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Midrange Control Chart under Non Normality

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Abstract

Control charts monitor ongoing manufacturing processes and are thus helpful in maintaining quality of the products. In this paper, a Shewhart type $\widetilde{M}$ control chart based on midrange for process location parameter is proposed. The performance of the newly developed control chart is investigated for some symmetric distributions including normal distribution and is evaluated in terms of its power, average run length (ARL) and standard deviation of run length (SDRL). The control chart is compared with its competitors and is illustrated through example.

Keywords: ARL, Control chart, Location parameter, Logistic distribution, Midrange, Power.

Mathematics subject classification: 62P30, 62G32

1. Introduction

A control chart is a statistical tool used in quality control and process management to monitor an ongoing process over time. It is widely adopted technique in various industries. The primary purpose of a control chart is to detect and highlight any variations or abnormalities in a process that could indicate the presence of special causes or changes. To maintain the quality of products during manufacturing, monitoring process parameters such as process location or process variation is essential. As process location represents central tendency, it is desirable to develop control charts to process average. Under non normality, considering process median as location parameter is advantageous. For instance, in manufacturing industries like semiconductor fabrication and pharmaceutical production, variables such as yield rates and drug potency respectively, may not adhere to a normal distribution due to complexities in processes and materials.

Shewhart (1931) pioneered in statistical process control by introducing $\bar{X}$ control chart for process average. This control chart is based on the assumption of normal distribution and its three sigma limits. Later, under the assumption of normality, a number of control charts were developed to monitor the process average or process location. Under non normality, Amin et al. (1995) developed Shewhart and CUSUM (cumulative sum) control charts to process median based on sign statistic and discussed on the effect of non-normality on Shewhart's $\bar{X}$ control chart. Montgomery (1996) studied in detail about various control charts for process variables, process attributes and various performance measures of control charts. Woodal and Montgomery (1999) discussed on various control charting methods. Chakraborti et al. (2001) gave an overview of nonparametric control charts for process improvement. Bakir (2004) developed control chart based on Wilcoxon signed-rank statistic. Zombade (2015) and Zombade and Ghute (2019) carried out a detailed study on nonparametric control charts for location based on sign statistic and run statistic (NP-S and NP-R control charts). They compared these control charts with Shewhart's $\bar{X}$ control chart. Pawar et al. (2018) discussed nonparametric moving average control charts using sign and signed rank statistics. Chakraborti and Graham (2019) emphasized the importance of nonparametric control charts over parametric control charts and discussed Shewhart, CUSUM, EWMA (exponentially weighted moving average) control charts under nonparametric setup.

Midrange being one of the intrinsic measures of location, is useful in developing control charts to process location. We propose Shewhart type control chart based on midrange to monitor process median under non normality. The rationale behind developing such a control chart is to make use of the sample midrange which is an average of only two extreme order statistics. Gumbel (1944) carried out elaborative discussion on the distribution of ranges and midranges for symmetric distributions. Cramer (1946) discussed the sampling distributions of extremes, range and midrange. Gumbel (1958) emphasized on the distributional properties of extremes. Kendall and Stuart (1969) presented the asymptotic variance of the midrange for few symmetric distributions. Sundheim (1974) gave a detailed discussion on consistency and efficiency of midrange estimator for symmetric distributions. Broffitt (1974) discussed about the order of sample size for different distributions of midrange. George and Rousseau (1987) carried out a detailed study on midrange of logistic distribution including its properties and applications.

In section 2, we discuss midrange and its significance in SPC (statistical process control). Section 3 describes proposed control chart under various distributions. Section 4 deals with performance of the control chart. In section 5, we illustrate the control chart and record our conclusions in section 6.

2. Midrange and its significance

Midrange, $\widetilde{M}$ is defined as the arithmetic mean of maximum and minimum values of the dataset. As computation of midrange involves only two extreme observations of the distribution, it is a quick measure of the central value of the distribution and always exists. It is a sensitive measure and is suitable for the dataset without outliers.

Suppose $X_1, X_2, X_3, \dots, X_n$ be n independent and identically distributed continuous random variables from the symmetric distribution $F(x)$. Let $X_{(1)}, X_{(2)}, \dots, X_{(i)}, \dots X_{(n)}$ be the sequence sorted in an ascending order, with $X_{(i)}$ being i^{th}order statistic. Then midrange is given by

$$\widetilde{M} = \frac{1}{2}\left(X_{(1)} + X_{(n)}\right). \tag{1}$$

The probability density function (pdf) of the midrange due to Arnold et al. (2008) is given by

$$f_{\widetilde{M}}(x) = 2n(n-1)\int_{-\infty}^{x}\{F(2x-y)\}^{n-2} f(y) f(2x-y)dy \quad -\infty < x < \infty, \tag{2}$$

where $X_{(1)}$ is realized by y.

Gumbel (1944) showed that, when random sample is taken from a symmetric distribution, the distribution of the $X_{(1)} + X_{(n)}$ follows logistic distribution given by

$$f_{\widetilde{M}}(x) = \frac{\alpha e^{-\alpha x}}{(1+e^{-\alpha x})^2} \tag{3}$$

where α is reciprocal of the scale parameter and $\alpha = n\xi$, $\xi = f\left(x_{(n)}\right)$ is the initial distribution at $x_{(n)}$, the maximum value. Cramer (1946) gave the expressions for asymptotic variance of midrange for some symmetric distributions with location parameter μ and scale parameter σ. Kendall and Stuart (1958) discuss about general approach to evaluate the variance of midrange for some symmetric standard distributions with zero location and unit scale. Treating extreme order statistics as independent for large n, they obtained the variance of midrange given by

$$Var(\widetilde{M}) = \frac{1}{12}\left(\frac{\pi}{n\xi}\right)^2. \tag{4}$$

3. $\widetilde{M}$ Control chart for process location

In this section, we develop $\widetilde{M}$ control chart when observations are from various distributions. The scale of the distributions under consideration other than Cauchy distribution are modified in such a way that its mean is μ and variance is λ^2. From Cramer (1946), we observe that $E(\widetilde{M}) = \mu$ and obtain variances of $\widetilde{M}$ under symmetric distributions with variance λ^2. The pdf and standard deviation, $sd(\widetilde{M})$ given by $\sigma_{\widetilde{M}}$ are furnished in exhibit 1. Here U, N, LG, L and C stand respectively for uniform, normal, logistic, Laplace and Cauchy distributions.

