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Selection of Cross Over Ratio Factor in Differential Evolution Algorithm for FIR Filter Design

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Abstract

In this work, the effect of cross-over ratio parameter in Differential evolution (DE) algorithm for finite impulse response (FIR) filter is presented. DE is a population-based optimizer that obtains the solution by iteratively improving the given measure of quality. In this work, DE has been applied in FIR Filter design to optimize the frequency response and the signed-power-of-two (SPT) terms of the filter coefficients. DE is influenced mainly by three design parameters, Strategy S, Mutant factor F and Cross-over probability Cr. The focus of this paper is the Cross-over step of DE which involves Cr. Simulations with all possible combinations of Cr show that for $Cr = 1$, the error value is optimum.

AMS subject classification:
Keywords: FIR filter, Differential Evolution Algorithm, Cross-over ratio.

1. Introduction

A digital filter is a system that uses discrete time signal as input and produce a discrete time output signal for the purpose of achieving a filtering objective [1]. The main advantage of digital filters is that the characteristics of filter can be altered easily by changing the discrete values which are stored in the registers. The digital filters are classified into Infinite Impulse Response (IIR) filters, and Finite Impulse Response (FIR)

filters. However, FIR filters are widely used in almost all modern digital applications such as image processing, speech processing, channel equalization, signal enhancement, digital audio etc., because of their inherent stable response due to absence of poles in the transfer function and easily achievable linear phase property. Linear phase response which is required in applications such as data communications and crossover filters can be achieved by having symmetric coefficient FIR Filters [1, 2]. The core operations in FIR filter involves multiplication and accumulation of filter coefficients with the input digital data and those can be realized using as many multipliers and adders as the number of filter coefficients, respectively. Since multipliers are power and area consuming circuits, it is a common practice to use a multiplier-less realization where the multipliers are replaced with shift and adder circuits. In this approach, the filter coefficients are represented as sums or differences of signed-power-of-two terms (SPT). The adder cost depends on the number of SPT terms present in the filter coefficients, hence the reduction in complexity of FIR filters can be achieved by reducing the number of SPT terms in filter coefficients [3]. The canonical signed digit (CSD) is one of the methods to achieve a reduction of 33% of SPT terms [4].

Heuristic optimization algorithms like genetic algorithms (GAs) are highly popular in digital filter design [5, 6]. They are most widely used as global optimization methods. But genetic algorithms are weaker in determining local minimum in terms of convergence speed. To overcome these shortcomings of genetic algorithms, Differential Evolution algorithm (DE) is used. As most filter design tasks can be explained as problems to meet some given constraints or a tolerance level, DE is highly useful in digital filter design [7]. DE finds the true global minimum of a model search space regardless of the initial parameter values. It also has a very high convergence speed and uses only a few control parameters. DE is especially useful in solving unconventional filter design problems.

Differential evolution (DE) algorithm is a stochastic, population-based optimization algorithm introduced by Storn and Price [8, 9]. DE is a powerful non-deterministic algorithm, which searches for a solution, by generating and refining the search spaces continuously. In every iteration the results are tested for their fitness to survive, and whether they can form a part of the solution. Each set of solutions is said to form one generation. The major steps in a DE algorithm for FIR filter design are Initialization, Evaluation, Mutation, Recombination, Evaluation and Selection [8].

The algorithm will search from a search space of random solutions defined in Initialization, and iteratively refines the search space. It will sample out some solutions randomly, calculate a vector difference of them and apply this to the current solution.This step is known as Mutation. DE also employs Recombination to complement the Mutation step. It uses cross over probability ratio to create a new set of solution by controlling the fraction of mutant in the new solution. Thus the current solution now gets mutated and hence may or may not be better than before. If it is found to yield a better output, it is retained, else it is discarded, known as Evaluation and Selection. The above sequence of steps is repeated for a sufficiently large number of times. Several studies have been done on the importance of variation in mutant strategies, population sizes or recombination step in DE algorithm for FIR filter design [10, 11]. However, in this work, the focus is on variations in cross-over ratio for different iterations and filter order.

The implementation of FIR filter using DE is explained in section II. The simulation results of various cross-over ratio combinations are analyzed in section III. Finally, the conclusions are discussed in section IV.

2. Implementation of FIR filter Using DE Algorithm

In this section, the optimization problem for implementing the FIR filter using DE Algorithm is presented. The filter is designed to minimize the error function without compromising on the frequency response.

