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"Solution of Telegraph Equation by Sadik Transform and Double Sadik Transform"

Mohd Zafar Saber
Associate Professor, Department of Mathematics, Kohinoor Arts, Commerce and Science College Khultabad 431101, Dist- Chh SambhajiNagar (MS), India
Email: zafarmaths1@gmail.com

Abstract
In this Paper we will presents Sadik transform and Double Sadik transform and its way to deal with one Dimensional time Dependent partial differential equations by comparison Method, also we will produce solution procedure of Telegraph equation by these two methods, application of these two methods over Telegraph equation are also given.

Keywords: Sadik Transform, Double Sadik transform, one Dimensional time Dependent partial differential equations, Telegraph equation.

Introduction

In recent years, many Mathematicians have paid attention to find the solution of linear partial differential equations of different order by using various methods. Among these are Laplace Transform, Sumudu Transform, double Laplace transform, and double Sumudu transform , there are various ways have been proposed recently to deal with these Linear partial differential equations, Double Laplace transform method has not received much attention unlike other methods. Now a days Sadik Transform, Double Sadik transform are going very popular because it is general form of many Integral transforms like Laplace.

Preliminaries

Sadik Transform of Partial Derivatives:

1) If $\mathbb{S}\left(x,\upsilon^{\alpha},\beta\right)$ is a Sadik transform of $u\left(x,t\right)$ and $u_t\left(x,t\right)$ is a first partial derivative of with respect to variable t then
$$\mathbb{S}\left[u_t\left(x,t\right)\right]=\upsilon^{\alpha}\mathbb{S}\left(x,\upsilon^{\alpha},\beta\right)-\upsilon^{-\beta}u\left(x,0\right)$$

2) If $\mathbb{S}\left(x,\upsilon^{\alpha},\beta\right)$ is a Sadik transform of $u\left(x,t\right)$, $u_t\left(x,t\right)$ and $u_{tt}\left(x,t\right)$ are first and second order partial derivative of $u\left(x,t\right)$ with respect to variable 't' then

$\mathbb{S}\left[u_{tt}(x,t)\right]=\upsilon^{2\alpha}\mathbb{S}\left(x,\upsilon^{\alpha},\beta\right)-\upsilon^{\alpha-\beta}u(x,0)-\upsilon^{-\beta}u_{t}(x,0)$

3) If $\mathbb{S}\left(x,\upsilon^{\alpha},\beta\right)$ is a Sadik transform of $u(x,t)$ and $u_x(x,t)$, $\mathbb{S}_x\left(x,\upsilon^{\alpha},\beta\right)$ are Partial derivatives of $u(x,t)$ and $\mathbb{S}\left(x,\upsilon^{\alpha},\beta\right)$ with respect to variable 'x' then

$\mathbb{S}\left[u_x(x,t)\right]=\mathbb{S}_x\left(x,\upsilon^{\alpha},\beta\right)$

4) If $\mathbb{S}\left(x,\upsilon^{\alpha},\beta\right)$ is a Sadik transform of $u(x,t)$ and $u_{x^{(n)}}(x,t)$ and $\mathbb{S}_{x^{(n)}}\left(x,\upsilon^{\alpha},\beta\right)$ are the n^{th} Partial derivatives of $u(x,t)$ and $\mathbb{S}\left(x,\upsilon^{\alpha},\beta\right)$ with respect to variable 'x' then

$\mathbb{S}\left[u_{x^{(n)}}(x,t)\right]=\mathbb{S}_{x^{(n)}}\left(x,\upsilon^{\alpha},\beta\right)$

