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Quantum Algorithm for 3-SAT Problem of 5, and 6 Variables by Shor's Fourier Transform with Repeat Qubits on QCEngine

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Abstract

A quantum algorithm for the 3-SAT problem of 5, and 6 variables by the Shor's Fourier transform with the repeat qubits on the QCEngine, and its example are reported. When there are 3 literals with 2 'OR's in each clause, a number of clauses is m, an r-th clause ($1 \le r \le m$) is $C_{u,r}$ (x_1, x_2, x_3, … , x_n) [u is $2^0x_1 + 2^1x_2 + 2^2x_3 + \ldots + 2^{n-1}x_n$. x_1, x_2, …, and x_n are the variables, and the repeat qubits.], and S(u) is $\Sigma_{r=1 \to m}$ $r \times C_{u,r}$ (x_1, x_2, x_3, … , x_n), mod(S(u)$_{\max}$) of S(u) [S(u)$_{\max}$ is the maximum value of S(u).] is computed, next, for u, the quantum Fourier transform is done. In this time, there are 5, and 6 variables, and m = 9, and 10, respectively. The complexity of this method is able to be several times.

Keywords: Quantum algorithm, 3-SAT problem, 5, and 6 variables, Shor's Fourier transform, repeat qubits, QCEngine.

AMS subject classification: Primary 81-08; Secondary 81-10, 68Q12.

1. Introduction

The complexity of the 3-SAT problem is discussed by Cook. [1] Quantum computer's example of the 3-SAT problem is reported by Johnston, Harrigan, and Gimeno-Segovia with QCEngine (free on-line quantum computer simulator). [2] Fujimura discussed a quantum algorithm for the 3-SAT problem by the Shor's Fourier transform with the RAM on the QCEngine. [3] Still more, Fujimura discussed a quantum algorithm for the 3-SAT problem of 2, 3, and 4 answers by the Shor's Fourier transform on the QCEngine. [4]

According to my advanced study, when the Shor's Fourier transform with the repeat qubits for the 3-SAT problem of 5, and 6 variables is used, the complexity of the 3-SAT problem of 5, and 6 variables is able to be several times.

Therefore, the quantum algorithm for the 3-SAT problem of 5, and 6 variables is examined by the Shor's Fourier transform with the repeat qubits on the QCEngine, and its result is reported.

2. 3-SAT Problem

In the 3-SAT problem, it is assumed that (i) each value of n variables becomes "TRUE", or "FALSE", "~" is "NOT", "V" is "OR", "&" is "AND", (ii) "V", "~", and 3 different variables are included in each parentheses (= clause) that are connected by "&". If a value of logical formula by the literals and the logical connectives is "TRUE", it is decided whether there is at least one combination of values of the variables or not. [1-4]

3. Quantum Algorithm

The following conditions are assumed. (I) Each value of variables x_1, x_2, x_3, … , and x_n becomes "TRUE" [= 1], or "FALSE" [= 0]. "~" is "NOT". "V" is "OR". "&" is "AND". For example, it is assumed in this algorithm that (1 V 1 V 1), (1 V 1 V 0), and (1 V 0 V 0) become 1, and (0 V 0 V 0) becomes 0. (II) "V", "~", and 3 different variables in x_1, x_2, x_3, … , and x_n are included in each clause, and then the clauses are connected by "&". In these conditions, if a value of logical formula by the literals, and the operators is "TRUE", it is searched whether there is at least one combination of values of the variables or not. It is assumed that n is number of qubits, u is $2^0x_1 + 2^1x_2 + 2^2x_3 + \ldots + 2^{n-1}x_n$, a number of clauses is m, an r-th clause

($1 \leq r \leq m$) is $C_{u,r}$ (x_1, x_2, x_3, … , x_n), S(u) is $\Sigma_{r = 1 \to m}$ $r \times C_{u,r}$ (x_1, x_2, x_3, … , x_n), and S(u)$_{max}$ is (the maximum value of S(u)) = (m + 1)m/2 = k.

First of all, query quantum registers $|x_i\rangle$[$1 \leq i \leq n$. i is an integer. n is the number of the variables in logical formula, and the repeat qubits.], work1 quantum registers $|w_{1,j}\rangle$ [$1 \leq j \leq t$. j, and t are integers. t is a necessary number for S(u)$_{max} \leq 2^t$.], work2 quantum registers $|w_{2,p}\rangle$ [$1 \leq p \leq t + 1$. p is an integer. +1 is a qubit for the negative integer. [2]], and ancilla quantum qubit $|a\rangle$ are prepared.