Exhibit1: pdf and $\sigma_{\widetilde{M}}$ of various distributions

Sl. no	Distribution	Pdf	$\sigma_{\widetilde{M}}$
1	$U(\mu - \sqrt{3}\lambda, \mu + \sqrt{3}\lambda)$	$f(x) = \frac{1}{2\sqrt{3}\lambda}$, $\mu - \sqrt{3}\lambda < x < \mu + \sqrt{3}\lambda$, $-\infty < \mu < \infty$, $\lambda > 0$	$\frac{\sqrt{6}\lambda}{\sqrt{(n+1)(n+2)}}$
2	$N(\mu, \lambda^2)$	$f(x) = \frac{1}{\lambda\sqrt{2\pi}} e^{-\frac{1}{2\lambda^2}(x-\mu)^2}$, $-\infty < x, \mu < \infty$, $\lambda > 0$	$\frac{\lambda\pi}{2\sqrt{6\log(n)}}$
3	$LG\left(\mu, \frac{\sqrt{3}\lambda}{\pi}\right)$	$f(x) = \frac{\pi e^{-\frac{\pi(x-\mu)}{\sqrt{3}\lambda}}}{\sqrt{3}\lambda\left[1+e^{-\frac{\pi(x-\mu)}{\sqrt{3}\lambda}}\right]^2}$, $-\infty < x, \mu < \infty$, $\lambda > 0$	$\frac{\lambda n}{2(n-1)}$
4	$L\left(\mu, \frac{\lambda}{\sqrt{2}}\right)$	$f(x) = \frac{1}{\sqrt{2}\lambda} e^{-\frac{\sqrt{2}\lvert x-\mu\rvert}{\lambda}}$, $-\infty < x, \mu < \infty$, $\lambda > 0$	$\frac{\lambda\pi}{2\sqrt{6}}$
5	$C(\mu, \lambda)$	$f(x) = \frac{1}{\lambda\pi\left[1+\left(\frac{x-\mu}{\lambda}\right)^2\right]}$, $-\infty < x, \mu < \infty$, $\lambda > 0$	$\frac{\lambda n}{2\sqrt{2}\pi}$

The control limits of $\widetilde{M}$ control chart is given by

$$UCL_{\widetilde{M}} = E(\widetilde{M}) + 3\sigma_{\widetilde{M}} \tag{5}$$

$$CL_{\widetilde{M}} = E(\widetilde{M}) \tag{6}$$

$$LCL_{\widetilde{M}} = E(\widetilde{M}) - 3\sigma_{\widetilde{M}} \tag{7}$$

where $UCL_{\widetilde{M}}$ is upper control limit, $CL_{\widetilde{M}}$ is center line and $LCL_{\widetilde{M}}$ is lower control limit of the $\widetilde{M}$ control chart. The $\widetilde{M}$ control chart under various distributions is given by substituting for $E(\widetilde{M})$ and $\sigma_{\widetilde{M}}$ in(5), (6) and (7) from Exhibit 1. The width, $w_{\widetilde{M}}$ of the control chart is

$$w_{\widetilde{M}} = UCL_{\widetilde{M}} - LCL_{\widetilde{M}} = 6\sigma_{\widetilde{M}}. \tag{8}$$

4. Performance of $\widetilde{M}$ control chart

In this section, we will examine the performance of the proposed $\widetilde{M}$ control chart under different distributions in terms of power, ARL and SDRL. The operating characteristic (OC) function of $\widetilde{M}$ control chart is presented in terms of OC curves. The ability of the control chart to identify changes in the process quality characteristic is explained by the OC function. For the distributions under discussion, λ is taken to be known and constant.

$$\beta_{\widetilde{M}} = p\left(LCL_{\widetilde{M}} \leq \widetilde{M} \leq UCL_{\widetilde{M}} \middle| \mu'\right) \tag{9}$$

which is OC function gives the β - risk or the chance of not detecting the shift a for the first subsequent sample, if the process location shifts from μ to $\mu' = \mu + k\lambda$.

Since $\widetilde{M} \sim LG(\mu, 2\alpha)$ for symmetric distributions,

$$\beta_{\widetilde{M}} = F_{LG}\left(\frac{UCL_{\widetilde{M}} - a}{\sigma_{\widetilde{M}}}\right) - F_{LG}\left(\frac{LCL_{\widetilde{M}} - a}{\sigma_{\widetilde{M}}}\right) \tag{10}$$

where $F_{LG}(.)$ denotes the cumulative distribution of standard logistic variate.