Double Sadik Transform

First we define Sadik Transform in terms of two variables ω,υ for function of two variable x and t respectively that is $u(x,t)$

$$\mathbb{S}\left(x,\upsilon^{\alpha},\beta\right)=\mathbb{S}_{\upsilon}\left[u(x,t)\right]=\frac{1}{\upsilon^{\beta}}\int_0^{\infty}e^{-\upsilon^{\alpha}t}u(x,t)\,dt$$

$$\&\qquad \mathbb{S}\left(t,\omega^{\alpha},\beta\right)=\mathbb{S}_{\omega}\left[u(x,t)\right]=\frac{1}{\omega^{\beta}}\int_0^{\infty}e^{-\omega^{\alpha}x}u(x,t)\,dx$$

if $u(x,t)$ be a function that can be express as convergent infinite series and let $(x,t)\in R^2$ then the **double Sadik transform** is denoted by $\mathbb{S}_x\mathbb{S}_t\left[u(x,t)\right]=\mathbb{S}(\omega,\upsilon)$ and defined by

$$\mathbb{S}_x\mathbb{S}_t\left[u(x,t)\right]=\mathbb{S}(\omega,\upsilon)=\frac{1}{\omega^{\beta}\upsilon^{\beta}}\int_0^{\infty}\int_0^{\infty}e^{-(\omega^{\alpha}x+\upsilon^{\alpha}t)}u(x,t)\,dx\,dt$$

where $x,t>0$ and ω, υ are Sadik transform variables for x and t respectively, α is any non-zero real number and β is any real number, whenever improper integral is convergent.

Double Sadik Transform of Partial Derivatives:

1) If $\mathbb{S}_x\mathbb{S}_t\left[u(x,t)\right]=\mathbb{S}(\omega,\upsilon)$ then Double Sadik transform of first order partial derivative with respect to x is defined by

$$\mathbb{S}_x\mathbb{S}_t\left[\frac{\partial}{\partial x}u(x,t)\right]=\omega^{\alpha}.\mathbb{S}(\omega,\upsilon)-\omega^{-\beta}\mathbb{S}_{\upsilon}\left[u(0,t)\right]$$

2) If $\mathbb{S}_x\mathbb{S}_t\left[u(x,t)\right]=\mathbb{S}(\omega,\upsilon)$ is double sadik transform of the function $u(x,t)$ then Double Sadik transform of first order partial derivative with respect to t is defined by

$$\mathbb{S}_x\mathbb{S}_t\left[\frac{\partial}{\partial t}u(x,t)\right]=\upsilon^{\alpha}.\mathbb{S}(\omega,\upsilon)-\upsilon^{-\beta}\mathbb{S}_{\omega}\left[u(x,0)\right]$$

.

3) If $\mathbb{S}_x\mathbb{S}_t\left[u(x,t)\right]=\mathbb{S}(\omega,\upsilon)$ is double Sadik transform of the function $u(x,t)$ then Double Sadik transform of second order partial derivative with respect to x is defined by

$$\mathbb{S}_x\mathbb{S}_t\left[\frac{\partial^2}{\partial x^2}u(x,t)\right]=\omega^{2\alpha}.\mathbb{S}(\omega,\upsilon)-\omega^{\alpha-\beta}\mathbb{S}_{\upsilon}\left[u(0,t)\right]-\omega^{-\beta}\mathbb{S}_{\upsilon}\left[\frac{\partial}{\partial x}u(0,t)\right]$$

4) If $\mathbb{S}_x\mathbb{S}_t\left[u(x,t)\right]=\mathbb{S}(\omega,\upsilon)$ is double sadik transform of the function $u(x,t)$ then Double Sadik transform of second order partial derivative with respect to t is defined by

$$\mathbb{S}_x\mathbb{S}_t\left[\frac{\partial^2}{\partial t^2}u(x,t)\right]=\upsilon^{2\alpha}.\mathbb{S}(\omega,\upsilon)-\upsilon^{\alpha-\beta}\mathbb{S}_{\omega}\left[u(x,0)\right]-\upsilon^{-\beta}\mathbb{S}_{\omega}\left[\frac{\partial}{\partial t}u(x,0)\right]$$