The frequency response $H(\omega)$ of a Type-II linear phase FIR filter is described in (1) as it is defined in [1]

$$H(\omega) = \sum_{n=0}^{N-1} h(n)\cos(\omega n) \tag{1}$$

The filter has positive symmetric coefficients h(n) and it can be expressed as in (2)

$$h(n) = h(N-n-1)\, for\ n = 0, 1, \ldots, N-1 \tag{2}$$

2.1. Problem Formulation

The FIR filter optimization problem for finding the filter coefficients is designed to optimize the magnitude error (E_{MAG}). Thus, the E_{MAG} optimization problem is described as follows:

$$\begin{aligned} &\text{minimize: } E_{MAG} = \delta_{PR} - \delta_{PD} \\ &\text{subject to: } 1 - \delta_P \leq H(\omega) \leq 1 + \delta_P \text{ for } \omega\epsilon[0, \omega_P] \\ &\qquad\qquad -\delta_S \leq H(\omega) \leq \delta_S \text{ for } \omega\epsilon[\omega_S, \pi] \end{aligned} \tag{3}$$

Here δ_{PD} is the difference of ideal peak pass band and peak stop band attenuation and δ_{PR} is the difference of peak pass band and peak stop band attenuation obtained in every iteration. The error function E_{MAG} as defined in (3) will be obtained in every iteration.

2.2. Methodology of FIR Filter Design

The Methodology of FIR Filter Design using DE is explained with the help of the flowchart shown in Fig.1. The inputs for the algorithm are defined by pass band frequency 'ω_P', stop band frequency 'ω_S', pass band attenuation 'δ_P' and stop band attenuation 'δ_S'. Along with these, the DE parameters like 'I' maximum number of iterations, 'F' mutant factor in the range [0,1], strategy 'S', *OCV*: Optimum cost value, N_P: Size of the population of coefficient vectors etc. are also defined. The order of the filter 'N' is obtained by using kaiser order formula from [12]. The algorithm starts with generation of an initial filter coefficient set with order 'N'. With Mutation and crossover steps, a new set of coefficient vectors will be generated. Scaling factor (sf) is evaluated for this each set of filter coefficient vectors. Scaling factor (sf) is the ratio of 2^B to maximum valued coefficient set, which are multiplied by wordlength B. Hence, the quantized filter

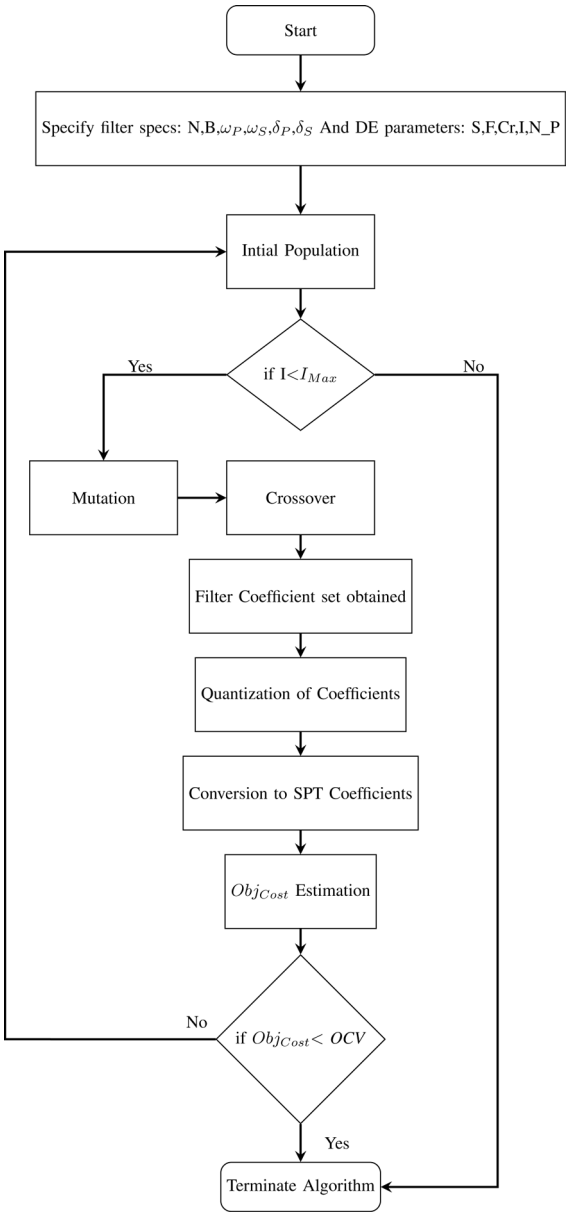

Figure 1: DE Flow Chart.

coefficients will be obtained by multiplying each set of coefficients with 2^B and its respective sf. The obj_{cost} will be evaluated using (4) and compared with OCV. The obj_{cost} value is estimated in each and every iteration untill it becomes less than OCV or the maximum number of iteration occurs.