Step 1: The r data are introduced to the RAM [2].

Step 2: Each qubit of $|x_i\rangle$, $|w_{1,j}\rangle$, $|w_{2,p}\rangle$, and $|a\rangle$ is set $|0\rangle$.

Step 3: The Hadamard gate $\boxed{\mathrm{H}}$ [2-9] acts on each qubit of $|x_i\rangle$. It changes them for entangled states.

Step 4: Each clause is presented by $|x_i\rangle$, $|w_{2,p}\rangle$, add gate, and quantum operators. For $|x_i\rangle$, RAM[r - 1] [RAM has r data of 0 → (m - 1).] is incremented in $|w_{2,p}\rangle$. In a function, S(u) = $\Sigma_{r = 1 \to m}$ $r \times C_{u,r}$ (x_1, x_2, x_3, … , x_n) is computed. This operation makes entangled data base. In this case, n is the number of variables in the logical formula, and repeat qubits.

Step 5: For $|w_{2,p}\rangle$, mod(k) [k = S(u)$_{max}$ = (m + 1)m/2] is done, where mod(k) is made by the subtraction, and addition in this program. [2] And then, work1 quantum registers are added work2 quantum registers, and the uncompute is done.

Step 6: For $|x_i\rangle$, the quantum Fourier transform (= QFT) [2-7] is done.

Step 7: For $|x_i\rangle$, and $|w_{1,j}\rangle$, the probes are done.

Step 8: For $|x_i\rangle$, the read is done.

Step 9: A number of spikes is estimated by the function (https: //oreilly-qc. github. io? p = 12-4 [2]), where the function estimate_num_spikes (spike, range) [spike: read value, range: 2^n] is used.

Step 10: From candidates of the number of spikes, the repeat period P is obtained.

Step 11: From u = P = $2^0x_1 + 2^1x_2 + 2^2x_3 + \ldots + 2^{n-1}x_n$, when there is S(P)$_{max}$ is $\Sigma_{r = 1 \to m}$ $r \times C_{P,r}$ (x_1, x_2, x_3, … , x_n) = k, it is the answer [one combination of (value of logical formula) = 1].

4. **Example of Numerical Computation**

4-1-1**. 6 Variables, and 3 Repeat qubits**

For example at $n = 9$ [6 variables, and 3 repeat qubits], it is assumed that logical formula : $(x_3 \mathrm{V} x_4 \mathrm{V} x_5)$ & $(\sim x_1 \mathrm{V} x_2 \mathrm{V} x_3)$ & $(\sim x_3 \mathrm{V} x_4 \mathrm{V} x_5)$ & $(x_3 \mathrm{V} \sim x_4 \mathrm{V} x_5)$ & $(\sim x_2 \mathrm{V} x_3 \mathrm{V} \sim x_5)$ & $(\sim x_3 \mathrm{V} \sim x_4 \mathrm{V} x_5)$ & $(\sim x_3 \mathrm{V} x_4 \mathrm{V} \sim x_5)$ & $(x_3 \mathrm{V} \sim x_4 \mathrm{V} \sim x_5)$ & $(\sim x_3 \mathrm{V} \sim x_4 \mathrm{V} \sim x_5)$ & $(x_4 \mathrm{V} \sim x_5 \mathrm{V} \sim x_6)$, each value of $x_{1\sim6}$: $x_1 = x_2 = x_3 = x_4 = x_6 = 0$, $x_5 = 1$, $m = 10$, $t = 6$, and $k = (m + 1)m/2 = 55$.

