International Journal of Applied Engineering Research
ISSN 0973-4562 Volume 10, Number 10 (2015) pp. 24917-24935

http://www.ripublication.com

Soret And Dufour Effects On Unsteady MHD Free Convection Flow Of Casson Fluid Past A Vertical Plate Embedded In A Porous Medium With Convective Boundary Condition

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Abstract

This article studies the Soret and Dufour effects on unsteady MHD free flow of Casson fluid over a vertical plate with a constant wall temperature. The fluid is electrically conducting and passing through the porous medium. The governing nonlinear partial differential equations have been reduced to the coupled nonlinear ordinary differential equations by the similarity transformations. The governing equations are solved numerically using MATLAB bvp4c routine and presented through graphs. The numerical values of skin-friction coefficient and Nusselt number are derived, discussed numerically for various values of physical parameters and presented through tables.

Keywords: Casson fluid, MHD flow, Porous medium, free convection, stretching vertical plate.

1. Introduction:

The unsteady magneto hydrodynamic (MHD) flow of a non-Newtonian fluid in porous medium has attracted the attention of many researchers. Of course, it is due to the fact that such phenomenon is mostly found in optimization of solidification process of metals and metal alloys, the geothermal sources investigation and nuclear fuel debris treatment. However non-Newtonian fluids are subtle compare to Newtonian fluids. Indeed the resulting equations of non-Newtonian fluids given

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highly nonlinear differential equations which are usually difficult to solve. These equations add further complexities when MHD flows of non-Newtonian fluid in a porous medium are encountered in irrigation problems, heat-storage beds, biological systems, process of petroleum, textile, and paper and polymer composite industries. Rajesh [1] has studied radiation effects on MHD free convection flow near a vertical plate with ramped wall temperature. Loganathan [2] studied the effects of thermal conductivity on unsteady MHD free convective flow over a semi infinite vertical plate. Palani and Srikanth [3] studied the MHD flow of an electrically conducting fluid over a semi-infinite vertical plate under the influence of the transversely applied magnetic field. Makinde [4] investigated the MHD boundary layer flow with the heat and mass transfer over a moving vertical plate in the presence of magnetic field and convective heat exchange at the surface.

When heat and mass transfer occur simultaneously in a moving fluid, the relation between the fluxes and the driving potential becomes complex. The Soret effect (or thermal diffusion) is mainly the occurrence of a diffusion flux due to a temperature gradient, whereas the Dufour effect is due to a heat flux as a result of chemical potential gradients. Such effects are significant in the fields of geosciences and chemical engineering. Hayat et al. [5] studied the magneto hydrodynamics flow of a Casson fluid over a stretching surface with Soret and Dufour effects. Mohamad [6] studied the Soret effect on the unsteady magneto hydrodynamics (MHD) free convection heat and mass transfer flow past a semi-infinite vertical plate in a Darcy porous medium in the presence of chemical reaction and heat generation. Shyam et al. [7] examined the Soret and Dufour effects on the MHD natural convection over a vertical surf ace embedded in a Darcy porous medium in the presence of thermal radiation. Numerical investigation of Dufour and Soret effects on unsteady MHD natural convection flow past vertical plate embedded in non-Darcy porous medium was investigated by Al-Odat and Al- Ghamdi [8]. Subhankar and Gangadhar [9] studied Soret and Dufour effects on MHD Free convection heat and mass transfer flow over a stretching vertical plate with suction and heat source or sink . Soret and Dufour Effects on mixed convection from an exponentially stretching surface was discussed by Srinivasacharya and RamReddy [10]. Md Enamul Karim et al. [11] have analyzed the Dufour and Soret effect on steady MHD flow in presence of heat generation and magnetic field past an inclined stretching sheet.

The purpose of the present paper is to study the unsteady MHD Casson fluid flow over an infinite vertical plate embedded in a saturated porous medium. The influence of the Soret and Dofour effect on the free convective heat and mass transfer on the unsteady flow over a plate is investigated. We diminish the system of nonlinear partial differential equations into the system of nonlinear ordinary differential equations. Nonlinear coupled equations are then attempted numerically to get the solutions and behaviors of non-dimensional corporal constraints namely Casson parameter β, Hartmann number M, slip parameter a, Soret number Sr, Dofour number D_f, Biot number Bi, Prandalt number Pr, Schimdt number Sc are exposed graphically and in tabular form.

2. FORMULATION OF THE PROBLEM

We consider the unsteady MDH free convection flow an incompressible Casson fluid over an infinite vertical flat plate embedded in a saturated porous medium. The flow being confined to $y > 0$, where y is the coordinate measured the normal direction of the plate. The fluid is assumed to be electrically conducting with a uniform magnetic field B, applied in a direction perpendicular to the plate. The fluid is assumed to be Newtonian and electrically conducting. Under the above assumptions the governing boundary layer equations of the flow field are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}=\nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2}{\rho}u+g\beta_T(T-T_\infty)+g\beta_c(C-C_\infty) \tag{2}$$

$$\frac{\partial T}{\partial t}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{D_m K_T}{C_s C_p}\frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$\frac{\partial C}{\partial t}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

The boundary conditions for the velocity, temperature and concentration fields are

$$\begin{aligned}
&u=0,\theta=0,\phi=0 \quad for\ t<0,\forall y>0 \\
&u=\frac{cx}{(1-\lambda t)}+K\frac{\partial u}{\partial y},\ -k\frac{\partial T}{\partial y}=h_f(T_f-T),\ C=C_w && \text{at } y=0, t\ge 0 \\
&u\to 0,\ T\to T_\infty,\ C\to C_\infty, && \text{as } y\to\infty, t\ge 0
\end{aligned} \tag{5}$$

where u, T and C are y −component of velocity, temperature and concentration, respectively, ν is the kinematic viscosity, ρ is the density, σ is the electric conductivity of the fluid, β_T and β_C are coefficients of thermal and concentration expansions, respectively, α is the thermal conductivity, C_∞ is the free stream concentration, B is the magnetic induction, D_m is the mass diffusivity and g is the gravitational acceleration, T_w is the temperature of the hot fluid at the surface of the plate, C_w is the species concentration at the plate surface . In the present problem we have introduce velocity slip condition and convective boundary condition at the surface of the plate. It is assumed that the bottom surface of the plate is heated by convection from a hot fluid of temperature T_f which related with the heat transfer coefficient $h_f = k(1-\nu t)^{-1/2}$ where k is the thermal conductivity. $K = K_0(1-\lambda t)^{1/2}$ is the slip constant. The continuity equation (1) is satisfied by the considering velocities in the form of stream function as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$.