The power of the $\widetilde{M}$ control chart is given by

$$P_{\widetilde{M}} = 1 - \beta_{\widetilde{M}} . \tag{11}$$

The expected number of samples required by the $\widetilde{M}$ control chart to signal that shift a in μ has occurred is given by

$$ARL_{\widetilde{M}} = \frac{1}{P_{\widetilde{M}}} . \tag{12}$$

The performance of the control chart is said to be better if ARL is small when there is a shift and is high for no shift in the process parameter. The $SDRL$ which measures dispersion of the run length distribution of $\widetilde{M}$ control chart is given by

$$SDRL_{\widetilde{M}} = \frac{\sqrt{\beta_{\widetilde{M}}}}{1 - \beta_{\widetilde{M}}} . \tag{13}$$

The values of $\beta_{\widetilde{M}}$, $P_{\widetilde{M}}$, $ARL_{\widetilde{M}}$ and $SDRL_{\widetilde{M}}$ for distributions under consideration except Cauchy distribution are computed by setting variance $\lambda^2 = 1$. For Cauchy distribution, the value of λ due to Arnold (1965) is taken as 0.2605, so that the distribution is scaled to give probability 0.05 above $\theta + 1.645$. The values of $\beta_{\widetilde{M}}$, $P_{\widetilde{M}}$, $ARL_{\widetilde{M}}$ and $SDRL_{\widetilde{M}}$ are presented respectively in Table 1, 2, 3 and 4 for various positive shifts in appendix. Figures 1, 2 and 3 are plotted respectively using Tables 1, 2 and 3.

