a given algebra in the universal algebraic sense and studied some (lattice) algebraic properties of the collection of all fuzzy subalgebras of the same type as the given algebra and showed that this set forms a complete lattice. In 2009 Aygunoglu-Aygun[12] introduced the notions of fuzzy soft group, normal fuzzy soft group and studied their algebraic properties.

In 2011 Manemaran[53] defined operations on fuzzy soft groups and proved some results of them. In 2011 Ghosh-Dinda-Samanta[30] introduced the notions of fuzzy soft ring, fuzzy soft ideal and studied their algebraic properties. In 2012 Varol-Aygunoglu-Aygun[82] introduced the notion of fuzzy soft ring, fuzzy soft ideal, idealistic fuzzy soft ring and studied their algebraic properties.

2.3 Soft Topological Spaces

In 1968 Chang was the first person who studied about the topological structures of Fuzzy set theory dealing with uncertainties. He introduced the notion of Fuzzy Topology and also studied some of its basic properties.


In 2012 Mahanta-Dass[48] studied in detail about semi open soft set, semi closed soft set, semi continuity, semi connectedness, semi compactness and semi separation axioms in Soft set theory. Das-Samanta [23] introduced a notion of soft metric, and alsoestablished completeness of soft metric spaces and Cantor’s intersection theorem in soft metric space setting. Nazmul-Samanta[74]introduced several notions such as soft topological soft groups, soft topological soft normal subgroups and soft topological soft factor groups and studied their properties. Aygünoğlu-Aygün[13] defined soft product topology and studied the properties of soft projection mappings. Also they defined soft compactness and generalized Alexander subbase theorem and Tychonoff theorem to the soft topological spaces. Kannan[41] introduced soft generalized closed sets in soft topological spaces which are defined over an initial universe with a fixed set of parameters and its basic properties are investigated.

In 2013 Georgiou[29] studied the theory of soft topological spaces and presented some new definitions. Chen[20,21] introduced some local properties by soft semi-open sets, soft semi-neighborhoods of the soft point and some of their properties are studied in the paper “Some Local Properties of Soft Semi-Open Sets”.
Wardowski[98] introduced a new notion of soft element of a soft set and established its natural relation with soft operations and soft objects in soft topological spaces. Krishnaveni-Sekar[47] introduced the soft semi separation of soft sets and exhibited some properties.

In 2014 Subhashinin-Sekar[96] introduced soft pre T1 space in the soft topological spaces. Peyghan-Samadi-Tayebi[84] considered the concepts of soft compactness and countably soft compact and obtained some results. Then they studied some soft separation axioms that have studied by Min-Naz, by constructing some examples to show that some classical results in general topology are not true about soft topological spaces, for instance every compact Hausdorff space need not be normal.

In 2013 Sai-Srinivasakumar[86,87] introduced Soft semi-topology on the collection of all Soft semi-open sets over a fixed universe set. The concept of soft semi-limit is introduced and its relationship with soft semi closure is discussed. The soft separability and soft semi separability are introduced and they are shown to be equivalent and the concept of soft relative topology is introduced. In 2014 [95], they constructed a soft topology over universe set $X$ by assigning a topological structure to $X$ and probed some interesting results in this context. Assigning some structure to the universe set and verifying its consequences in the resulting soft set theory as a new approach and this area has a scope for further research. In 2016[85], they presented the two approaches of soft topology and the comparison between them. They first presented Cagman’s[17] approach of soft topology and later they discussed Shabir-Naz’s[92] theory of soft topological space. Finally they compared these two approaches. Further they presented some topological properties and introduced the concept of soft second countability.

CONCLUSION

In this paper we have presented a detailed theoretical study on soft sets. We successfully observed algebraic operations, some algebraic structures, soft topological spaces and their basic properties on soft sets which are established by so many authors. This paper stating soft mathematical concepts of structures that are based on soft set theoretic operations. Hence we can expect that some research teams will be actively working on soft algebraic and topological structures.

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