To transform equations (2) - (4) into a set of ordinary differential equations, the following similarity transformations and dimensionless variables are introduced.

$$\eta = y\sqrt{\frac{c}{\nu(1-\lambda t)}},\ \psi = x\sqrt{\frac{c}{\nu(1-\lambda t)}}\, f(\eta),\ T = T_\infty + T_w\left(\frac{cx}{2\nu(1-\lambda t)^2}\right)\theta(\eta),\ C = C_\infty + C_w\left(\frac{cx}{2\nu(1-\lambda t)^2}\right)\phi(\eta), \tag{6}$$

Using the above transformation Esq. (2)-(4) can be written as

$$\left(1+\frac{1}{\beta}\right) f''' - \frac{A}{2}\eta f'' - Af' + Gr\,\theta + Gc\,\phi - Mf' = 0, \tag{7}$$

$$\theta'' - \Pr\left(\frac{A}{2}\theta' + 2A\theta - D_f \phi''\right) = 0, \tag{8}$$

$$\phi'' - Sc\left(\frac{A}{2}\phi' + 2A\phi - D_f \theta''\right) = 0, \tag{9}$$

The corresponding boundary conditions are

$$\begin{aligned} &f = 0,\ f' = 1 + a_0 f'',\ \theta' = -Bi(1-\theta(0)),\ \phi = 0 && \text{at } \eta = 0 \\ &f' \to 1,\ \theta \to 0, \phi \to 0 && \text{as } \eta \to \infty \end{aligned} \tag{10}$$

where $M = \frac{\sigma {B_0}^2(1-\lambda t)}{\rho c}$ is the magnetic parameter, $Gr = \frac{g\beta_T T_w}{2cv}$ is the thermal Grashof number, $Gc = \frac{g\beta_c C_w}{2cv}$ is the solutal Grashof number, $D_f = \frac{D_m K_T C_w}{C_s C_p T_w v}$ is the Dufour number, $Sr = \frac{D_m K_T T_w}{v\, T_m C_w}$ is the Soret number, $Pr = \frac{v}{\alpha}$ is the Prandtl number, , $Sc = \frac{v}{D_m}$ is the Schmidt number, $A = \frac{\lambda}{c}$ is the unsteadiness parameter, $Bi = \frac{h_f}{k}\sqrt{\frac{v}{c}}$ is the Biot number.

Skin friction, heat and mass transfer coefficients

The quantities of practical interest in this study are the skin friction coefficient, heat transfer rate and mass transfer rate, which is respectively defined as

$$C_f = \frac{\tau_w}{\rho U_\infty^2},\ \mathrm{Nu}_x = \frac{x q_w}{k(T_f - T_\infty)},\ \mathrm{Sh}_x = \frac{x q_m}{D_s(C_w - C_\infty)} \tag{11}$$

The shear stress at the surface of the wall τ_w is defined as

$$\tau_w = \left(\mu_B + \frac{\tau_y}{\sqrt{2\pi_c}}\right)\frac{\partial u}{\partial y}\bigg|_{y=0} = \left(\mu_B + \frac{\tau_y}{\sqrt{2\pi_c}}\right)\left(\frac{a^3}{\nu}\right)^{1/2} x f''(0) \tag{12}$$

Using the above equation, the skin friction is written as

$$\mathrm{Re}_x^{1/2} C_f = \left(1+\frac{1}{\beta}\right) f''(0) \tag{13}$$

The surface flux at the wall is defined as

$$q_w = -k\frac{\partial T}{\partial y}\bigg|_{y=0} = -k(T_w - T_\infty)\left(\frac{a}{\nu}\right)^{1/2}\theta'(0) \tag{14}$$

Using the above equation, the Local Nusselt number Nu_x is written as

$$\mathrm{Re}_x^{1/2}\,\mathrm{Nu}_x = -\theta'(0) \tag{15}$$

The mass flux at the surface is written as

$$q_m = -D_s\frac{\partial C}{\partial y}\bigg|_{y=0} = -D_s(C_w - C_\infty)\left(\frac{a}{\nu}\right)\phi'(0) \tag{16}$$

Using the above equation, the local Sherwood number Sh_x written as

$$\mathrm{Re}_x^{-1/2}\,\mathrm{Sh}_x = -\phi'(0) \tag{17}$$

Results and discussions

In the present study, we solved the system of differential equations (7)-(9) with boundary condition (10) to quantify the effects of various physical parameters. We used similarity variables to transform the governing partial differential equations into coupled ordinary differential equations and then solved by using 'bvp4c' Matlab package. A comparison is made with Khalid et al. [12] to justify our numerical method and it shows a very good agreement which provides a confidence on our results. The effect of the different parameters on the velocity, temperature and concentration of the fluid and skin friction, local Nusselt number and local Sherwood number are shown by graphs and in tabular form.