An example of program on the QCEngine is the following.

```
10 var a = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]; // RAM_a
20 var query_qubits = 9;
30 var work1_qubits = 6;
40 var work2_qubits = 7;
50 var ancilla_qubit = 1;
60 qc.reset(query_qubits + work1_qubits + work2_qubits + ancilla_qubit);
70 var query = qint.new(query_qubits, 'query');
80 var work1 = qint.new(work1_qubits, 'work1');
90 var work2 = qint.new(work2_qubits, 'work2');
100 var ancilla = qint.new(ancilla_qubit, 'ancilla');
110 qc.label('q'); // set query
120 query.write(0);
130 query.hadamard();
140 qc.label(' ');
150 qc.label('w1'); // set work1
160 work1.write(0);
170 qc.label('w2'); // set work2
180 work2.write(0);
190 qc.label('a'); // set ancilla
```

```
ancilla.write(0);
qc.print(' RAM before increment : ' + a + '¥n');
var query16 = 16;
var k = 55;
var work1_0 = 0;
qc.label('increment');
qc.not(query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
work2.add(a[0],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
qc.not(query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
qc.not(query.bits(0x2)|query.bits(0x4));
work2.add(a[1],query.bits(0x1)|query.bits(0x2)|query.bits(0x4));
qc.not(query.bits(0x2)|query.bits(0x4));
qc.not(query.bits(0x8)|query.bits(0x10));
work2.add(a[2],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
qc.not(query.bits(0x8)|query.bits(0x10));
qc.not(query.bits(0x4)|query.bits(0x10));
work2.add(a[3],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
qc.not(query.bits(0x4)|query.bits(0x10));
qc.not(query.bits(0x4));
work2.add(a[4],query.bits(0x2)|query.bits(0x4)|query.bits(0x10));
qc.not(query.bits(0x4));
qc.not(query.bits(0x10));
work2.add(a[5],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
qc.not(query.bits(0x10));
qc.not(query.bits(0x8));
```

```
work2.add(a[6],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
qc.not(query.bits(0x8));
qc.not(query.bits(0x4));
work2.add(a[7],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
qc.not(query.bits(0x4));
work2.add(a[8],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
qc.not(query.bits(0x8));
work2.add(a[9],query.bits(0x8)|query.bits(0x10)|query.bits(0x20));
qc.not(query.bits(0x8));
qc.label('mod(' + k + ')');
work2.subtract(k);
qc.cnot(ancilla.bits(0x1), work2.bits(0x40));
work2.add(k,ancilla.bits(0x1));
work1.add(work2);
qc.label('uncompute');
work2.subtract(k,ancilla.bits(0x1));
qc.cnot(ancilla.bits(0x1),work2.bits(0x40));
work2.add(k);
qc.not(query.bits(0x8));
work2.subtract(a[9],query.bits(0x8)|query.bits(0x10)|query.bits(0x20));
qc.not(query.bits(0x8));
work2.subtract(a[8],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
qc.not(query.bits(0x4));
work2.subtract(a[7],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
qc.not(query.bits(0x4));
```

```
qc.not(query.bits(0x8));
work2.subtract(a[6],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
qc.not(query.bits(0x8));
qc.not(query.bits(0x10));
work2.subtract(a[5],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
qc.not(query.bits(0x10));
qc.not(query.bits(0x4));
work2.subtract(a[4],query.bits(0x2)|query.bits(0x4)|query.bits(0x10));
qc.not(query.bits(0x4));
qc.not(query.bits(0x4)|query.bits(0x10));
work2.subtract(a[3],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
qc.not(query.bits(0x4)|query.bits(0x10));
qc.not(query.bits(0x8)|query.bits(0x10));
work2.subtract(a[2],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
qc.not(query.bits(0x8)|query.bits(0x10));
qc.not(query.bits(0x2)|query.bits(0x4));
work2.subtract(a[1],query.bits(0x1)|query.bits(0x2)|query.bits(0x4));
qc.not(query.bits(0x2)|query.bits(0x4));
qc.not(query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
work2.subtract(a[0],query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
qc.not(query.bits(0x4)|query.bits(0x8)|query.bits(0x10));
qc.label('QFT');
query.QFT();
var prob16 = 0;
prob16 += query.peekProbability(query16);
```

```
950 // Print output query-Prob
960 qc.print(' Prob_query16: ' + prob16);
970 var prob0 = 0;
980 prob0 += work1.peekProbability(work1_0);
990 // Print output work1-Prob
1000 qc.print(' Prob_work1_0: ' + prob0);
1010 //read
1020 qc.label('Rq');
1030 var b2 = query.read();
1040 // Print output result
1050 qc.print(' Read query = ' + b2 +'.');
1060 // end
```

When this program is copied on Programming Quantum Computers https: //oreilly-qc. github. io/# [free on-line quantum computation simulator QCEngine] [2], you can run it. [Caution!: Please delate the line numbers.]