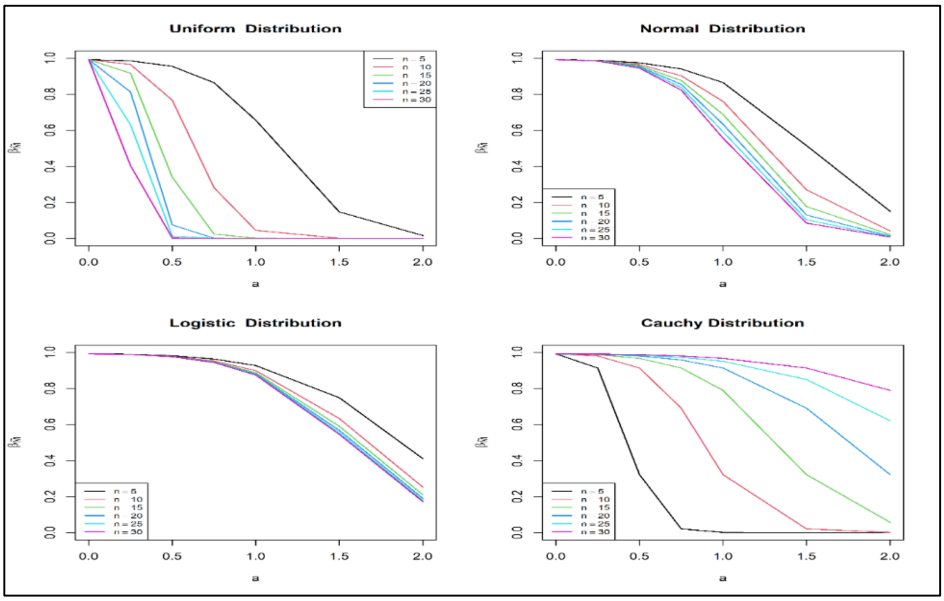

Figure 1: OC curves of $\widetilde{M}$ control chart for various distributions

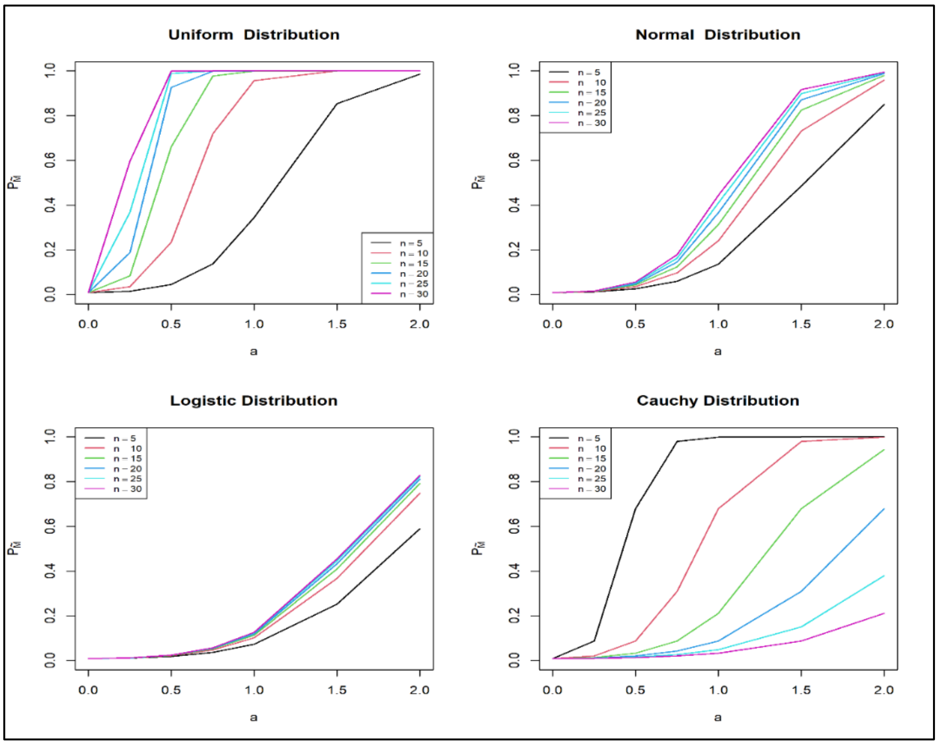

Figure 2: $P_{\widetilde{M}}$ for various distributions

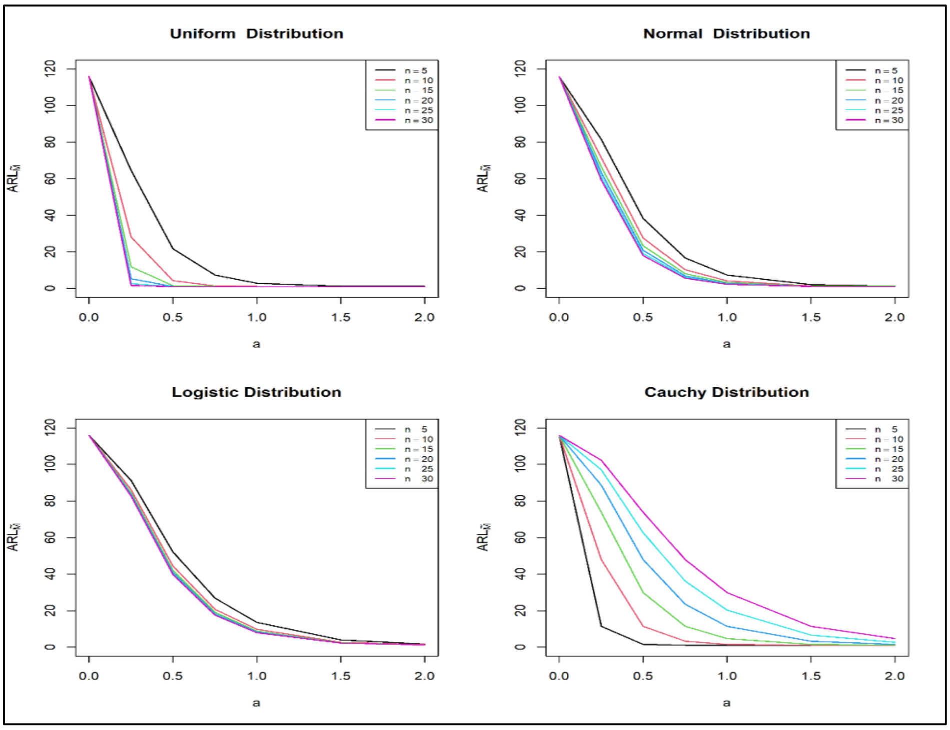

Figure 3: $ARL_{\widetilde{M}}$ for various distributions

From Figure 1 and Table 1, we observe that, for a specified shift a, as n increases, $\beta_{\widetilde{M}}$ decreases for uniform, normal and logistic distribution, whereas it increases for Cauchy distribution. For fixed n, $\beta_{\widetilde{M}}$ is decreasing for increasing a for all the distributions under consideration. For $n = 5$, $\beta_{\widetilde{M}}$ is minimum for Cauchy distribution for all the values of a. From Figure 2 and Table 2, we see that, for a specified n, $P_{\widetilde{M}}$ increases as a increases for all the distributions, $P_{\widetilde{M}}$ is high for Cauchy distribution when $n = 5$ and is high for uniform distribution when $n \geq 10$.

From Figure 3 and Table 3, we observe that, the ARL $_{\widetilde{M}}$ is approximately 116 for all the distributions under consideration when there is no shift. For fixed a, as n increases, ARL$_{\widetilde{M}}$ decreases for uniform, normal and logistic distributions and increases for Cauchy distribution. Among the distributions considered, ARL$_{\widetilde{M}}$ is minimum for Cauchy distribution when $n = 5$ and is minimum for uniform distribution when $n \geq 10$. Table 4 reveals that, for specified n and increasing a, $SDRL_{\widetilde{M}}$ decreases. As the performance measures viz., $\beta_{\widetilde{M}}$, $P_{\widetilde{M}}$, $ARL_{\widetilde{M}}$ and $SDRL_{\widetilde{M}}$ are independent of sample size n for Laplace distribution, from table 5, we see that, as a increases, $\beta_{\widetilde{M}}$, $ARL_{\widetilde{M}}$, $SDRL_{\widetilde{M}}$ decrease and $P_{\widetilde{M}}$ increases.

From Tables 1, 2, 3, 4 and 5, we note that, for $a = 0.5$, $\beta_{\widetilde{M}}$ for Laplace distribution is larger than that of $\beta_{\widetilde{M}}$ for uniform, normal, Cauchy distributions and smaller than that of logistic distribution for $n = 5$.

On comparing the $\widetilde{M}$ control chart with respect to $\bar{X}$, NP-S, NP-R control charts in terms of ARL, for $n = 5$, we note that, ARL$_{\widetilde{M}}$ is higher than ARL of $\bar{X}$, NP-S, NP-R control charts for $a = 0$ under uniform, normal and Laplace distributions. Also, ARL$_{\widetilde{M}}$ is lower than ARL of $\bar{X}$, NP-S, NP-R control charts under Laplace distribution for $a \geq 0.4$.

5. Illustration

In this section, we illustrate the $\widetilde{M}$ control chart with an example given in Alloway and Raghavachari (1991). The example deals with primer thickness data and is given in Exhibit 2. The control limits and $w_{\widetilde{M}}$ of the $\widetilde{M}$ control chart are computed. The proposed control chart is plotted in Figures 4 and 5.

Since, μ and $\sigma_{\widetilde{M}}$ are not known, they are estimated from the samples. Suppose n samples are taken m times, μ is estimated by taking the average of midrange of these samples. That is, $\hat{\mu}_{\widetilde{M}} = \frac{1}{m}\sum_{i=1}^{m} \widetilde{M}_i$, where $\widetilde{M}_i$ is midrange of i^{th} sample. And $\sigma_{\widetilde{M}}$ is estimated by obtaining $\hat{\sigma}_{\widetilde{M}}$ under various distributions by substituting $\delta\hat{\lambda}$ where $\delta = \sqrt{\frac{n-1}{2}}\left(\frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}\right)$ for λ. Here $\hat{\lambda} = \frac{1}{m}\sum_{i=1}^{m} s_i$, where $s_i^2 = \frac{1}{(n-1)}\sum_{i=1}^{n}(x_i - \bar{x})^2$.

