Fuzzy Wavelet Neural Network Design for Air Pollution Modeling in Mataram City, Lombok, West Nusa Tenggara

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Abstract

Air pollution is a global problem, which is a matter of concern to researchers in terms of identification and measurement, impact analysis, including the level of modeling and forecasting. In regards to modeling and forecasting air pollution problem, various techniques and methods have been developed based on analysis, physical and numerical simulations. In this study, a hybrid model was developed, which combines neural network methods, wavelets and fuzzy techniques to model air pollution problems based on the influence of meteorological and pollutant variables. The Fuzzy Wavelet Neural Network (FWNN) hybrid model developed, is a deep learning model that raises the parallel aggregate advantages of B-spline and Morlet wavelets as an activation function and Takagi-Sugeno-Kang (TSK) type fuzzy inference techniques for information aggregation processes in the Feed Forward Neural Network (FF-NN) architecture. It was used for modeling and forecasting air pollution caused by three types of pollutants, which include sulfur dioxide (SO₂), carbon monoxide (CO) and particulate matter (PM10). Based on the Root Mean Square Error (RMSE) indicator, the results showed that the numerical simulation of the
FWNN model implementation towards the three types of pollutants was quite significant. Furthermore, the model showed a better performance on the same data, compared to the Wavelet Neural Network (WNN) model developed in previous studies.

**Keywords:** fuzzy wavelet neural network, pollutant, deep learning, time series prediction, adaptive neural network and hybrid model.

1. **BACKGROUND**

The progress of science and technology, which is a sign of civilization has affected the development and growth of the industrial world. Although, it is a gift from God for humans, it also has negative impacts such as air pollution and global warming. The fundamental problem related to air pollution was shown by (Tiwary & Colls, 2010), in relation to the identification and measurement of pollutant sources, including (Huang et al., 2018) and (Lee et al., 2018), by examining the effects of the air pollution. Furthermore, several researchers, such as (Miguel et al., 2017), (Chalabi et al., 2017), (Vitolo et al., 2018) and (Gómez-Losada et al., 2018), have discussed air pollution modeling. A study related to the application of the adaptive neural network (ANN) method in the form of a standard neural network model for air pollution modeling has also been carried out by (Rynkiewicz et al., 2002), (Nunnary & Nucifora, 2002) and (Krajny et al., 2002). In addition, a study with a more complex model has been carried out by (Bahri et al., 2019), which utilized the advantages of the wavelet method at the data pre-processing stage and simultaneously using two types of wavelet functions as an activation function in the neural network learning process and obtained quite good results.

In accordance with (Bahri et al., 2019), since 1990, studies have been carried out to develop soft computing-based methods for time series analysis problems. The soft computing technique referred to include the neural network method, genetic algorithm, fuzzy technique and fuzzy inference, wavelet method and its transformation, including a combination of several soft computing methods. In terms of handling the problem of time series data analysis, soft computing techniques both partially and hybrid have been successfully used ((Mendivel et al., 2008) and (Lineesh et al., 2010)).

Partially, ANN, the wavelet method and its transformation, including fuzzy techniques have been widely used and recognized by many studies to solve time series data problems. This is because, it has several advantages, which include adaptive capabilities, learning algorithm by self, generalization and being able to solve complex nonlinear problems. However, the wavelet method and its transformation are preferred
because of their superiority in de-noising, compression and multi-resolution processes. Meanwhile, fuzzy and inferential techniques are preferred because of their ability to handle uncertain and stochastic data.

The hybrid method developed from the combination of several soft computing technique, such as ANN and wavelet, wavelet and fuzzy, ANN and fuzzy, or a combination of ANN, wavelet and fuzzy, is believed to be able to provide better results. The combination of the ANN and wavelet methods, known as the wavelet neural network (WNN) method has been used to solve various problems, which include nonlinear time series analysis (Minu et al., 2010), image processing (Amar & Jemai, 2007), WNN model design and its application for various economic data ((Bruzda, 2004) and (Bahri et al., 2016)). Furthermore, two parallel wavelet functions in the ANN model, has been used to solve air pollution modeling problems (Bahri et al., 2019).