A result of this program is the following.

The probability probe value of $|w_{1,j}\rangle = 0$: 0.015625 (= 1/64).

The probability probe value of $|x_i\rangle = 16$: ≈ 0.088819.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 32, 16, 32, 0, 8, 288, 320, 416, 48, 16, 160, 496, 16, 368, 464, 32, 304, 368, 480, 496, 480, 8, 32, 496, 80, 80, 16, 464, 416, 440, 0, 448, 32, 96, 496, 32, 160, 480, 480, 16, 488, 16, 416, 48, 208, 32, 464, 16, 160, 48. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^9 = 512$]] : R_q → candidates ; 32 → 16, … ; 16 → 32, … ; 0 → nothingness ; 8 → 64, … ; 288 → 2, 5, 7, 9, 16, … ; 320 → 3, 5, 8, 16, … ; 416 → 5, 11, 16, … ; 48 → 11, 21, … ; 160 → 3, 6, 10, 13, 16, … ; 496 → 32, … ; 368 → 4, 7, 14, 18, ... ; 464 → 11, 21, …; 304 → 3, 5, 10, 15, 17, … ; 480 → 16, … ; 80 → 6, 13, 19, … ; 440 → 7, 14, 21, … ; 448 → 8, 16, … ; 96 → 5, 11, 16,

… ; 488 → 21, … ; 208 → 3, 5, 10, 15, 17, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 + 2^8x_9 = 2^0\times0 + 2^1\times0 + 2^2\times0 + 2^3\times0 + 2^4\times1 + 2^5\times0 + 2^6\times0 + 2^7\times0 + 2^8\times0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

4-1-2. **6 Variables, and 2 Repeat qubits**

For example at $n = 8$ [6 variables, and 2 repeat qubits], it is assumed that logical formula : same logical formula of the section 4-1-1, each value of $x_{1\sim6}$: $x_1 = x_2 = x_3 = x_4 = x_6 = 0$, $x_5 = 1$, $m = 10$, $t = 6$, and $k = (m + 1)m/2 = 55$.

A result of this problem is the following.

The probability probe value of $|w_{1,j}\rangle = 0$: 0.015625 (= 1/64).

The probability probe value of $|x_i\rangle = 16$: ≈ 0.065482.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 8, 0, 16, 200, 240, 16, 236, 48, 16, 240, 0, 8, 248, 232, 0, 248, 208, 12, 8, 96, 0, 32, 104, 0, 8, 12, 240, 200, 236, 88, 16, 160, 16, 8, 216, 248, 60, 112, 248, 0, 160, 176, 16, 240, 208, 16, 88, 32, 24, 208. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^8 = 256$]] : R_q → candidates ; 8 → 32, … ; 0 → nothingness ; 16 → 16, … ; 200 → 5, 9, 18, … ; 240 → 16, … ; 236 → 13, 26, … ; 48 → 5, 11, 16, … ; 248 → 32, … ; 232 → 11, 21, … ; 208 → 5, 11, 16, … ; 12 → 21, … ; 96 → 3, 5, 8, 16, … ; 32 → 8, 16, … ; 104 → 3, 5, 10, 15, 17, … ; 88 → 3, 6, 9, 12, 15, 17, … ; 160 → 3, 5, 8, 16, … ; 216 → 6, 13, 19, … ; 60 → 4, 9, 13, 17, … ; 112 → 2, 5, 7, 9, 16, … ; 176 → 3, 6, 10, 11, 13, 16, … ; 24 → 11, 21, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 = 2^0\times0 + 2^1\times0 + 2^2\times0 + 2^3\times0 + 2^4\times1 + 2^5\times0 + 2^6\times0 + 2^7\times0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

4-1-3. **6 Variables, and 1 Repeat Qubit**

For example at $n = 7$ [6 variables, and 1 repeat qubit], it is assumed that logical formula : same logical formula of the section 4-1-1, each value of $x_{1\sim6}$: $x_1 = x_2 = x_3 = x_4 = x_6 = 0$, $x_5 = 1$,

m = 10, t = 6, and $k = (m + 1)m/2 = 55$.

A result of this problem is the following.