Also, $\hat{\lambda}$ can be computed using sample midrange as the central value given by $\hat{\lambda}' = \frac{1}{m}\sum_{i=1}^{m} t_i$, where $t_i^2 = \frac{1}{(n-1)}\sum_{i=1}^{n}(x_i - \widetilde{M}_i)^2$.

Example: The example due to Alloway and Raghavachari (1991) deals with the measurement of primer thickness (Mils) of Ford Motor Company. This data has 10 samples taken 20 times.

Exhibit 2

n / m	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	$\widetilde{M}_i$	s_i^2	s_i	t_i^2	t_i
1	1.3	1.1	1.2	1.25	1.05	0.95	1.1	1.16	1.37	0.98	1.16	0.0186	0.1363	0.0188	0.1371
2	1.01	1.1	1.15	0.97	1.25	1.12	1.1	0.9	1.04	1.08	1.075	0.0096	0.0981	0.0096	0.0981
3	1.22	1.05	0.93	1.08	1.15	1.27	0.95	1.11	1.12	1.1	1.1	0.0112	0.1057	0.0112	0.1057
4	1.08	1.12	1.11	1.28	1	0.95	1.15	1.14	1.12	1.31	1.13	0.012	0.1094	0.0120	0.1094
5	0.98	1.3	1.31	1.12	1.08	1.1	1.15	1.35	1.12	1.26	1.165	0.0146	0.1207	0.0147	0.1213
6	1.12	1.3	1.01	1.2	1.11	0.93	1.02	1.25	1.05	1.1	1.115	0.0131	0.1145	0.0132	0.1147
7	0.92	1.1	1.13	1.02	0.93	1.17	1.24	0.98	1.34	1.12	1.13	0.0184	0.1355	0.0197	0.1404
8	1.04	1.14	1.18	1.12	1	1.02	1.05	1.34	1.12	1.05	1.17	0.0101	0.1006	0.0147	0.1211
9	1.08	0.92	1.14	1.2	1.02	1.04	0.94	1.05	1.12	1.06	1.06	0.0073	0.0856	0.0073	0.0857
10	1.2	1.13	1.19	1.16	1.03	1.25	1.2	1.24	1.1	1.03	1.14	0.0063	0.0792	0.0065	0.0803
11	1.25	0.91	0.96	1.04	0.93	1.08	1.29	1.42	1.1	1	1.165	0.0288	0.1698	0.0338	0.1839
12	1.24	1.34	1.4	1.26	1.13	1.15	1.08	1.02	1.05	1.18	1.21	0.0155	0.1246	0.0162	0.1273
13	1.13	1.16	1.12	1.22	1.12	1.07	1.04	1.28	1.12	1.1	1.16	0.0049	0.07	0.0055	0.0744
14	1.08	1.31	1.12	1.18	1.15	1.17	0.98	1.05	1	1.26	1.145	0.0114	0.1066	0.0116	0.1077
15	1.08	1.26	1.13	0.94	1.3	1.15	1.07	1.02	1.22	1.18	1.12	0.0123	0.111	0.0126	0.1121
16	1.14	1.02	1.14	0.94	1.3	1.08	0.94	1.12	1.15	1.36	1.15	0.0187	0.1367	0.0197	0.1405
17	1.06	1.12	0.98	1.12	1.2	1.02	1.19	1.03	1.02	1.09	1.09	0.0055	0.0744	0.0056	0.0748
18	1.14	1.22	1.18	1.27	1.17	1.26	1.15	1.07	1.02	1.36	1.19	0.0098	0.0992	0.0099	0.0994
19	1.07	1.05	0.97	1.05	1.16	1.02	1.02	1.14	1.07	1	1.065	0.0035	0.0591	0.0036	0.0600
20	1.13	0.9	1.12	1.04	1.4	1.12	1.15	1.01	1.3	1.14	1.15	0.0198	0.1406	0.0202	0.1420

Here, $n = 10$, $\hat{\mu}_{\widetilde{M}} = \frac{1}{n}\sum_{i=i}^{n} \widetilde{M} = 1.1345$, $\hat{\lambda} = \frac{1}{20}\sum_{i=1}^{20} s_i = 0.1089$, $\delta = 1.0281$,

$\hat{\lambda}' = \frac{1}{20}\sum_{i=1}^{20} t_i = 0.1118$. We compute $\hat{\sigma}_{\widetilde{M}}$, control limits and $w_{\widetilde{M}}$ of $\widetilde{M}$ control chart for various distributions under consideration in Exhibit 3.

Exhibit 3

$\hat{\sigma}_{\widetilde{M}}$ obtained by substituting $\delta\hat{\lambda}$					
Distributions	$\hat{\sigma}_{\widetilde{M}}$	$UCL_{\widetilde{M}}$	$CL_{\widetilde{M}}$	$LCL_{\widetilde{M}}$	$W_{\widetilde{M}}$
Uniform	0.0239	1.2061	1.1345	1.0629	0.1432
Normal	0.0473	1.2765	1.1345	0.9926	0.2839
Logistic	0.0622	1.3211	1.1345	0.9479	0.3732
Laplace	0.0718	1.3499	1.1345	0.9191	0.4308
Cauchy	0.0328	1.2330	1.1345	1.0360	0.1969
$\hat{\sigma}_{\widetilde{M}}$ obtained by substituting $\delta\hat{\lambda}'$					
Distributions	$\hat{\sigma}_{\widetilde{M}}$	$UCL_{\widetilde{M}}$	$CL_{\widetilde{M}}$	$LCL_{\widetilde{M}}$	$W_{\widetilde{M}}$
Uniform	0.0245	1.2080	1.1345	1.0610	0.1470
Normal	0.0486	1.2802	1.1345	0.9888	0.2915
Logistic	0.0639	1.3261	1.1345	0.9429	0.3831
Laplace	0.0737	1.3556	1.1345	0.9134	0.4423
Cauchy	0.0337	1.2356	1.1345	1.0334	0.2022