5) If $\mathbb{S}_x\mathbb{S}_t\left[u(x,t)\right]=\mathbb{S}(\omega,\upsilon)$ is double sadik transform of the function $u(x,t)$ then Double Sadik transform of mix partial derivative with respect to x and t is defined by

$$\mathbb{S}_x\mathbb{S}_t\left[\frac{\partial^2}{\partial x\partial t}u(x,t)\right]=\omega^{\alpha}\upsilon^{\alpha}.\mathbb{S}(\omega,\upsilon)-\upsilon^{\alpha}\omega^{-\beta}\mathbb{S}_{\upsilon}\left[u(0,t)\right]-\upsilon^{-\beta}\omega^{\alpha}\mathbb{S}_{\omega}\left[u(x,0)\right]-\upsilon^{\beta}\omega^{\beta}\left[u(0,0)\right]$$

6) If $\mathbb{S}_x\mathbb{S}_t\left[u(x,t)\right]=\mathbb{S}(\omega,\upsilon)$ is double sadik transform of the function $u(x,t)$ then show that Double Sadik transform of nth order partial derivative with respect to t is defined by

$$\mathbb{S}_x\mathbb{S}_t\left(\frac{\partial^n}{\partial t^n}u(x,t)\right)=\upsilon^{n\alpha}\mathbb{S}(\omega,\upsilon)-\sum_{k=0}^{n-1}\upsilon^{k\alpha-\beta}\mathbb{S}_{\omega}\left[\frac{\partial^{(n-k-1)}}{\partial t^{(n-k-1)}}u(x,0)\right]$$

7) If $\mathbb{S}_x\mathbb{S}_t\left[u(x,t)\right]=\mathbb{S}(\omega,\upsilon)$ is double sadik transform of the function $u(x,t)$ then Double Sadik transform of nth order partial derivative with respect to x is defined by,

$$\mathbb{S}_x\mathbb{S}_t\left(\frac{\partial^m}{\partial x^m}u(x,t)\right)=\omega^{m\alpha}\mathbb{S}(\omega,\upsilon)-\sum_{i=0}^{m-1}\omega^{i\alpha-\beta}\mathbb{S}_{\upsilon}\left[\frac{\partial^{(m-i-1)}}{\partial x^{(m-i-1)}}u(0,t)\right]$$

Main Result

Theorem 1: Show that solution of Linear, one-dimensional, time-dependent partial differential equation is defined as

$$\sum_{n=0}^{N} a_n \frac{\partial^n}{\partial t^n} u(x,t) = \sum_{m=1}^{M} b_m \frac{\partial^m}{\partial x^m} u(x,t) + f(x,t), \quad (x,t) \in \mathbb{R}_+^2 \tag{1}$$

where a_n and b_m are given coefficients and N, M are positive integers and $f(x,t)$ is the source term with initial conditions

$$\frac{\partial^n}{\partial t^n} u(x,0) = g_n(x), \; n = 0,1,2, \ldots \ldots \ldots N\text{-}1, \; x \in \mathbb{R}_+ \tag{2}$$

and boundary conditions

$$\frac{\partial^m}{\partial x^m} u(0,t) = h_m(t), \; m = 0,1,2, \ldots \ldots \ldots, M\text{-}1, \; t \in \mathbb{R}_+ \tag{3}$$

exist by Sadik integral transform.

Proof: Applying Sadik Transform on (1) we get

$$\mathbb{S}\left[\sum_{n=0}^{N} a_n \frac{\partial^n}{\partial t^n} u(x,t)\right] = \mathbb{S}\left[\sum_{m=1}^{M} b_m \frac{\partial^m}{\partial x^m} u(x,t)\right] + \mathbb{S}\left[f(x,t)\right]$$