Figure (1) to (10) describes the effect of different parameters on the velocity profile. It is now a well established fact that the magnetic field presents a damping effect on the velocity field by creating drag force that opposes the fluid motion, causing the velocity to decrease with increase of the magnetic field. A backflow is observed near to the wall for non-dimension axial direction between Fig. 2 to Fig. 8. The momentum boundary layer thickness decreases with increase in the Biot number as shown in Fig. 2, which appear due to convection. Fig. 3 shows the variation of the velocity boundary-layer with the unsteadiness parameter. It is found that the velocity boundary layer thickness decreases with an increase in the unsteadiness parameter. In Fig.4, the fluid velocity is found to increase with increase of Casson parameter β. Fig. 5 shows the variation of the velocity boundary-layer with the Dufour number. It is observed that the velocity boundary layer thickness decreases with an increase in the Dufour number near to the wall while as goes away from the surface the velocity boundary layer thickness increases with an increase in the Dufour number. Fig.6 illustrates the effect of the thermal Grashof number on the velocity field. The thermal Grashof number signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force. The flow is accelerated due to the enhancement in buoyancy force corresponding to an increase in the thermal Grashof number, i.e. free convection effects. It is noticed that the thermal Grashof number influence the velocity field almost in the boundary layer when compared to far away from the plate. It is seen that as the thermal Grashof number increases, the velocity field increases up to zero. Then it is decreases with increases the value of Gr. The effect of solutal Grashof number on the velocity is illustrated in Fig.7. The solutal Grashof number defines the ratio of the species buoyancy force to the viscous hydrodynamic force. It is noticed that the velocity increases with increasing values of the solutal Grashof number. Fig.8 shows that decrease in the fluid velocity near the vertical plate is observed with an increase in Prandtl number. Fig.9 illustrates the effect of the Schmidt number on the velocity. The Schmidt number embodies the ratio of the momentum diffusivity to the mass (species) diffusivity. It physically relates the relative thickness of the hydrodynamic boundary layer and mass-transfer (concentration) boundary layer. It is noticed that as Schmidt number increases the velocity field decreases of the positive quadrant. Fig.10 shows that the velocity of the fluid decreases with increase of the Soret number.

As per the boundary conditions of the flow field under consideration, the fluid temperature attains its maximum value at the plate surface and decreases exponentially to the free stream zero value away from the plate. This is observed in Figs. 11-14. The effect of unsteadiness parameter on the temperature field is illustrated Fig.11. As the unsteadiness parameter increases the thermal boundary layer is found to be decreasing near the surface then it increases. Fig. 12 shows the variation of the thermal boundary-layer with different Dufour number. It is noticed that the thermal boundary layer thickness increases with an increase in the Dufour number near the surface. Fig. 13 illustrates the effect of Schmidt number on the temperature. It is noticed that as the Schmidt number increases, an decreasing trend in the temperature field near the surface but the difference is not significant for higher value of Schmidt number. From Fig.14, it is evident that with the Prandtl number increases, the thermal boundary layer thickness decreases.

Figs. 15 - 20 depict chemical species concentration against span wise coordinate η for varying values of physical parameters in the boundary layer. The species concentration is highest at the plate surface and decrease to zero far away from the plate satisfying the boundary condition. Fig.15 depicted that the concentration boundary layer thickness decreases with increases in the unsteadiness parameter. Fig. 16 illustrates the effect of Biot number on the concentration. It is noticed that as the Biot number increases, there is a decreasing trend in the concentration field. The concentration boundary layer thickness decreases with an increase in the Dufour number as shown in Fig. 17. It is observed the concentration boundary layer thickness decreases with an increase in the Prandtl number in Fig. 18 From Fig. 19, it is noticed that as the Schmidt number increases, there is a decreasing trend in the concentration field. Fig. 20 shows the variation of the concentration boundary-layer with the Soret number. It is found that the concentration boundary layer thickness increases with an increase in the Soret number near the surface. It is decreases with an increase the Soret number far away from the surface.

In order to benchmark our numerical results, the present results for the skin -friction, Nusselt number and Sherwood number for different parameter. Table.2 shows that The skin-friction decreases with increases the different parameters thermal Grashof number and solute Grashof number. It was notice that if the Casson parameters increases the skin-friction and local Sherwood number increases, while local heat and mass transfer decreases. As the Prandtl number increases, both the skin-friction and Sherwood number increase, whereas the Nusselt number decreases. It was found that the local heat and mass transfer rate at the plate increases, skin-friction coefficient increases, local Sherwood number increases with an increase in the Schmidt number. It is observed that the local heat and mass transfer rate at the plate decreases, whereas Skin-friction coefficient and Sherwood number increases with an increase of the unsteadiness parameter. For the case of Biot number, it was found that with increases the Biot number the local skin-friction and heat and mass transfer rate decreases while local Sherwood number increases. The effect of the Soret number is to increase the Skin-friction and the Sherwood number, and to decrease the Nusselt number.

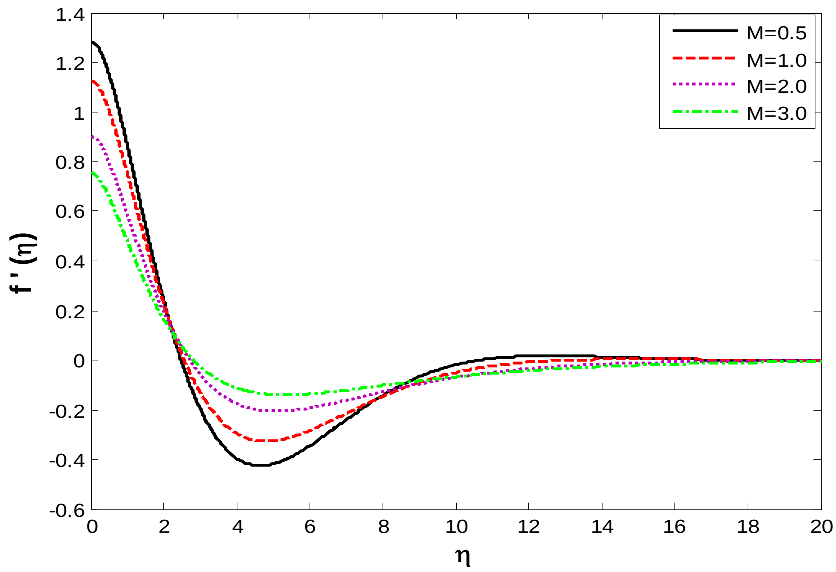

Fig.1.Variation of the velocity for different magnetic parameter.