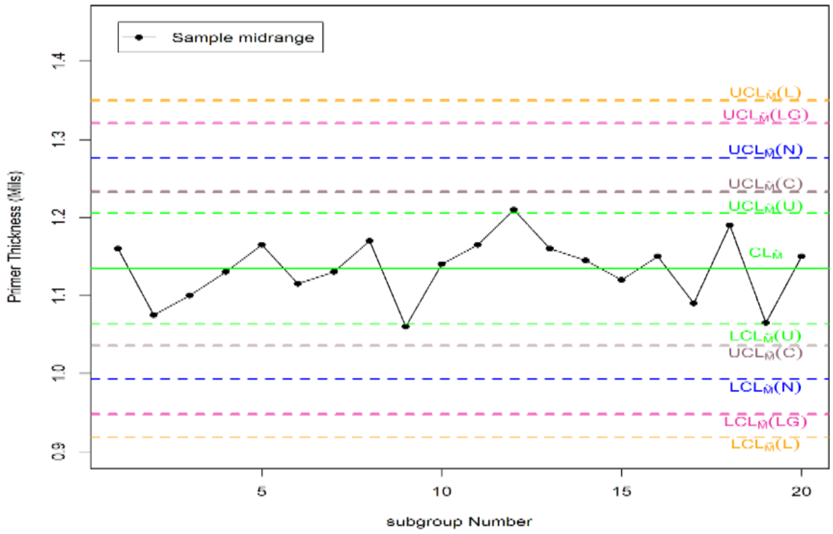

Figure 4: $\tilde{M}$ Control chart under various distributions for exhibit 2 using $\hat{\lambda}$

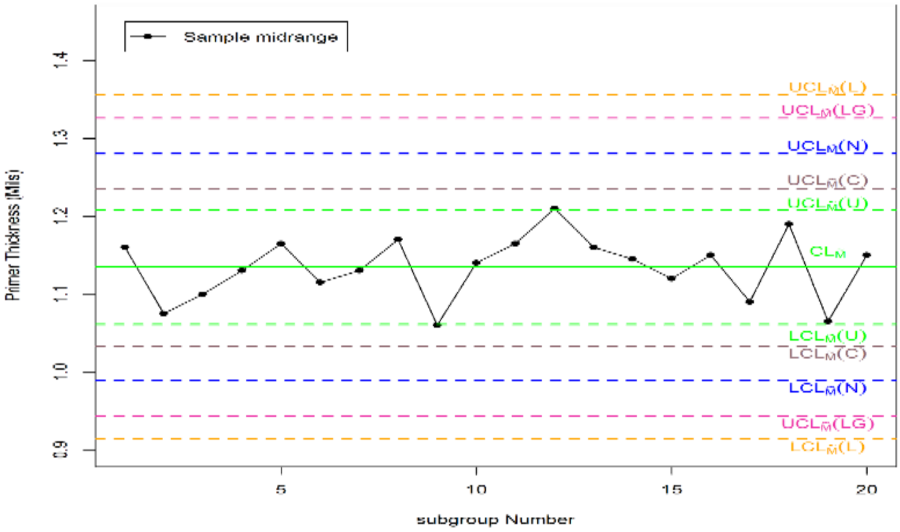

Figure 5: $\tilde{M}$ Control chart under various distributions for exhibit 2 using $\hat{\lambda}'$

From Exhibit 3, Figures 4 and 5, we observe that, $w_{\tilde{M}}$ obtained from $\hat{\lambda}$ and $\hat{\lambda}'$ are wider for Laplace distribution and narrower for uniform distribution. Also, $w_{\tilde{M}}$ based on $\hat{\lambda}'$ is marginally increasing when compared to $w_{\tilde{M}}$ based on $\hat{\lambda}$ for all the distributions.

CONCLUSIONS

In this section, we provide conclusions on proposed $\tilde{M}$ control chart based on our observations.

- A control chart based on midrange, $\widetilde{M}$ is developed for the process median for symmetric distributions including normal distribution.
- When random samples are from symmetric distribution the distribution of $\widetilde{M}$ follows logistic distribution with reciprocal of the scale parameter being 2α.
- For a specified shift and increasing n, the OC function $\beta_{\widetilde{M}}$ increases for Cauchy distribution, remains constant for Laplace distribution and decreases for normal, logistic and uniform distributions.
- For a specified shift and increasing sample, power of $\widetilde{M}$ control chart increases for uniform, normal and logistic distributions, remains constant for Laplace distribution and decreases for Cauchy distribution.
- When there is no shift in process location parameter, the ARL of the proposed control chart, $\text{ARL}_{\widetilde{M}}$ is approximately 116 for all the distributions.
- $\widetilde{M}$ control chart performs better when random samples are from Cauchy distribution as its ARL is lesser for smaller shifts and smaller sample sizes when compared to other distributions.
- $\widetilde{M}$ control chart performs better than its competitors given in the literature for smaller sample size under Laplace distribution.
- The width of the $\widetilde{M}$ control chart becomes wider for heavier tailed distributions, with Cauchy distribution being an exception.
- The control chart with $\hat{\sigma}_{\widetilde{M}}$ computed using deviation from $\widetilde{M}$ results in wider width than $\hat{\sigma}_{\widetilde{M}}$ computed using deviation from $\bar{x}$.
- Midrange control chart is useful when the data is free from outliers or is available only in terms of maximum and minimum values.