$$\sum_{n=0}^{N} a_n \left[\upsilon^{n\alpha}\mathbb{S}_\upsilon[u(x,t)] - \sum_{k=0}^{n-1} \upsilon^{k\alpha-\beta} \frac{\partial^{n-k-1}}{\partial t^{n-k-1}} u(x,t)\right] = \sum_{m=1}^{M} b_m \left[\mathbb{S}_\upsilon\left[\frac{\partial^m}{\partial x^m} u(x,t)\right]\right] + \mathbb{S}\left[f(x,t)\right]$$

$$\sum_{n=0}^{N} a_n \left[\upsilon^{n\alpha}\mathbb{S}_\upsilon\left[u(x,t)\right] - \sum_{k=0}^{n-1} \upsilon^{k\alpha-\beta} \frac{\partial^{n-k-1}}{\partial t^{n-k-1}} u(x,0)\right] = \sum_{m=1}^{M} b_m \frac{\partial^m}{\partial x^m}\left[\mathbb{S}_\upsilon\left[u(x,t)\right]\right] + \mathbb{S}\left[f(x,t)\right] \tag{4}$$

by applying boundary conditions (2) to (4) we get ordinary differential equation of order m of dependent variable $\mathbb{S}_\upsilon\left[u(x,t)\right]$ and independent variable x, by finding particular integral of this differential equation we get solution in terms of $\mathbb{S}_\upsilon[u(x,t)]$, applying Sadik inverse Transform we get the solution $u(x,t)$.

Theorem 2: Show that solution of (1) exists by Sadik Double Transform.

Proof: Applying Sadik Double transform on (1) we get

$$\mathbb{S}_x\mathbb{S}_t\left[\sum_{n=0}^{N} a_n \frac{\partial^n}{\partial t^n} u(x,t)\right] = \mathbb{S}_x\mathbb{S}_t\left[\sum_{m=1}^{M} b_m \frac{\partial^m}{\partial x^m} u(x,t)\right] + \mathbb{S}_x\mathbb{S}_t\left[f(x,t)\right]$$

$$\sum_{n=0}^{N} a_n \left[\mathbb{S}_x\mathbb{S}_t\left(\frac{\partial^n}{\partial t^n} u(x,t)\right)\right] = \sum_{m=1}^{M} b_m \left[\mathbb{S}_x\mathbb{S}_t\left(\frac{\partial^m}{\partial x^m} u(x,t)\right)\right] + \mathbb{S}_x\mathbb{S}_t\left[f(x,t)\right]$$

$$\sum_{n=0}^{N} a_n \left[\upsilon^{n\alpha} \mathbb{S}(\omega,\upsilon) - \sum_{k=0}^{n-1} \upsilon^{k\alpha-\beta} \mathbb{S}_{\omega} \left[\frac{\partial^{n-k-1}}{\partial t^{n-k-1}} u(x,0) \right] \right] =$$

$$\sum_{m=1}^{M} b_m \left[\omega^{m\alpha} \mathbb{S}(\omega,\upsilon) - \sum_{i=0}^{m-1} \omega^{i\alpha-\beta} \mathbb{S}_{\upsilon} \left[\frac{\partial^{m-i-1}}{\partial x^{m-i-1}} u(0,t) \right] \right] + \mathbb{S}_x \mathbb{S}_t \left[f(x,t) \right] \tag{5}$$

$$\sum_{n=0}^{N} a_n \left[\upsilon^{n\alpha} \mathbb{S}(\omega,\upsilon) - \sum_{k=0}^{n-1} \upsilon^{k\alpha-\beta} \mathbb{S}_{\omega} \left[g_{n-k-1}(x) \right] \right] =$$

$$\sum_{m=1}^{M} b_m \left[\omega^{m\alpha} \mathbb{S}(\omega,\upsilon) - \sum_{i=0}^{m-1} \omega^{i\alpha-\beta} \mathbb{S}_{\upsilon} \left[h_{m-i-1}(t) \right] \right] + \mathbb{S}_x \mathbb{S}_t \left[f(x,t) \right] \tag{6}$$

Equation (6) is an algebraic equation in $\mathbb{S}(\omega,\upsilon)$. Solving this algebraic equation and by taking inverse **Double Sadik Transform** of $\mathbb{S}(\omega,\upsilon)$, we get an exact solution $u(x,t)$.