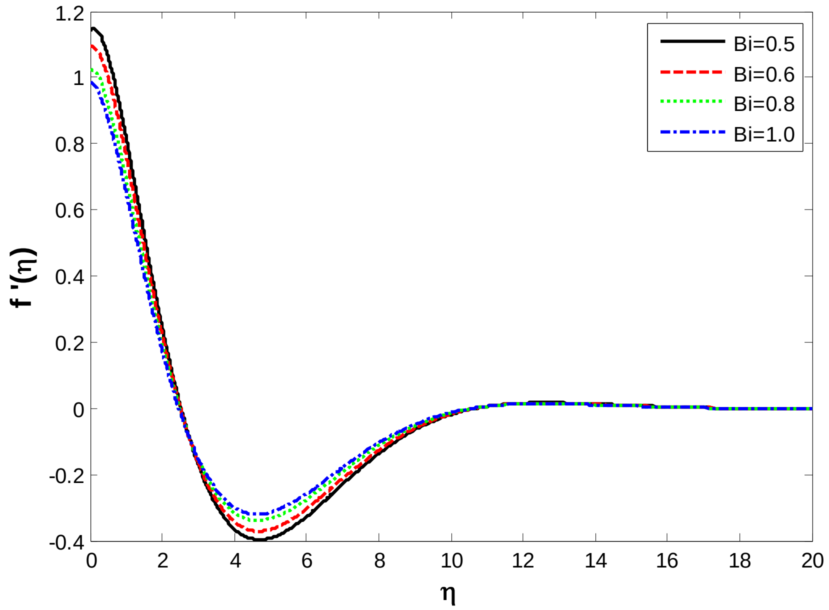

Fig.2: Variation of the velocity for different Biot number.

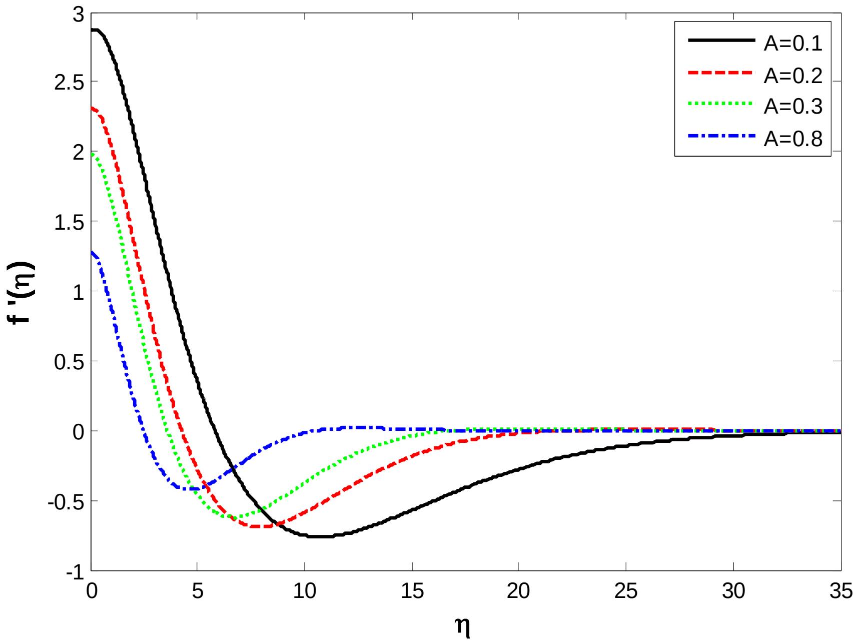

Fig.3: Variation of the velocity for different unsteadiness parameter.

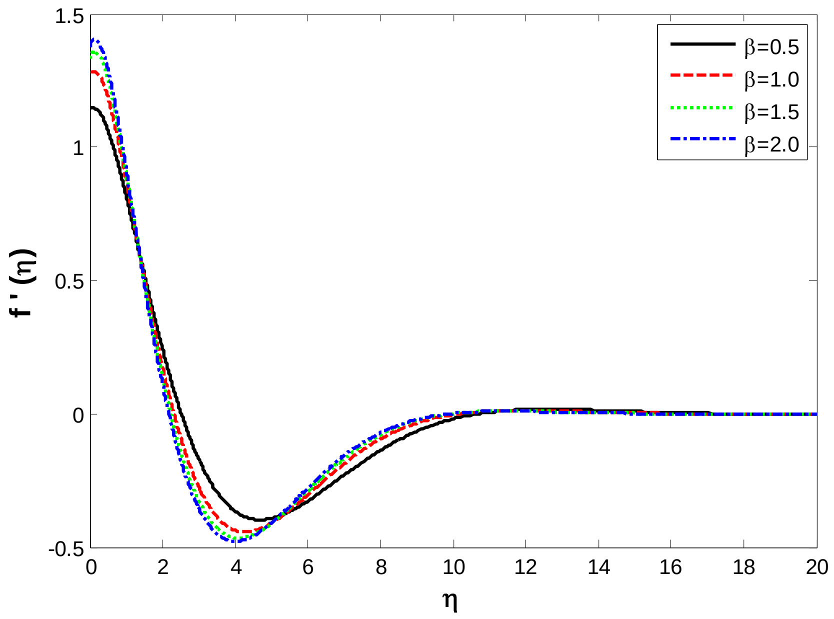

Fig.4: Variation of the velocity for different Casson parameter.