The obj_{cost} estimation is done using (4).

$$\begin{aligned} obj_{cost} &= obj_{cost} + \eta \quad \text{if } E_{MAG} < 0 \\ &= obj_{cost} \ \text{ Otherwise;} \end{aligned} \tag{4}$$

Here η is a real number. The flowchart shown in Fig. 2 explains the obj_{cost} estimation for one iteration.

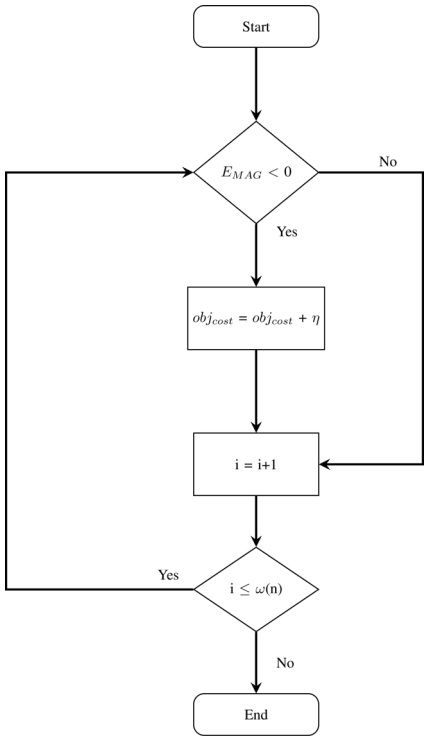

Figure 2: obj_{cost} Estimation.

For the obj_{cost} calculation in every iteration, the frequency ω is split into 'n' number of samples between $[0, \pi]$. Where is the index and the loop is run from i = 1 to 'n' samples. The obj_{cost} value is the number of frequency samples at which the obtained response has not matched the ideal response.The obj_{cost} is incremented by η for all those frequency samples for which either E_{MAG} is negative. Hence, lower the obj_{cost}, more are the number of points which have the actual response equal to or better than the desired response.

3. Results

In this section various examples are given to illustrate the proposed DE algorithm. The filter specifications are described in Table 1. Three filters namely DE_F1, DE_F2, DE_F3 with orders 30,40,50 respectively are defined. The passband and stopband attenuations along with the ripple values are kept constant for these filters. These filter specifications are given as an input to the DE algorithm while the design parameters of DE like 'F', 'S', and 'Cr' for different values of 'I'. The other DE parameter N_P are kept constant. The major steps in a DE algorithm for FIR filter design are Initialization, Mutation,

Table 1: Specification of Filter.

Filters	**N**	ω_p	ω_s	δ_p	δ_s
DE_F1	30	0.2π	0.4π	0.00316	0.00316
DE_F2	40	0.2π	0.4π	0.00316	0.00316
DE_F3	50	0.2π	0.4π	0.00316	0.00316

Recombination, Evaluation and Selection. Among the above steps, Recombination is the main focus in this work.

3.1. Recombination

DE maintains two population vectors S_x and S_m both of size N_P with each row having L elements. Here, L is the number of parameters to be determined. The current population, given by S_x is composed of vectors $X_{i,g}$ which is the initial population. Each $X_{i,g}$ vector consists of $x_{j,i,g}$ elements with j varying from 0 to $L-1$. Here g represents the current generation. This step is used to create the trial vectors for S_c population. It complemented the mutation step. Crossover basically mixes up the two populations based on the crossover probability (Cr). It is pre-defined by the user like F and decides which of the two populations S_x or S_m supplies the trial vector. If the random number generated in the range (0, 1) was less than Cr, then the mutant element was copied otherwise the element from S_x was taken. Also, in addition, the trial parameter with randomly chosen index, $rand_j$, was taken from the mutant to ensure that the trial vector does not duplicate $X_{i,g}$

$$\begin{aligned} C_{j,i,g} &= m_{j,i,g} \quad \text{if } rand_j(0,1) \leq Cr \text{ or } j = j_{rand} \\ &= x_{i,j,g} \quad \text{otherwise.} \end{aligned} \tag{5}$$

In DE, various strategies 'S' have been established to counter different types of filter design tasks. These various strategies are basically variations in the mutation step of the algorithm. In total, six strategies have been devised and each strategy has a different method of doing mutation [8, 9]. In one of our earlier works, we concluded that $S = 2$ and $F = 0.7$ are suitable for FIR filter design [13]. Hence, the Cr values are chosen in such a way that, they recombine the population vectors effectively and converge to a better solution. Since DE is sensitive to Cr, selection of this parameters require detailed analysis.