Corollary 1: Show that solution of Telegraph equation

$$a_1 \frac{\partial}{\partial t} u(x,t) + \frac{\partial^2}{\partial t^2} u(x,t) = b_2 \frac{\partial^2}{\partial x^2} u(x,t) + f(x,t), \quad (x,t) \in \mathbb{R}_+^2 \tag{7}$$

subject to initial and boundary conditions

$$u(x,0) = g_0(x), \ \frac{\partial}{\partial t} u(x,0) = g_1(x), \ u(0,t) = h_0(t), \ \frac{\partial}{\partial x} u(0,t) = h_1(t) \tag{8}$$

exist by **Sadik Integral Transform**.

Proof: We consider linear, one-dimensional, time-dependent partial differential equation of the form (1) in which put $N=2, M=2, a_0=0, b_1=0$ and $a_2=1$ we get linear Telegraph equation (7) applying Sadik transform with respect to t to the equation (7) with initial and boundary conditions

$$a_1 \upsilon^{\alpha} \mathbb{S}_{\upsilon}\left[u(x,t)\right] - a_1 \upsilon^{-\beta} g_0(x) + \upsilon^{2\alpha} \mathbb{S}_{\upsilon}\left[u(x,t)\right] - \upsilon^{\alpha-\beta} g_0(x) - \upsilon^{-\beta} g_1(x)$$

$$= b_2 \frac{\partial^2}{\partial x^2} \mathbb{S}_{\upsilon}\left[u(x,t)\right] + \mathbb{S}_{\upsilon}\left[f(x,t)\right]$$

$$b_2 \frac{\partial^2}{\partial x^2} \mathbb{S}_{\upsilon}\left[u(x,t)\right] - \upsilon^{2\alpha} \mathbb{S}_{\upsilon}\left[u(x,t)\right] - a_1 \upsilon^{\alpha} \mathbb{S}_{\upsilon}\left[u(x,t)\right]$$

$$= -\mathbb{S}_{\upsilon}\left[f(x,t)\right] - a_1 \upsilon^{-\beta} g_0(x) - \upsilon^{\alpha-\beta} g_0(x) - \upsilon^{-\beta} g_1(x)$$

this equation is in the form of ordinary differential equation therefor using particular integral we get

$$\mathbb{S}_{\upsilon}\left[u(x,t)\right]=\frac{-\left(\mathbb{S}_{\upsilon}\left[f(x,t)\right]+a_1\upsilon^{-\beta}g_0(x)+\upsilon^{\alpha-\beta}g_0(x)+\upsilon^{-\beta}g_1(x)\right)}{\left(b_2D^2-\upsilon^{2\alpha}-a_1\upsilon^{\alpha}\right)} \tag{9}$$

after finding particular integral we can apply inverse sadik transform we get solution.

Corollary 2: Show that solution of Telegraph equation (7) subject to initial and boundary conditions (8) exist by **Double Sadik Integral Transform**.

Proof: Using preliminaries on equation (7) with (8) we get

$$\begin{aligned}a_1\upsilon^{\alpha}.\mathbb{S}(\omega,\upsilon)-a_1\upsilon^{-\beta}\mathbb{S}_{\omega}\left[g_0(x)\right]+\upsilon^{2\alpha}.\mathbb{S}(\omega,\upsilon)-\upsilon^{\alpha-\beta}\mathbb{S}_{\omega}\left[g_0(x)\right]-\upsilon^{-\beta}\mathbb{S}_{\omega}\left[g_1(x)\right]\\ = b_2\omega^{2\alpha}.\mathbb{S}(\omega,\upsilon)-b_2\omega^{\alpha-\beta}\mathbb{S}_{\upsilon}\left[h_0(t)\right]-b_2\omega^{-\beta}\mathbb{S}_{\upsilon}\left[h_1(t)\right]+\mathbb{S}_x\mathbb{S}_t\left[f(x,t)\right]\end{aligned}$$