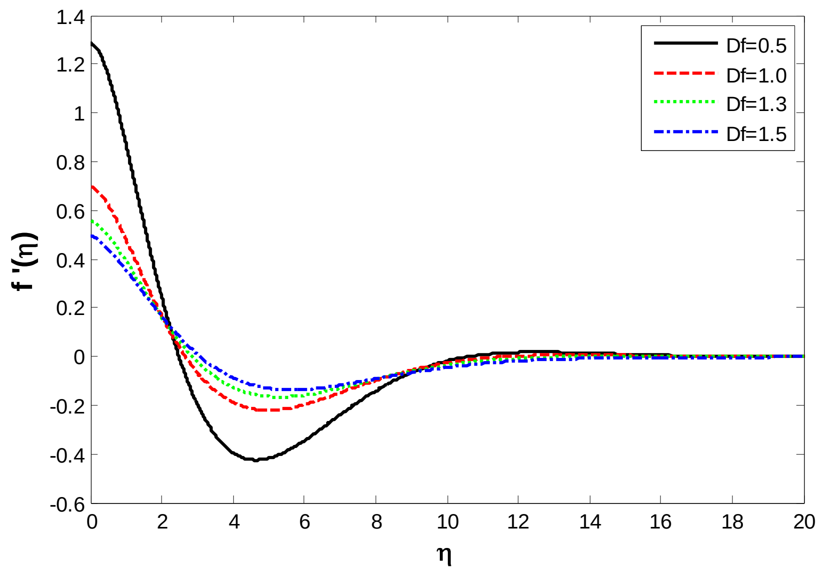

Fig.5: Variation of the velocity for different Dofour number.

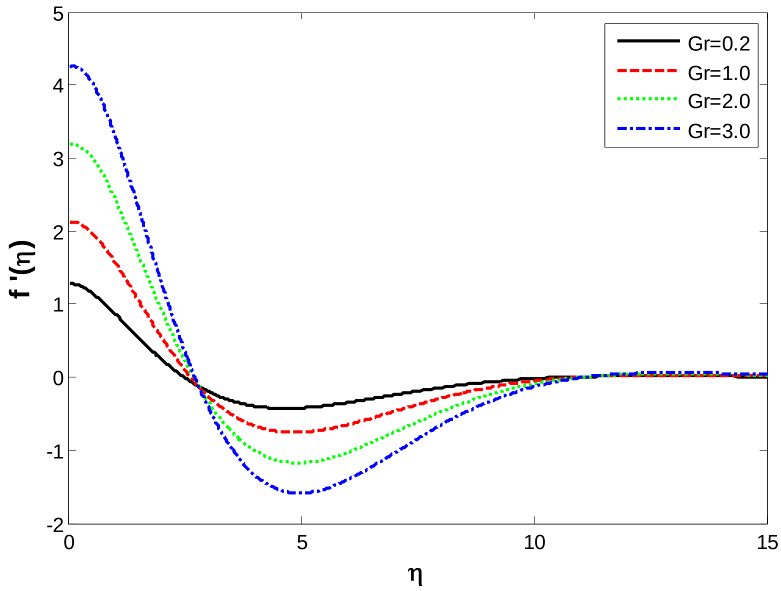

Fig.6: Variation of the velocity for different thermal Grashof number.

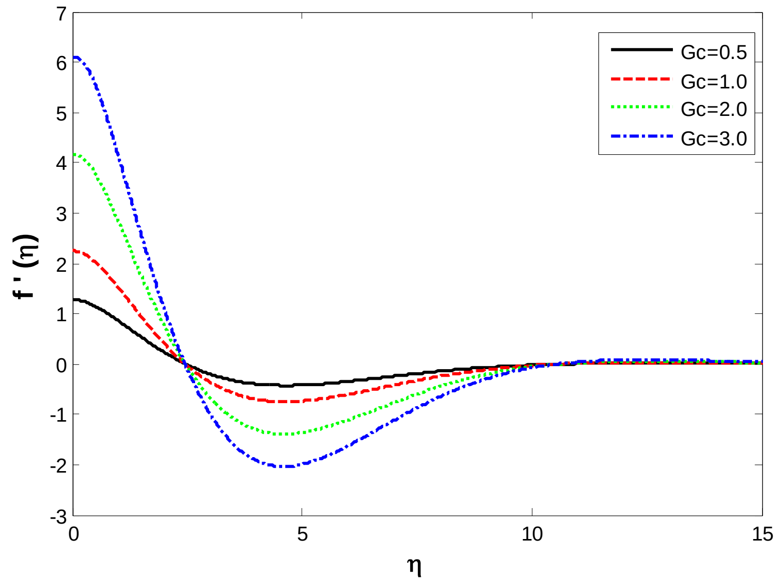

Fig.7: Variation of the velocity for different solutal Grashof number.

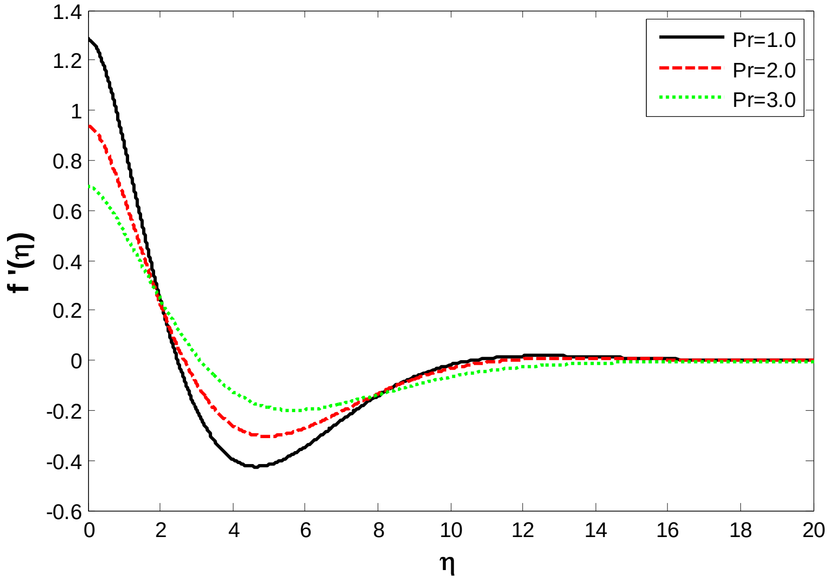

Fig.8: Variation of the velocity for different Prandtl number.