This study is a continuation of (Bahri et al., 2019) work, which developed a hybrid model by combining ANN models, wavelet methods and fuzzy techniques to increase the model accuracy in its application to model and predict air pollution problems, influenced by meteorological factors and several types of related pollutants. Therefore, air pollution modeling was carried out using ANN as the core model. Subsequently, the role of the transformed wavelet (wavelet B-spline and wavelet Morlet) was implemented at the data pre-processing stage as an activation function, while the fuzzy inference advantage was used to aggregate information from the activation results using these two types of wavelet functions. Furthermore, the model developed was used to model and forecast air pollution problems towards three types of pollutants, which include (i) sulfur dioxide (SO$_2$), (ii) carbon monoxide (CO) and (iii) particulate matter (PM10) in Mataram City, Lombok, Indonesia.

2. RESEARCH METHODOLOGY

In this study, the hybrid national network model developed is a combination model of the neural network model, the wavelet method and the fuzzy technique. The advantages of the wavelet were implemented in the data pre-processing stage, while the B-spline wavelet and its advantage as the parallel aggregate from the B-spline and Morlet wavelets was implemented as an activation function. Furthermore, the advantages of fuzzy technique were accommodated in the feed forward neural network (FF-NN) architecture for the information aggregation process based on TSK type fuzzy inference.
2.1. Proposed Wavelet Neural Network Architecture

The implementation of the fuzzy wavelet neural network (FWNN) model proposed in this study was based on the architecture depicted in Figure 1.

\[ \tilde{x}_i = \frac{x'_i - x'_{\min}}{x'_{\max} - x'_{\min}}, \]

with \( x'_i \) represents the i-th original data, while \( x_i \) represents the results of the data normalization in \( x'_i \), \( x'_{\min} = \min \{x'_i\} \), and \( x'_{\max} = \min \{x'_i\} \). Furthermore, the data normalization results \( \tilde{x}_i \), was transformed by using Daubechies wavelet transform order 8 level 3, into data \( x_i, i = 1, 2, \ldots, n \).
At layer 3, the parallel pre-processed data were added by weight of \( w'_{ik} \) and \( w''_{ik} \) for \( i, j = 1, 2, \cdots n \) and \( k = 1, 2, \cdots, m \) where \( j \) and \( k \) represent the number of inputs and \( k \) denotes the number of input clusters:
\[
u'_{ik} = \sum_{i=1}^{m} w'_{ik} x_i \quad \text{and} \quad \nu''_{ik} = \sum_{i=1}^{m} w''_{ik} x_i.
\] (2)

Furthermore, at layer 4, \( \nu'_k \) and \( \nu''_k \) values were activated respectively, using the B-Spline wavelet order 3 (Unser, 1997) and Morlet wavelet given by Equations (3) and (4) below:
\[
z''(\nu'_k) = \frac{4b^{p+1}}{\sqrt{2\pi(p+1)}} \cos(2\pi f_0 (2\nu'_k - 1)) \exp\left(\frac{-(2\nu'_k - 1)^2}{2\sigma_w^2 (p+1)}\right),
\] (3)

\[
z''(\nu''_k) = \exp(-\nu''_k^2) \cos(5\nu''_k)
\] (4)

Where \( p \) denotes wavelet B-spline order, constant \( b = 0.657066, \quad f_0 = 0.409177 \), and \( \sigma_w^2 = 0.561145 \).

At layer 5, \( z' \) and \( z'' \) values with successive weights of \( \nu'_k \) including \( k = 1, 2, \cdots, m \) were inferred using TSK type fuzzy inference with rules:

If \( z'_{1} \in A_{11} \) and \( z'_{2} \in A_{12} \) and \( \cdots \) and \( z'_{m} \in A_{1m} \) then
\[
y_1 := \alpha_1 \times \min\{z'_k : k = 1, 2, \cdots, m\} \times \sum_{k=1}^{m} \nu'_k z'_k
\] (5)

and

If \( z''_{1} \in A_{21} \) and \( z''_{2} \in A_{22} \) and \( \cdots \) and \( z''_{m} \in A_{2m} \) then
\[
y_2 := \alpha_2 \times \min\{z''_k : k = 1, 2, \cdots, m\} \times \sum_{k=1}^{m} \nu''_k z''_k
\] (6)

for a real constant of \( \alpha_1 \) and \( \alpha_2 \).