In this work, the filters described in Table 1 are designed for various combinations of 'Cr'. 'F' and 'S' are kept as '0.7' and '2' respectively. Population 'N_P' is kept as ten times of the filter order, for all the filter designs. Initially, DE_F1 is designed for different values of 'I' = 50, 75, 100. As mentioned in section II, lower the obj_{cost} value, better the frequency response of the filter. Hence, the obj_{cost} is estimated using the method described in Fig. 2 for Cr = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1 for each I. The other parameters namely η, 'n', OCV are kept as '1' ,'8196' and '1' respectively. The value of 'n' as 8196, signifies the fact that the obj_{cost} value will vary from the best value

0 to the worst value 8196. The estimated values of the obj_{cost} for 'I' = 50, 75, 100 are given in Table 2 respectively.

Table 2: Cross Over Comparision.

N	Iteration	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
DE_F1	50	3880	4550	5045	4575	4483	4617	4757	4370	4249	2813
	75	4411	4295	4842	4723	4904	4697	4567	4793	3978	2426
	100	4473	4508	4390	5064	4827	4795	4546	4294	3608	2511
DE_F2	50	5412	5369	5386	5385	5504	5351	5325	5414	4708	3405
	75	4292	4722	4757	4498	5009	5130	4877	5235	3945	2806
	100	4864	4864	4644	4904	4723	4342	4342	3936	4098	1721
DE_F3	50	5217	5237	5011	4755	5025	5720	4906	4846	4715	3387
	75	4749	5106	4795	5181	5174	4893	4967	5115	4470	3514
	100	4912	4589	4659	4859	4743	4846	5106	4803	4018	2461

By referring to Table 2, for $N = 30$, it is observed that the minimum obj_{cost} value is achieved for $Cr = 1$. Table 2 shows that for $Cr = 1$ gives lower obj_{cost} values for $I = 75$ as compared to $I = 50$. The values decrease due to increase in number of iterations resulting in a better solution. With the further increase in number of iterations to 100, it is observed from Table 2 that the obj_{cost} value is further minimized for $Cr = 1$. However, Cr values less than 0.6, are not suitable for as each of them give high obj_{cost} values for all three values of I. Also, $Cr = 0.7$ and 0.8 have high obj_{cost} values for any value of I. Unlike other Cr, the obj_{cost} value does not reduce for any value of Cr with increase in I, pointing to the fact that they have low convergence speed and need more number of iterations to find an optimum solution. From these results, for $Cr = 1$, the obj_{cost} value is consistently minimized with the increase in number of iterations which can also be seen in Fig. 3.

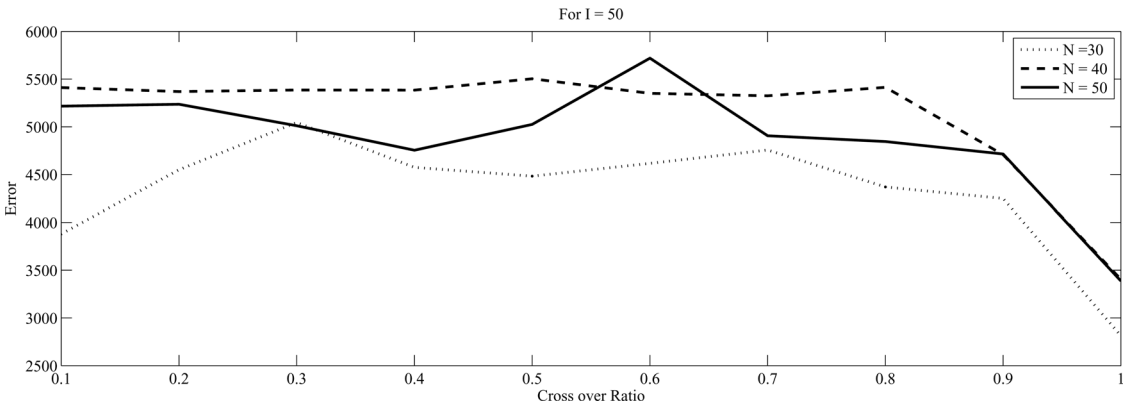

Figure 3: Comparison of 'Cr' for $N = 30, 40, 50$ with $I = 50$.