The probability probe value of $|w_{1,j}\rangle = 0$: 0.015625 (= 1/64).

The probability probe value of $|x_i\rangle = 16$: ≈ 0.030343.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 12, 32, 84, 120, 40, 4, 120, 124, 8, 110, 36, 124, 16, 84, 4, 32, 8, 4, 18, 2, 16, 0, 16, 12, 4, 124, 76, 44, 4, 122, 112, 8, 12, 24, 124, 88, 124, 22, 28, 8, 52, 124, 4, 8, 8, 0, 8, 4, 4, 124. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^7 = 128$]] : R_q → candidates ; 12 → 11, 21, … ; 32 → 4, 8, 12, 16, … ; 84 → 3, 6, 9, 12, 15, 17, … ; 120 → 16, … ; 40 → 3, 6, 10, 13, 16, … ; 4 → 32, … ; 124 → 32, … ; 8 → 16, … ; 110 → 7, 14, 21, … ; 36 → 4, 7, 14, 18, … ; 16 → 8, 16, … ; 84 → 3, 6, 9, 12, 15, 17, … ; 32 → 4, 8, 12, 16, … ; 18 → 7, 14, 21, … ; 2 → 64 ; 0 → nothingness ; 76 → 3, 5, 10, 15, 17, … ; 44 → 3, 6, 9, 12, 15, 17, … ; 122 → 21, … ; 112 → 8, 16, … ; 88 → 3, 6, 10, 13, 16, … ; 22 → 6, 12, 17, … ; 28 → 5, 9, 18, … ; 52 → 3, 5, 10, 15, 17, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 = 2^0{\times}0 + 2^1{\times}0 + 2^2{\times}0 + 2^3{\times}0 + 2^4{\times}1 + 2^5{\times}0 + 2^6{\times}0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

4-1-4. **6 Variables, and 0 Repeat Qubits**

For example at n = 6 [6 variables, and 0 repeat qubit], it is assumed that logical formula : same logical formula of the section 4-1-1, each value of $x_{1\sim6}$: $x_1 = x_2 = x_3 = x_4 = x_6 = 0$, $x_5 = 1$, m = 10, t = 6, and $k = (m + 1)m/2 = 55$.

A result of this problem is the following.

The probability probe value of $|w_{1,j}\rangle = 0$: 0.015625 (= 1/64).

The probability probe value of $|x_i\rangle = 16$: ≈ 0.0097656.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 8, 40, 9, 59, 0, 5, 7, 2, 60, 58, 63, 40, 44, 13, 24, 62, 40, 6, 58, 16, 57, 42, 63, 2, 62, 46, 4, 58, 38, 3, 50, 62, 51, 16, 10, 60, 2, 2, 2, 6, 0, 44, 4, 4, 2, 0, 58, 16, 22, 2. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^6 = 64$]] : R_q → candidates ; 8 → 8, 16, … ; 40 → 3, 5, 8, 16, … ; 9 → 7, 14, 21, … ; 59 → 13, 26, … ; 0 → nothingness ; 5 → 13, 26, … ; 7 → 9, 18, … ; 2 → 32 ; 60 → 16, … ; 58 → 11, 21, … ; 63 → nothingness ; 44 → 3, 6, 10, 13,16, … ; 13 → 5, 10, 15, 20, … ; 24 → 3, 5, 8, 16, … ; 62 → 32 ; 6 → 11, 21, … ; 16 → 4, 8, 12, 16, … ; 57 → 9, 18, … ; 42 → 3, 6, 9, 12, 15, 17, … ; 46→ 4, 7, 14, 18, … ; 4 → 16, … ; 38 → 3, 5, 10, 15, 17, … ; 3 → 21, … ; 50 → 5, 9, 18, … ; 51 → 5, 10, 15, 20, … ; 10 → 6, 13, 19, … ; 22 → 3, 6, 9, 12, 15, 17, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 = 2^0{\times}0 + 2^1{\times}0 + 2^2{\times}0 + 2^3{\times}0 + 2^4{\times}1 + 2^5{\times}0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

4-2-1. **5 Variables, and 4 Repeat Qubits**

For example at $n = 9$ [5 variables, and 4 repeat qubits], it is assumed that logical formula : (x_3 V x_4 V x_5) & (~x_1 V x_2 V x_3) & (~x_3 V x_4 V x_5) & (x_3 V ~x_4 V x_5) & (~x_2 V x_3 V ~x_5) & (~x_3 V ~x_4 V x_5) & (~x_3 V x_4 V ~x_5) & (x_3 V ~x_4 V ~x_5) & (~x_3 V ~x_4 V ~x_5), each value of $x_{1\sim5}$: $x_1 = x_2 = x_3 = x_4 = 0$, $x_5 = 1$, $m = 9$, $t = 6$, and $k = (m + 1)m/2 = 45$.