Finally, at the sixth layer, the output of the FWNN model was obtained based on Equation (7) below.
\[
y_{FWNN} = \beta \times y_1 \times y_2 + \gamma
\] (7)

for a constant of \( \beta, \gamma \in \mathbb{R} \).
2.2. Learning Parameters of the Proposed WNN

The FWNN model developed in this study was a feed forward FWNN using the supervised training learning process type, which was carried out to minimize the following cost functions:

\[
J = \frac{1}{2NJR} \sum_{i=1}^{N} \left( y_i - y'_i \right)^2
\]  

Where \( N \) represents the number of row data, \( JR = \max_{i \leq N} \left\{ y'_i \right\} - \min_{i \leq N} \left\{ y'_i \right\} \), \( y_i \) is the output of the \( i \)-th WNN model, and \( y'_i \) is the \( i \)-th actual data (target output). The optimization of weight parameters on \( w_i \) and \( v_i \), for \( i = 1, 2, \ldots, N \) was carried out using the gradient descent with momentum (GDM) algorithm. Furthermore, updating \( w_i \) and \( v_i \), parameters for \( i = 1, 2, \ldots, N \) was carried out according to Equation (9) and (10).

\[
w_i(t+1) = m_w w_i(t) + (1 - m_c) \eta_w \frac{\partial J}{\partial w_i}, \tag{9}
\]

\[
v_i(t+1) = m_v v_i(t) + (1 - m_c) \eta_v \frac{\partial J}{\partial v_i}, \tag{10}
\]

Where \( \eta_w \) dan \( \eta_v \) represent the learning parameters for the weights \( W \), while \( V \), \( m_c \) states the momentum coefficient on GDM and the partial derivative value of \( \frac{\partial J}{\partial w'_{jk}} \), \( \frac{\partial J}{\partial w''_{jk}} \), \( \frac{\partial J}{\partial v'_{k}} \) and \( \frac{\partial J}{\partial v''_{k}} \) is given by Equations (11) - (14) below.

\[
\frac{\partial J}{\partial w'_{jk}} = \frac{\partial J}{\partial y_{FWNN}} \frac{\partial y_{FWNN}}{\partial y_i} \frac{\partial y_i}{\partial z'_k} \frac{\partial z'_k}{\partial u'_k} \frac{\partial u'_k}{\partial w'_{jk}}, \tag{11}
\]

\[
\frac{\partial J}{\partial w''_{jk}} = \frac{\partial J}{\partial y_{FWNN}} \frac{\partial y_{FWNN}}{\partial y_2} \frac{\partial y_2}{\partial z''_k} \frac{\partial z''_k}{\partial u''_k} \frac{\partial u''_k}{\partial w''_{jk}}. \tag{12}
\]

\[
\frac{\partial J}{\partial v'_{k}} = \frac{\partial J}{\partial y_{FWNN}} \frac{\partial y_{FWNN}}{\partial y_i} \frac{\partial y_i}{\partial v'_{k}}. \tag{13}
\]

\[
\frac{\partial J}{\partial v''_{k}} = \frac{\partial J}{\partial y_{FWNN}} \frac{\partial y_{FWNN}}{\partial y_2} \frac{\partial y_2}{\partial v''_{k}}. \tag{14}
\]
The selection of $\eta_\alpha$ dan $\eta_\beta$ values to ensure the convergence of the FWNN model was determined based on the (Banakar & Azeem, 2006) as follows:

$$0 \leq \eta_\rho \leq \frac{2}{\max_{\rho} \left| \frac{\partial y(t)}{\partial \eta_\rho} \right|}, \rho = w \text{ dan } v. \quad (15)$$

3. SIMULATION AND DISCUSSION

This section shows the numerical results of sulfate dioxide (SO$_2$), carbon monoxide (CO) and PM10 particles modeling using predictor variables of meteorological factors in Mataram, Lombok, West Nusa Tenggara. The meteorological factors used in this modeling consisted of daily average of rainfall data ($x_1$), wind speed ($x_2$), solar radiation ($x_5$), air temperature ($x_6$), air pressure ($x_9$), and humidity ($x_6$). Meanwhile, the air population parameters observed were daily average concentrations of sulfur dioxide-SO$_2$ ($y_1$), carbon monoxide-CO ($y_2$), and particulate matter PM10 ($y_3$).