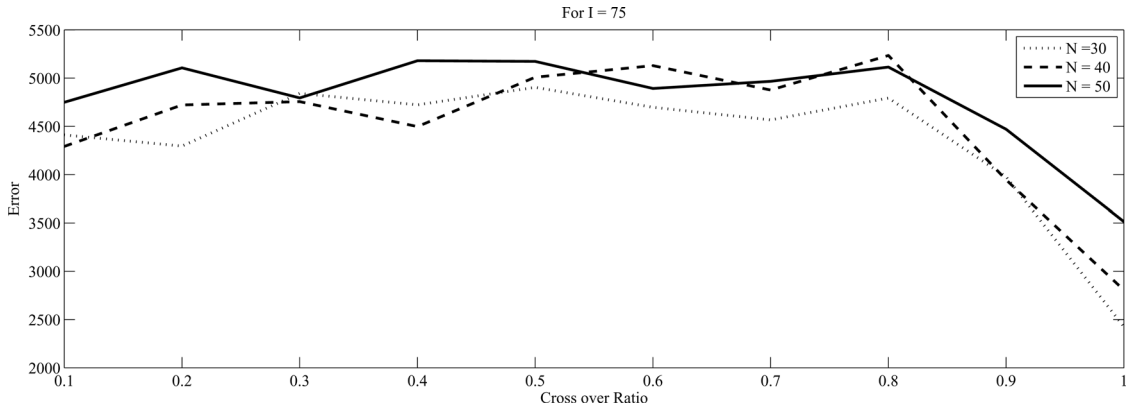

Figure 4: Comparison of 'Cr' for $N = 30, 40, 50$ with $I = 75$.

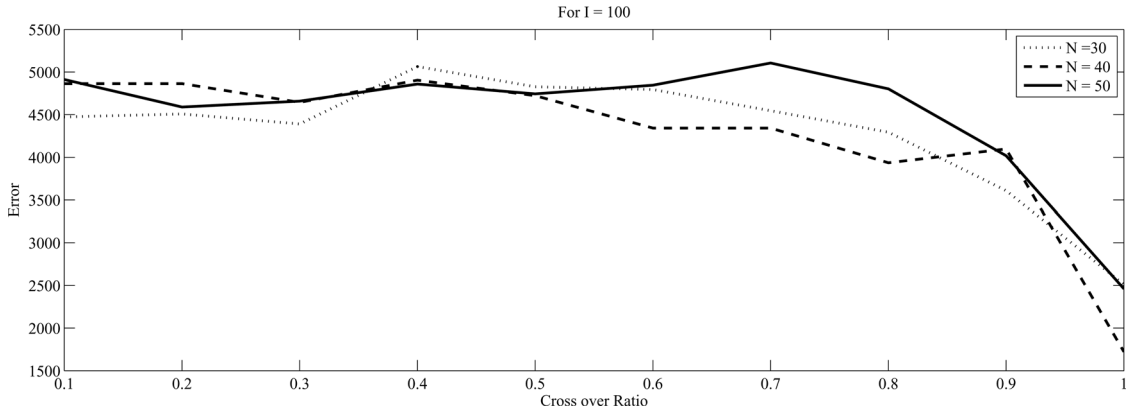

Figure 5: Comparison of 'Cr' for $N = 30, 40, 50$ with $I = 100$.

The filter coefficients obtained for DE_F1 filter in each iteration obtains the frequency response and matches with the ideal frequency response. The coefficients obtained are of positive symmetry as given in [1]. With frequency response being achieved by Cr, the measure of optimization is considered, which is, minimizing the obj_{cost}. The obj_{cost} is estimated for the coefficients obtained in each iteration. This estimation shows that, DE_F1 designed with Cr =1 gives minimum obj_{cost}. However, the obj_{cost} is also depends on number of iterations I. As I increases, the obj_{cost} value decreases. This can be observe from Table 2. For DE_F1 filter as the number of iterations increases the obj_{cost} is decreases. To extend this analysis to higher order filters, DE_F2 and DE_F3 with filter order 40 and 50 respectively were also designed for various Cr values. The obj_{cost} values for each of these designs are detailed in Table 2. For $I = 75$ and 100 the obj_{cost} values aare plotted against the Cr values in Fig. 4 and 5 respectively. It can be observed from Table 2 that, the obj_{cost} values for $Cr = 1$ are very minimum for all the

three filter designs. This means that $Cr = 1$ gives nearly the same frequency response as the order increases up till 50.

4. Conclusion

In this paper, a detailed analysis on cross-over ratio in DE algorithm is carried out for the design of FIR filter. The error value is minimized to achieve optimized filter design. Three filters with different orders are designed for various values of cross-over ratio from 0.1 to 1 respectively. The results show that the frequency response of the filters designed with cross-over ratio $Cr = 1$, match the ideal frequency response with minimum error value.

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