Appendix

Table 1*: $\beta_{\widetilde{M}}$ of proposed control chart under various distributions*

Distribution	n / a	05	10	15	20	25	30
Uniform	0.00	0.9914	0.9914	0.9914	0.9914	0.9914	0.9914
	0.25	0.9845	0.9644	0.9157	0.8118	0.6310	0.4039
	0.50	0.9540	0.7663	0.3397	0.0747	0.0125	0.0020
	0.75	0.8631	0.2811	0.0237	0.0015	0.0001	0
	1.00	0.6553	0.0445	0.0011	0	0	0
	1.50	0.1472	0.0007	0	0	0	0
	2.00	0.0154	0	0	0	0	0
Normal	0.00	0.9914	0.9914	0.9914	0.9914	0.9914	0.9914
	0.25	0.9877	0.9860	0.9850	0.9842	0.9836	0.9831
	0.50	0.9739	0.9638	0.9570	0.9519	0.9477	0.9441
	0.75	0.9396	0.9021	0.8754	0.8543	0.8368	0.8218
	1.00	0.8644	0.7594	0.6871	0.6332	0.5907	0.5560
	1.50	0.5147	0.2696	0.1765	0.1299	0.1024	0.0845
	2.00	0.1499	0.0414	0.0205	0.0127	0.0089	0.0068

Logistic	0.00	0.9914	0.9914	0.9914	0.9914	0.9914	0.9914
	0.25	0.9890	0.9884	0.9881	0.9880	0.9879	0.9879
	0.50	0.9808	0.9775	0.9762	0.9755	0.9751	0.9748
	0.75	0.9627	0.9519	0.9476	0.9453	0.9439	0.9430
	1.00	0.9266	0.8980	0.8864	0.8802	0.8763	0.8736
	1.50	0.7480	0.6327	0.5897	0.5676	0.5542	0.5452
	2.00	0.4103	0.2519	0.2091	0.1898	0.1789	0.1719
Cauchy	0.00	0.9914	0.9914	0.9914	0.9914	0.9914	0.9914
	0.25	0.9126	0.9791	0.9865	0.9887	0.9897	0.9902
	0.50	0.3218	0.9126	0.9665	0.9791	0.9840	0.9865
	0.75	0.0211	0.6902	0.9126	0.9573	0.9723	0.9791
	1.00	0.0010	0.3218	0.7886	0.9126	0.9507	0.9665
	1.50	0	0.0211	0.3218	0.6902	0.8492	0.9126
	2.00	0	0.0010	0.0569	0.3218	0.6205	0.7886

Table 2: $P_{\tilde{M}}$ *of proposed control chart under various distributions*

Distribution	n / a	05	10	15	20	25	30
Uniform	0.00	0.0086	0.0086	0.0086	0.0086	0.0086	0.0086
	0.25	0.0155	0.0356	0.0843	0.1882	0.3690	0.5961
	0.50	0.0460	0.2337	0.6603	0.9253	0.9875	0.9980
	0.75	0.1369	0.7189	0.9763	0.9985	0.9999	1.0000
	1.00	0.3447	0.9555	0.9989	1.0000	1.0000	1.0000
	1.50	0.8528	0.9993	1.0000	1.0000	1.0000	1.0000
	2.00	0.9846	1.0000	1.0000	1.0000	1.0000	1.0000
Normal	0.00	0.0086	0.0086	0.0086	0.0086	0.0086	0.0086
	0.25	0.0123	0.0140	0.0150	0.0158	0.0164	0.0169
	0.50	0.0261	0.0362	0.0430	0.0481	0.0523	0.0559
	0.75	0.0604	0.0979	0.1246	0.1457	0.1632	0.1782
	1.00	0.1356	0.2406	0.3129	0.3668	0.4093	0.4440
	1.50	0.4853	0.7304	0.8235	0.8701	0.8976	0.9155
	2.00	0.8501	0.9586	0.9795	0.9873	0.9911	0.9932
Logistic	0.00	0.0086	0.0086	0.0086	0.0086	0.0086	0.0086
	0.25	0.0110	0.0116	0.0119	0.0120	0.0121	0.0121
	0.50	0.0192	0.0225	0.0238	0.0245	0.0249	0.0252
	0.75	0.0373	0.0481	0.0524	0.0547	0.0561	0.0570
	1.00	0.0734	0.1020	0.1136	0.1198	0.1237	0.1264
	1.50	0.2520	0.3673	0.4103	0.4324	0.4458	0.4548
	2.00	0.5897	0.7481	0.7909	0.8102	0.8211	0.8281
Cauchy	0.00	0.0086	0.0086	0.0086	0.0086	0.0086	0.0086
	0.25	0.0874	0.0209	0.0135	0.0113	0.0103	0.0098
	0.50	0.6782	0.0874	0.0335	0.0209	0.0160	0.0135
	0.75	0.9789	0.3098	0.0874	0.0427	0.0277	0.0209
	1.00	0.9990	0.6782	0.2114	0.0874	0.0493	0.0335
	1.50	1.0000	0.9789	0.6782	0.3098	0.1508	0.0874
	2.00	1.0000	0.9990	0.9431	0.6782	0.3795	0.2114