3.1. Sulfur Dioxide (SO$_2$) Modeling

Modeling the concentration of pollutant levels on sulfur dioxide in the air is mathematically written in Equation (17) below:

$$y_1(t) = f(x_1(t-l_{p1}), x_2(t-1), x_3(t-1), x_4(t-l_{p4}), x_5(t-1), x_6(t-l_{p6}),$$

$$y_1(t-l_{r1}), y_2(t-1), y_3(t-l_{r3})) \quad (17)$$

where $l_{p1} = 1, 2, 3, 4, l_{p4} = 1, 5, 7, l_{p6} = 1, 3, l_{r1} = 1, 7$ and $l_{r3} = 1, 2, 3$.

The learning process used the FWNN model with a supervised learning type of 20,000 iterations, 295 in-sample data sharing (91.33%) and 28 out-sample data (8.67%) and the results is shown in Figure 2.
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Figure 2. Graph on the distribution pattern of sulfur dioxide (SO$_2$) data generated by the FWNN model (blue) compared to actual data (red)

From Figure 2, it is seen that the SO$_2$ pollutant graph has one outlier on the 180-th data. Furthermore, based on the autocorrelation analysis of SO$_2$ data, it is seen that the t-data was influenced by the (t-1) and (t-7) data. The facts which show the distribution of data around the 180-th data is as follows:

Figure 3. The distribution of SO$_2$ data around the outlier point, the 180-th data.

Based on Figure 3, it was assumed that there is an anomaly in the outlier data. This is because, there was a sharp jump from the previous data, especially from the 173-rd data.
and the 179-th data according to the results of the autocorrelation analysis. This is either due to a certain event on the d-day, which either triggered a surge of SO$_2$ concentrate in the air or an error might have occurred when recapping the data.

### Table 1. FWNN Performance Model proposed for sulfur dioxide modeling.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>In-sample Statistics Parameter</th>
<th>Performance (Root of MSE)</th>
<th>Out-sample Statistics Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>min</td>
<td>mean</td>
<td>max</td>
</tr>
<tr>
<td>FWNN Model</td>
<td>-2.724</td>
<td>35.7893</td>
<td>303.2332</td>
</tr>
<tr>
<td>Original Data</td>
<td>0</td>
<td>37.7182</td>
<td>567.3958</td>
</tr>
<tr>
<td>WNN* Model</td>
<td>-4.729</td>
<td>42.5346</td>
<td>168.2065</td>
</tr>
</tbody>
</table>

*) (Bahri et al., 2019)

The statistical parameters in Table 1. show that the minimum, mean and maximum data provided by the developed FWNN model was closer to the actual value (actual data) and the model's performance towards the RMSE indicator, both for in-sample and out-sample data, was better than the WNN* model.

### 3.2. Carbon Monoxide (CO) Modeling

Modeling the concentration of carbon monoxide (CO) pollutant levels in the air, is mathematically written in Equation (18) below:

$$ y_2(t) = f[x_1(t-l_{p1}), x_2(t-l_{p2}), x_3(t-1), x_4(t-l_{p4}), x_5(t-1), x_6(t-l_{p6}),$$

$$ y_2(t-l_{r1}), y_2(t-1), y_3(t-1)] $$

Where $l_{p1} = 1, 2, \cdots, 5$, $l_{p2} = 1, 2, l_{p4} = 1, 5, 7, l_{p6} = 1, 3$ and $l_{r1} = 1, 7$. 


The learning process used the FWNN model with a supervised learning type of 20,000 iterations, 233 in-sample data sharing (72.14%) and 90 out-sample data (27.86%) and the results are shown in Figure 4.

![Figure 4](image)

**Figure 4.** Graph on the distribution pattern of carbon monoxide (CO) data generated by the FWNN model (blue) compared to actual data (red)

Based on Figure 4, in the first 5 days, there it is seen that there was a significant difference between the carbon monoxide data from the developed FWNN model and the actual data (Figure 5). However, in total, the data pattern formed on the in-sample data was quite good, except for some between the 160-th data and the 215-th data which tend to be lower.