A result of this problem is the following.

The probability probe value of $|w_{1,j}\rangle = 0$: 0.031250 (= 1/32).

The probability probe value of $|x_i\rangle = 16$: ≈ 0.099363.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 368, 496, 400, 496, 432, 464, 384, 64, 16, 448, 80, 48, 192, 0, 400, 288, 16, 464, 48, 144, 464, 32, 352, 48, 0, 0, 448, 480, 320, 336, 16, 160, 448, 464, 464, 448, 160, 336, 496, 128, 448, 96, 64, 192, 160, 80, 0, 16, 80, 0. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^9 = 512$]] : R_q → candidates ; 368 → 4, 7, 14, 18, … ; 496 → 32, … ; 400 → 5, 9, 18, … ; 432 → 6, 13, 19, … ; 464 → 11, 21, … ; 384 → 4, 8, 12, 16, … ; 64 → 8, 16, … ; 16 → 32, … ; 448 → 8, 16, … ; 80 → 6, 13, 19, … ; 48 → 12, 24, ... ; 192 → 3, 5, 8,16, …; 0 → nothingness ; 288 → 2, 5, 7, 9, 16, … ; 144 → 4, 7, 14, 18, … ; 32 → 16, … ; 352 → 3, 6, 10, 13, 16, … ; 480 → 16, … ; 320 → 3, 5, 8, 16, … ; 336 → 3, 6, 9, 12, 15, 17, … ; 160 → 3, 6, 10, 13, 16, … ; 128 →

4, 8, 12, 16, … ; 96 → 5, 11, 16, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 + 2^8x_9 = 2^0\times0 + 2^1\times0 + 2^2\times0 + 2^3\times0 + 2^4\times1 + 2^5\times0 + 2^6\times0 + 2^7\times0 + 2^8\times0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

4-2-2. **5 Variables, and 3 Repeat Qubits**

For example at $n = 8$ [5 variables, and 3 repeat qubits], it is assumed that logical formula : same logical formula of the section 4-1, each value of $x_{1\sim5}$: $x_1 = x_2 = x_3 = x_4 = 0$, $x_5 = 1$, $m = 9$, $t = 6$, and $k = (m + 1)m/2 = 45$.

A result of this problem is the following.

The probability probe value of $|w_{1,j}\rangle = 0$: 0.031250 (= 1/32).

The probability probe value of $|x_i\rangle = 16$: ≈ 0.075654.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 8, 160, 240, 224, 144, 248, 72, 240, 248, 16, 16, 8, 16, 128, 0, 16, 152, 80, 8, 240, 0, 48, 72, 32, 32, 168, 48, 0, 224, 8, 48, 0, 192, 8, 16, 0, 16, 144, 88, 8, 248, 224, 240, 8, 200, 32, 232, 232, 240, 8. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^8 = 256$]] : R_q → candidates ; 8 →32, … ; 160 → 3, 5, 8, 16, … ; 240 → 16, … ; 224 → 8, 16, … ; 144 → 2, 5, 7, 9, 16, … ; 248 → 32, … ; 72 → 4, 7, 14, 18, … ; 16 → 16, … ; 128 → 16, … ; 0 → nothingness ; 152 → 3, 5, 10, 15, 17, ... ; 80 → 3, 6, 10, 13,16, …; 48 → 5, 11, 16, … ; 32 → 8, 16, … ; 168 → 3, 6, 9, 12, 15, 17, … ; 192 → 4, 8, 12, 16, … ; 88 → 3, 6, 9, 12, 15, 17, … ; 200 → 5, 9, 18, … ; 232 → 11, 21, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 + 2^7x_8 = 2^0\times0 + 2^1\times0 + 2^2\times0 + 2^3\times0 + 2^4\times1 + 2^5\times0 + 2^6\times0 + 2^7\times0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

4-2-3. **5 Variables, and 2 Repeat qubits**

For example at $n = 7$ [5 variables, and 2 repeat qubits], it is assumed that logical formula : same logical formula of the section 4-2-1, each value of $x_{1\sim5}$: $x_1 = x_2 = x_3 = x_4 = 0$, $x_5 = 1$, m

= 9, t = 6, and $k = (m + 1)m/2 = 45$.