Table 3: $ARL_{\tilde{M}}$ of proposed control chart under various distributions

Distribution	n / a	05	10	15	20	25	30
Uniform	0.00	115.8823	115.8823	115.8823	115.8823	115.8823	115.8823
	0.25	64.5857	28.0959	11.8663	5.3140	2.7099	1.6776
	0.50	21.7586	4.2783	1.5144	1.0807	1.0127	1.0020
	0.75	7.3044	1.3909	1.0243	1.0015	1.0001	1.0000
	1.00	2.9010	1.0466	1.0011	1.0000	1.0000	1.0000
	1.50	1.1726	1.0007	1.0000	1.0000	1.0000	1.0000
	2.00	1.0157	1.0000	1.0000	1.0000	1.0000	1.0000
Normal	0.00	115.8823	115.8823	115.8823	115.8823	115.8823	115.8823
	0.25	81.4374	71.4667	66.5096	63.3156	61.0019	59.2088
	0.50	38.2847	27.5978	23.2828	20.7951	19.1232	17.8979
	0.75	16.5650	10.2118	8.0238	6.8644	6.1283	5.6117
	1.00	7.3734	4.1555	3.1961	2.7259	2.4430	2.2524
	1.50	2.0607	1.3692	1.2143	1.1493	1.1141	1.0923
	2.00	1.1764	1.0432	1.0209	1.0129	1.0090	1.0068
Logistic	0.00	115.8823	115.8823	115.8823	115.8823	115.8823	115.8823
	0.25	91.1920	86.0672	84.3389	83.4730	82.9532	82.6065
	0.50	52.1602	44.3730	42.0039	40.8612	40.1888	39.7460
	0.75	26.8178	20.7780	19.0866	18.2946	17.8357	17.5365
	1.00	13.6267	9.8000	8.8005	8.3439	8.0828	7.9140
	1.50	3.9683	2.7229	2.4371	2.3125	2.2430	2.1988
	2.00	1.6957	1.3368	1.2644	1.2343	1.2179	1.2076
Cauchy	0.00	115.8823	115.8823	115.8823	115.8823	115.8823	115.8823
	0.25	11.4381	47.9242	73.8049	88.5028	96.9974	102.1916
	0.50	1.4745	11.4381	29.8451	47.9242	62.6115	73.8049
	0.75	1.0215	3.2278	11.4381	23.4395	36.1577	47.9242
	1.00	1.0010	1.4745	4.7298	11.4381	20.2751	29.8451
	1.50	1.0000	1.0215	1.4745	3.2278	6.6317	11.4381
	2.00	1.0000	1.0010	1.0603	1.4745	2.6351	4.7298

Table 4: $SDRL_{\tilde{M}}$ of proposed control chart under various distributions

Distribution	n / a	05	10	15	20	25	30
Uniform	0.00	115.3812	115.3812	115.3812	115.3812	115.3812	115.3812
	0.25	64.0838	27.5914	11.3553	4.7879	2.1526	1.0662
	0.50	21.2528	3.7450	0.8826	0.2954	0.1133	0.0447
	0.75	6.7860	0.7374	0.1577	0.0389	0.0097	0.0024
	1.00	2.3484	0.2208	0.0339	0.0053	0.0008	0.0001
	1.50	0.4498	0.0257	0.0016	0.0001	0	0
	2.00	0.1261	0.0031	0.0001	0	0	0
Normal	0.00	115.3812	115.3812	115.3812	115.3812	115.3812	115.3812
	0.25	80.9358	70.9649	66.0077	62.8136	60.4998	58.7067
	0.50	37.7814	27.0932	22.7774	20.2890	18.6165	17.3908
	0.75	16.0572	9.6989	7.5072	6.3447	5.6061	5.0872
	1.00	6.8552	3.6211	2.6494	2.1690	1.8776	1.6795

	1.50	1.4785	0.7109	0.5101	0.4142	0.3566	0.3175
	2.00	0.4555	0.2122	0.1461	0.1144	0.0954	0.0827
Logistic	0.00	115.3812	115.3812	115.3812	115.3812	115.3812	115.3812
	0.25	90.6906	85.5658	83.8374	82.9715	82.4517	82.1050
	0.50	51.6578	43.8702	41.5009	40.3581	39.6857	39.2428
	0.75	26.3131	20.2718	18.5799	17.7876	17.3285	17.0291
	1.00	13.1172	9.2865	8.2854	7.8279	7.5663	7.3971
	1.50	3.4321	2.1659	1.8715	1.7422	1.6698	1.6235
	2.00	1.0862	0.6710	0.5782	0.5378	0.5152	0.5007
Cauchy	0.00	115.3812	115.3812	115.3812	115.3812	115.3812	115.3812
	0.25	10.9266	47.4215	73.3032	88.0014	96.4961	101.6904
	0.50	0.8364	10.9266	29.3408	47.4215	62.1094	73.3032
	0.75	0.1482	2.6815	10.9266	22.9340	35.6542	47.4215
	1.00	0.0312	0.8364	4.2001	10.9266	19.7688	29.3408
	1.50	0.0014	0.1482	0.8364	2.6815	6.1112	10.9266
	2.00	0.0001	0.0312	0.2529	0.8364	2.0757	4.2001

Table 5: $\boldsymbol{\beta_{\tilde{M}}}$, $\boldsymbol{P_{\tilde{M}}}$, $\boldsymbol{ARL_{\tilde{M}}}$ ***and*** $\boldsymbol{SDRL_{\tilde{M}}}$ ***for Laplace distribution***

Shift	$\boldsymbol{\beta_{\tilde{M}}}$	$\boldsymbol{P_{\tilde{M}}}$	$\boldsymbol{ARL_{\tilde{M}}}$	$\boldsymbol{SDRL_{\tilde{M}}}$
0.00	0.9914	0.0086	115.8823	115.3812
0.25	0.9892	0.0108	92.2150	91.7136
0.50	0.9814	0.0186	53.8656	53.3633
0.75	0.9646	0.0354	28.2422	27.7377
1.00	0.9314	0.0686	14.5848	14.076
1.50	0.7682	0.2318	4.3148	3.7819
2.00	0.4463	0.5537	1.8061	1.2066

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