![Figure 5](image)

**Figure 5.** The distribution of CO pollutant data in the first 10 days
Table 2. Performance of the proposed FWNN model for carbon monoxide modeling

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Statistics Parameter</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>In-sample</td>
<td>out-sample</td>
</tr>
<tr>
<td></td>
<td>min</td>
<td>mean</td>
</tr>
<tr>
<td>FWNN Model</td>
<td>46.3000</td>
<td>456.8000</td>
</tr>
<tr>
<td>Original Data</td>
<td>0.0000</td>
<td>461.6000</td>
</tr>
</tbody>
</table>

*) (Bahri et al., 2019)

Based on Table 2, for in-sample data, the minimum, mean and maximum data provided by the developed FWNN model were closer to the actual data. Therefore, the data pattern formed was better compared to the WNN* model outside of the first 5 data. Conversely, based on the RMSE indicator, the FWNN model performed no better than the WNN* model. This is presumable due to the striking difference in the first 5 data, where the data produced by the FWNN model was far below the actual data, especially in the 5th data.

Conversely, on the out-sample data, towards the mean indicator, the developed FWNN model had a lower accuracy compared to the WNN* model, but based on the RMSE indicator, it was more accurate.

3.3. PM10 Particle Modeling

Modeling the concentration of PM10 particulate pollutant levels in the air is mathematically written in Equation (19) below:

\[
y_3(t) = f(x_1(t-l_{p1}), x_2(t-l_{p2}), x_3(t-1), x_4(t-l_{p4}), x_5(t-1), x_6(t-l_{p6}), y_1(t-l_{r1}), y_2(t-l_{r2}), y_3(t-1))
\]  

(19)

Where \( l_{p1} = 1, 2, \cdots, 7, \) \( l_{p2} = 1, 2, l_{p4} = 1, 5, 7, l_{p6} = 1, 3, \) \( l_{r1} = 1, 2, 3 \) and \( l_{r2} = 1, 2, 3. \)

The learning process used the FWNN model with a supervised learning type of 20,000 iterations, 275 in-sample data sharing (85.14%) and 48 out-sample data (14.86%) and the results are shown in Figure 6 below.
Figure 6. The graph of the PM10 particle data distribution pattern produced by the FWNN model (blue) compared to the actual data (red)

Table 3. The performance of the proposed FWNN model for PM10 particles modeling

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Statistics Parameter</th>
<th>Performance (Root of MSE)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>In-sample</td>
<td>out-sample</td>
</tr>
<tr>
<td></td>
<td>min</td>
<td>mean</td>
</tr>
<tr>
<td>Original Data</td>
<td>4.7708</td>
<td>20.8548</td>
</tr>
</tbody>
</table>

*) (Bahri et al., 2019)

Based on Table 3, especially for in-sample data, reviewed based on the RMSE indicator, the developed FWNN model had better accuracy compared to the WNN* model. However, for out-sample data, the WNN* model performed better compared to the developed FWNN model.

4. CONCLUSION

Air pollution modeling using the developed fuzzy wavelet neural network is stated to have better accuracy compared to the WNN* model (Bahri et al., 2019). These results support the hypothesis by (Bahri et al., 2019), which stated that the accuracy of the
WNN model is improvable by accommodating fuzzy inference in the previous model design. Furthermore, for the sulfur dioxide (SO\textsubscript{2}) data simulation, the accuracy of the developed FWNN model was better compared to the WNN* model based on the RMSE indicator, both for in-sample and out-sample data.

In addition, based on the simulation on carbon monoxide (CO) data, with the RMSE indicator, the FWNN model developed provided better accuracy compared to the WNN* model for out-sample data. However, the data distribution pattern of the developed FWNN model was better compared to WNN* model except for the first 5 data. This was also supported by the minimum, mean and maximum data indicators generated by the developed FWNN model, which were more accurate than the WNN* model.

Lastly, for the PM10 particulate data simulation, based on the RMSE indicator, the developed FWNN model provided better accuracy compared to the WNN* model especially for in-sample data. Therefore, it was concluded that the developed FWNN model is more appropriate for studying the characteristics of the model than a forecasting model.

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