A result of this problem is the following.

The probability probe value of $|w_{1,j}\rangle = 0$: 0.031250 (= 1/32).

The probability probe value of $|x_i\rangle = 16$: ≈ 0.035630.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 72, 0, 0, 0, 80, 96, 0, 16, 48, 120, 124, 124, 0, 116, 4, 16, 116, 120, 40, 72, 100, 112, 120, 0, 108, 16, 120, 124, 8, 120, 64, 124, 0, 92, 16, 4, 80, 124, 8, 0, 116, 116, 104, 24, 124, 20, 4, 8, 84. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^7 = 128$]] : R_q → candidates ; 72 → 2, 5, 7, 9, 16, … ; 0 → nothingness ; 80 → 3, 5, 8, 16, … ; 96 → 4, 8, 12, 16, … ; 16 → 8, 16, … ; 48 → 3, 5, 8, 16, … ; 120 → 16, … ; 124 → 32, … ; 116 → 11, 21, … ; 4 → 32, … ; 40 → 3, 6, 10, 13, 16, … ; 72 → 2, 5, 7, 9, 16, … ; 100 → 5, 9, 18, … ; 112 → 8, 16, … ; 108 → 6, 13, 19, … ; 8 → 16, … ; 64 → 2, 4, 6, 8, 10, 12, 14, 16, … ; 92 → 4, 7, 14, 18, … ; 104 → 5, 11, 16, … ; 24 → 5, 11, 16, … ; 20 → 6, 13, 19, … ; 84 → 3, 6, 9, 12, 15, 17, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 + 2^6x_7 = 2^0\times0 + 2^1\times0 + 2^2\times0 + 2^3\times0 + 2^4\times1 + 2^5\times0 + 2^6\times0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

4-2-4. **5 Variables, and 1 Repeat Qubit**

For example at n = 6 [5 variables, and 1 repeat qubit], it is assumed that logical formula : same logical formula of the section 4-1, each value of $x_{1\sim5}$: $x_1 = x_2 = x_3 = x_4 = 0$, $x_5 = 1$, m = 9, t = 6, and $k = (m + 1)m/2 = 45$.

A result of this problem is the following.

The probability probe value of $|w_{1,j}\rangle = 0$: 0.031250 (= 1/32).

The probability probe value of $|x_i\rangle = 16$: ≈ 0.011719.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 8, 60, 0, 12, 12, 2, 0, 22, 62, 0, 14, 62, 8, 8, 8, 62, 18, 22, 0, 54, 20, 6, 38, 56, 62, 58, 60, 60, 26, 16, 2, 38, 10, 58, 4, 30, 60, 52, 42, 0, 4, 2, 4, 62, 58, 6, 60, 10, 60, 0. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^6 = 64$]] : R_q → candidates ; 8 → 8, 16, … ; 60 → 16, … ; 0 → nothingness ; 12 → 5, 11, 16, … ; 2 → 32 ; 22 → 3, 6, 9, 12, 15, 17, … ; 62 → 32 ; 14 → 5, 9, 18, … ; 18 → 4, 7, 14, 18, … ; 54 → 6, 13, 19, … ; 20 → 3, 6, 10, 13, 16, … ; 6 → 11, 21, … ; 38 → 3, 5, 10, 15, 17, … ; 56 → 8, 16, … ; 58 → 11, 21, … ; 26 → 3, 5, 10, 15, 17, … ; 16 → 4, 8, 12, 16, … ; 10 → 6, 13, 19, … ; 4 →16, … ; 30 → 2, 4, 6, 9, 11, 13, 15, 17, … ; 52 → 5, 11, 16, … ; 42 → 3, 6, 9, 12, 15, 17, … .

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 + 2^5x_6 = 2^0×0 + 2^1×0 + 2^2×0 + 2^3×0 + 2^4×1 + 2^5×0 = 16$), the value of logical formula is 1. Therefore, it is the answer.