$$\mathbb{S}(\omega,\upsilon)=\frac{\begin{pmatrix}\mathbb{S}_x\mathbb{S}_t\left[f(x,t)\right]+a_1\upsilon^{-\beta}\mathbb{S}_{\omega}\left[g_0(x)\right]+\upsilon^{\alpha-\beta}\mathbb{S}_{\omega}\left[g_0(x)\right]+\upsilon^{-\beta}\mathbb{S}_{\omega}\left[g_1(x)\right]\\ -b_2\omega^{\alpha-\beta}\mathbb{S}_{\upsilon}\left[h_0(t)\right]-b_2\omega^{-\beta}\mathbb{S}_{\upsilon}\left[h_1(t)\right]\end{pmatrix}}{\left(a_1\upsilon^{\alpha}+\upsilon^{2\alpha}-b_2\omega^{2\alpha}\right)}$$

applying inverse of double Sadik transform

$$u(x,t)=\mathbb{S}_x^{-1}\mathbb{S}_t^{-1}\left(\frac{\begin{pmatrix}\mathbb{S}_x\mathbb{S}_t\left[f(x,t)\right]+a_1\upsilon^{-\beta}\mathbb{S}_{\omega}\left[g_0(x)\right]+\upsilon^{\alpha-\beta}\mathbb{S}_{\omega}\left[g_0(x)\right]+\\ \upsilon^{-\beta}\mathbb{S}_{\omega}\left[g_1(x)\right]-b_2\omega^{\alpha-\beta}\mathbb{S}_{\upsilon}\left[h_0(t)\right]-b_2\omega^{-\beta}\mathbb{S}_{\upsilon}\left[h_1(t)\right]\end{pmatrix}}{\left(a_1\upsilon^{\alpha}+\upsilon^{2\alpha}-b_2\omega^{2\alpha}\right)}\right) \tag{10}$$

Applications of the Method

EXAMPLE 1: Consider the Telegraph Equation

$$3\frac{\partial}{\partial t}u(x,t)+\frac{\partial^2}{\partial t^2}u(x,t)=\frac{\partial^2}{\partial x^2}u(x,t)+3(x^2+t^2+1),\quad (x,t)\in\mathbb{R}_+^2 \tag{11}$$

with initial and boundary conditions

$$u(x,0)=g_0(x)=x,\ \frac{\partial}{\partial t}u(x,0)=g_1(x)=1+x^2$$

$$u(0,t)=h_0(t)=t+\frac{t^3}{3},\ \frac{\partial}{\partial x}u(0,t)=h_1(t)=t$$

SOLUTION: By the result (9) that is by Sadik transform

$$\mathbb{S}_\upsilon\left[u(x,t)\right]=\frac{-\left(3(x^2+1)\frac{1}{\upsilon^{\alpha+\beta}}+\frac{6}{\upsilon^{3\alpha+\beta}}+3\upsilon^{-\beta}x+\upsilon^{\alpha-\beta}x+\upsilon^{-\beta}(1+x^2)\right)}{\left(D^2-\upsilon^{2\alpha}-3\upsilon^{\alpha}\right)}$$

$$\mathbb{S}_\upsilon\left[u(x,t)\right]=\frac{\upsilon^{-\beta}}{\upsilon^{2\alpha}}\left(1+\left(\frac{D^2}{\upsilon^{\alpha}(3+\upsilon^{\alpha})}\right)-\ldots\right)\left[\frac{6}{\upsilon^{2\alpha}(3+\upsilon^{\alpha})}+\upsilon^{\alpha}x+\left(x^2+1\right)\right]$$