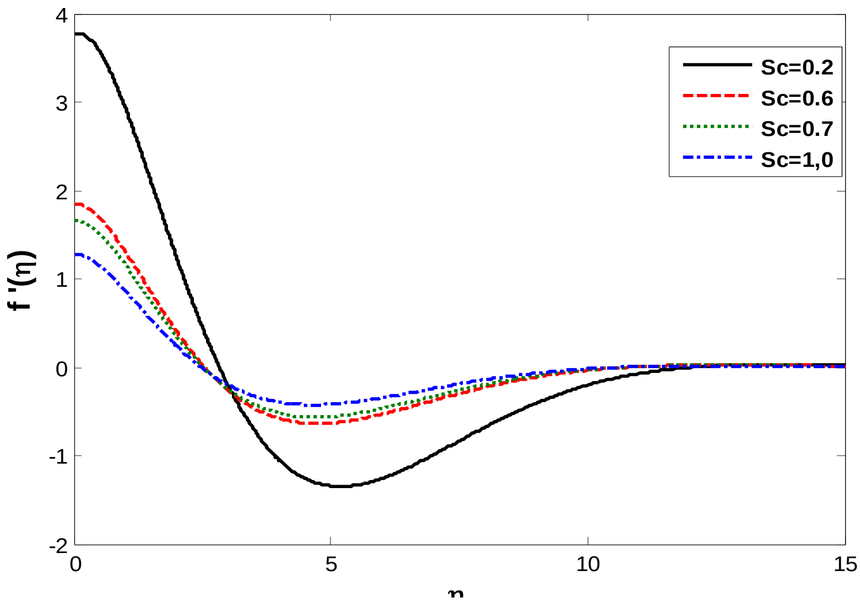

Fig.9: Variation of the velocity for different Schmidt number.

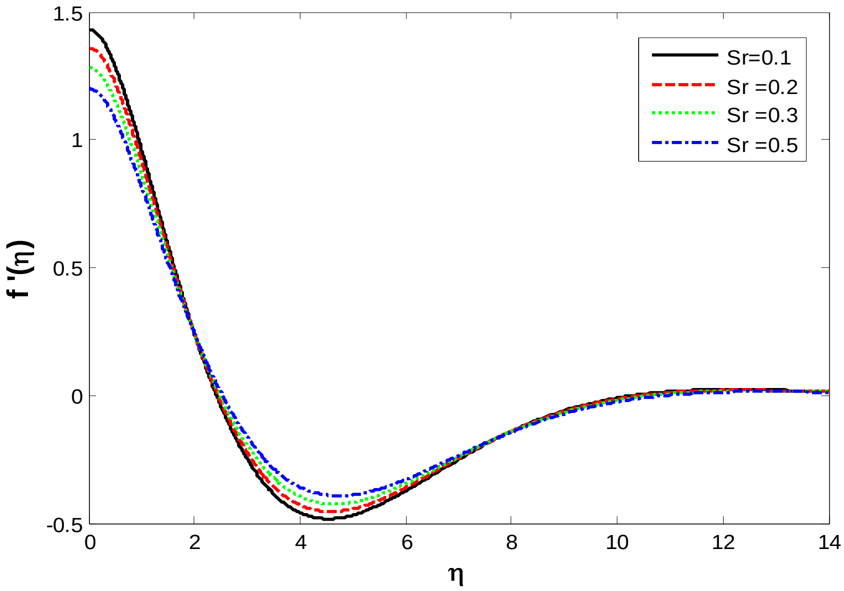

Fig.10: Variation of the velocity for different Soret number.

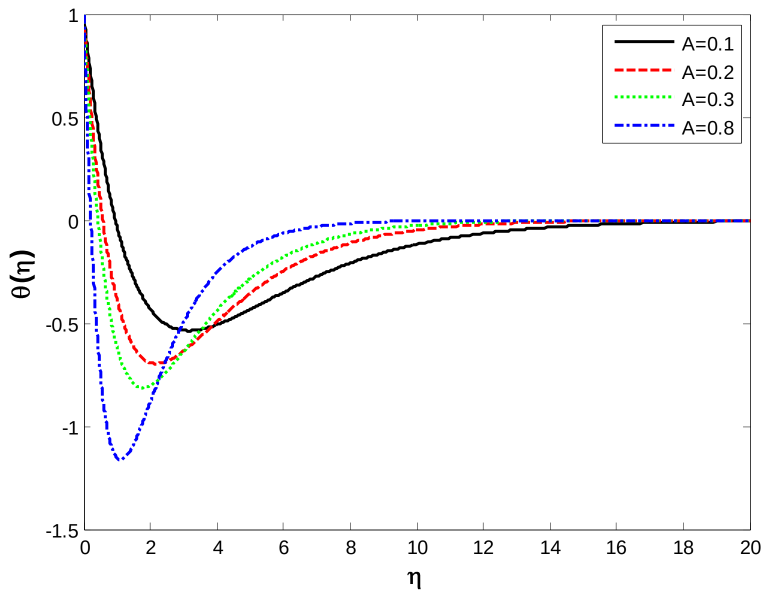

Fig.11: Variation of the temperature for different unsteadiness parameter.

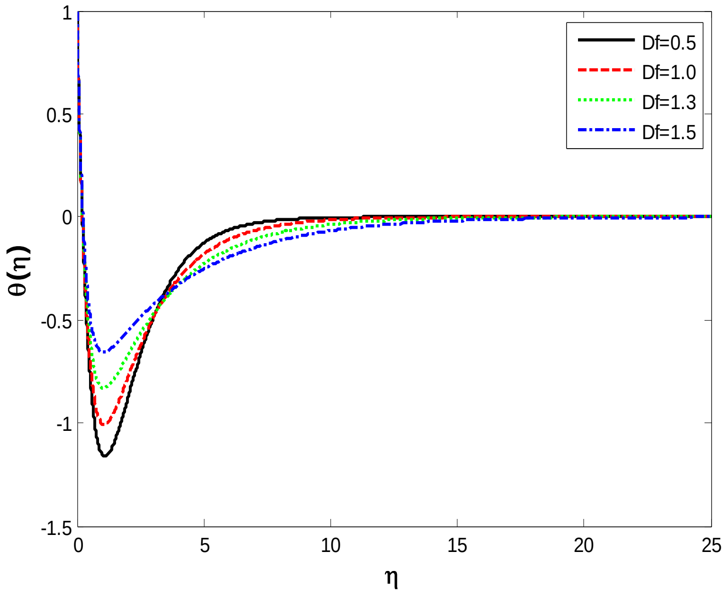

Fig.12: Variation of the temperature for different Dofour number.