4-2-5. **5 Variables, and 0 Repeat Qubit**

For example at $n = 5$ [5 variables, and 0 repeat qubit], it is assumed that logical formula : same logical formula of the section 4-2-1, each value of $x_{1\sim5}$: $x_1 = x_2 = x_3 = x_4 = 0$, $x_5 = 1$, $m = 9$, $t = 6$, and $k = (m + 1)m/2 = 45$.

A result of this problem is the following.

The probability probe value of $|w_{1,j}\rangle = 0$: 0.031250 (= 1/32).

The probability probe value of $|x_i\rangle = 16$: ≈ 0.0078125.

The example of 50 times test : The read value of $|x_i\rangle$; R_q = 0, 5, 1, 30, 0, 0, 12, 0, 0, 3, 3, 2, 0, 3, 3, 31, 7, 0, 18, 28, 5, 15, 0, 0, 30, 2, 30, 31, 1, 0, 22, 1, 21, 30, 0, 21, 28, 25, 3, 0, 28, 22, 3, 29, 2, 29, 3, 31, 26, 0. (=spike)

The candidates of number of spikes are estimated by the function [the function estimate_num_spikes (spike, range) [spike : read value, range : $2^n = 2^5 = 32$]] : R_q → candidates ; 0 → nothingness ; 5 → 6, 13, 19 ; 1 → nothingness ; 30 → 16 ; 12 → 3, 5, 8, 16 ; 3 → 11, 12 ; 2 → 16 ; 31 → nothingness ; 7 → 5, 9, 18, … ; 18 → 2, 5, 7, 9, 16 ; 28 → 8, 16, … ; 15 → 2, 4, 6, 9, 11, 13, 15 ; 22 → 3, 6, 10, 13, 16 ; 21 → 3, 6, 9, 12, 15, 17 ; 25 → 5, 9, 18, … ; 29 → 11, 21 ; 26 → 5, 11, 16.

When u is 16 ($2^0x_1 + 2^1x_2 + 2^2x_3 + 2^3x_4 + 2^4x_5 = 2^0×0 + 2^1×0 + 2^2×0 + 2^3×0 + 2^4×1 = 16$), the value of logical formula is 1. Therefore, it is the answer.

5. Discussion

In the 3-SAT problem, when [(the logical formula) = 1] is obtained, there is only one combination.

5-1. 6 Variables, and 3, 2, 1, and 0 Repeat Qubits

In the section 4-1, there are 6 variables. And then, in 6 variables, [(the logical formula) = 1] of combination of variables is selected, and the computation of repeats is done. When N is 2^6, in the Grover's method, the complexity is $N^{1/2} = 2^{6/2} \approx 8$, in the Shor's Fourier transform, for (variables, repeat qubits) = (6, 0), it is $50/15 \approx 3$, for (6, 1), it is $50/17 \approx 3$, for (6, 2), it is $50/22 \approx 2$, and for (6, 3), it is $50/20 \approx 3$.

In this range, the Shor's Fourier transform is less than the complexity of the Grover's method, and then, in for (6, 2), the probability is the maximum value 44%.

5-2. 5 Variables, and 4, 3, 2, 1, and 0 Repeat Qubits

In the section 4-2, there are 5 variables. And then, in 5 variables, [(the logical formula) = 1] of combination of variables is selected, and the computation of repeats is done. When N is 2^5, in the Grover's method, the complexity is $N^{1/2} = 2^{5/2} \approx 6$, in the Shor's Fourier transform, for (variables, repeat qubits) = (5, 0), it is $50/15 \approx 3$, for (5, 1), it is $50/19 \approx 3$, for (5, 2), it is $50/24 \approx 2$, for (5, 3), it is $50/26 \approx 2$, and for (5, 4), it is $50/20 \approx 3$.

In this range, the Shor's Fourier transform is less than the complexity of the Grover's method, and then, in for (5, 3), the probability is the maximum value 52%.

6. Summary

The quantum algorithm for the 3-SAT problem of 5, and 6 variables by the Shor's Fourier transform with the repeat qubits on the QCEngine, and its example are reported.

The complexity of this method is several times, and then, in for (6, 2), the probability is the maximum value 44%, and in for (5, 3), the probability is the maximum value 52%.

I will apply this method for other problems.

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