$$\mathbb{S}_\upsilon\left[u(x,t)\right]=\left[\frac{2\upsilon^{-\beta}}{\upsilon^{4\alpha}}+x\frac{\upsilon^{-\beta}}{\upsilon^{\alpha}}+\left(x^2+1\right)\frac{\upsilon^{-\beta}}{\upsilon^{2\alpha}}\right]$$

applying inverse Sadik Transform we get

$$u(x,t)=\frac{t^3}{3}+x^2t+x+t$$

now by the result (10) that is by **Double Sadik transform**

$$\because \mathbb{S}_x\mathbb{S}_t\left[f(x,t)\right]=\frac{6}{\omega^{3\alpha+\beta}\upsilon^{\alpha+\beta}}+\frac{6}{\omega^{\alpha+\beta}\upsilon^{3\alpha+\beta}}+\frac{3}{\omega^{\alpha+\beta}\upsilon^{\alpha+\beta}}$$

$$u(x,t)=\mathbb{S}_x^{-1}\mathbb{S}_t^{-1}\left(\frac{1}{\left(3\upsilon^{\alpha}+\upsilon^{2\alpha}-\omega^{2\alpha}\right)}\left(\begin{array}{l}\frac{6}{\omega^{3\alpha+\beta}\upsilon^{\alpha+\beta}}+\frac{6}{\omega^{\alpha+\beta}\upsilon^{3\alpha+\beta}}+\frac{3}{\omega^{\alpha+\beta}\upsilon^{\alpha+\beta}}+\upsilon^{-\beta}\mathbb{S}_\omega\left(1+x^2\right)\\+\left(3\upsilon^{-\beta}+\upsilon^{\alpha-\beta}\right)\mathbb{S}_\omega(x)-\omega^{\alpha-\beta}\mathbb{S}_\upsilon\left(t+\frac{t^3}{3}\right)-\omega^{-\beta}\mathbb{S}_\upsilon(t)\end{array}\right)\right)$$

$$u(x,t)=\mathbb{S}_x^{-1}\mathbb{S}_t^{-1}\left((\omega\upsilon)^{-\beta}\left(\begin{array}{l}\frac{6}{\omega^{3\alpha}\upsilon^{\alpha}}+\frac{6}{\omega^{\alpha}\upsilon^{3\alpha}}+\frac{3}{\omega^{\alpha}\upsilon^{\alpha}}+3\frac{1}{\omega^{2\alpha}}+\upsilon^{\alpha}\frac{1}{\omega^{2\alpha}}+\frac{1}{\omega^{\alpha}}+\frac{2}{\omega^{3\alpha}}\\-\frac{\omega^{\alpha}}{\upsilon^{2\alpha}}-\frac{2\omega^{\alpha}}{\upsilon^{4\alpha}}-\frac{1}{\upsilon^{2\alpha}}\end{array}\right)\right)$$

simplifying we get

$$u(x,t)=\mathbb{S}_x^{-1}\mathbb{S}_t^{-1}\left((\omega\upsilon)^{-\beta}\left(\frac{1}{\omega^{2\alpha}\upsilon^{\alpha}}+\frac{1}{\omega^{\alpha}\upsilon^{2\alpha}}+\frac{2}{\omega^{3\alpha}\upsilon^{2\alpha}}+\frac{2}{\omega^{\alpha}\upsilon^{4\alpha}}\right)\right)$$

now applying Inverse Double Sadik Transform

$$u(x,t)=\frac{t^3}{3}+x^2t+x+t$$

Conclusion

we get same solution of Telegraph Equation by Sadik Transform and Double Sadik Transform; we observe that by Sadik transform we get a differential equation of order 2 in terms of $\mathbb{S}_\upsilon\left[u(x,t)\right]$ but by Sadik Double transform we get an algebraic equation in terms of two variables ω,υ so Sadik Double Transform completely transfer partial differential equations in terms of algebraic equation which is easy to solve.

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