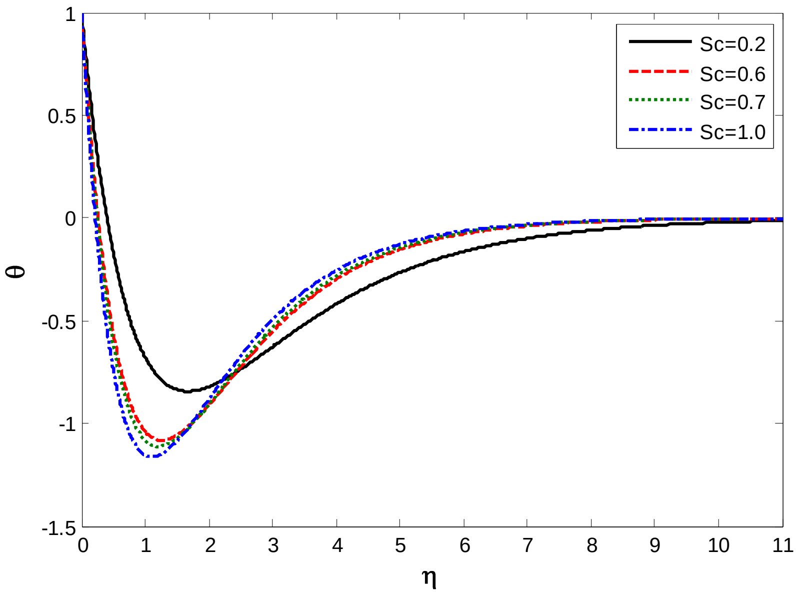

Fig.13: Variation of the temperature for different Schmidt number.

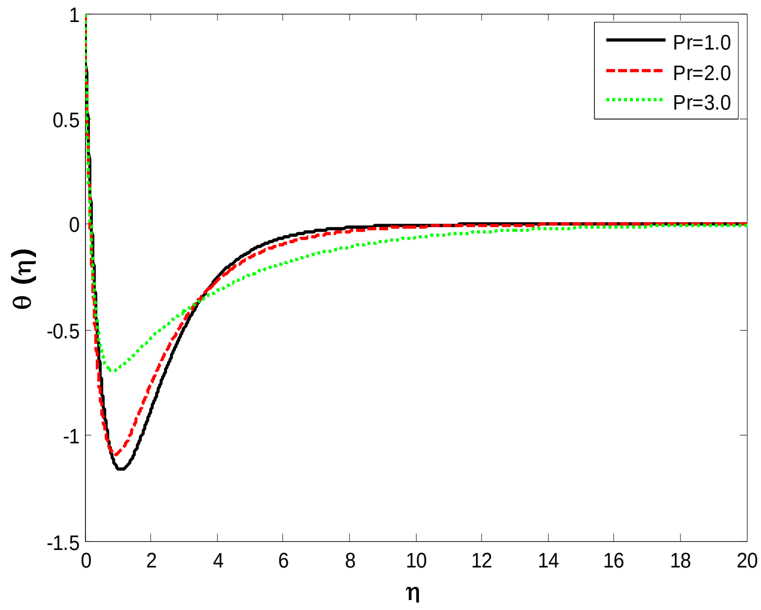

Fig.14: Variation of the temperature for different Prandtl number.

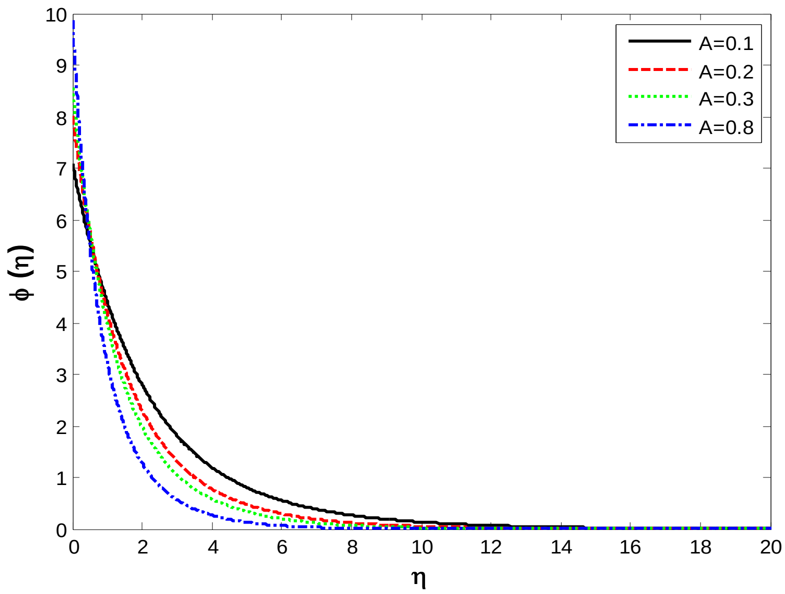

Fig.15: Variation of the concentration for different unsteadiness parameter.

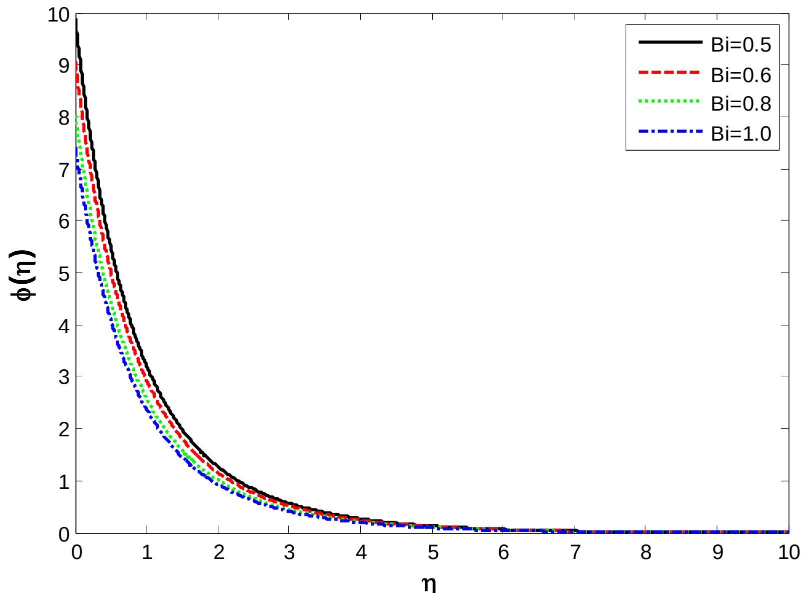

Fig.16: Variation of the concentration for different Biot number.

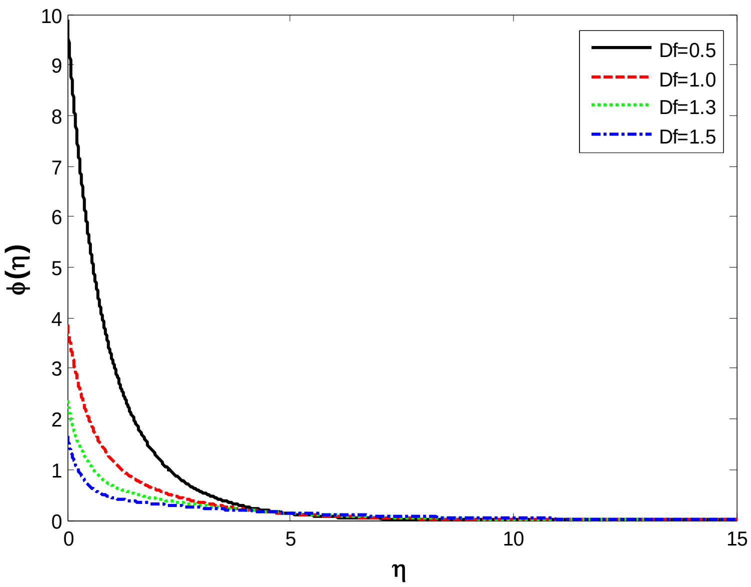

Fig.17: Variation of the concentration for different Dofour number.

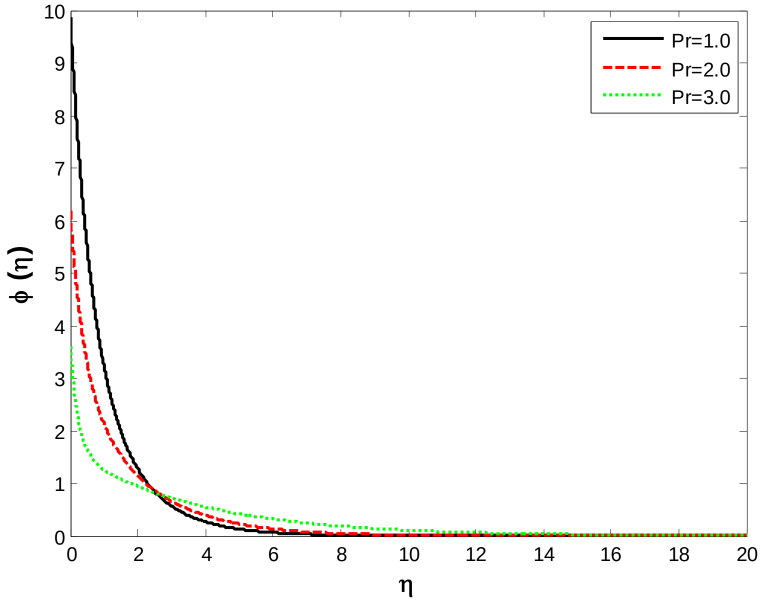

Fig.18: Variation of the concentration for different Prandtl number.

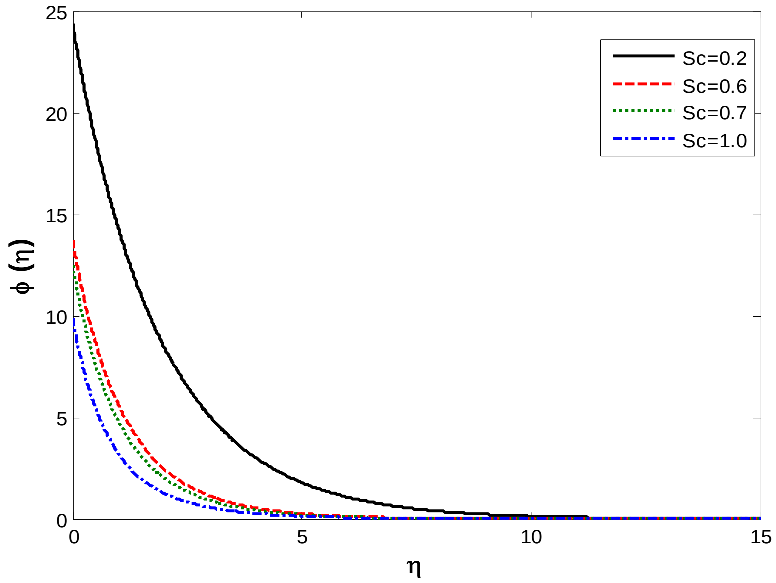

Fig.19: Variation of the concentration for different Schmidt number.

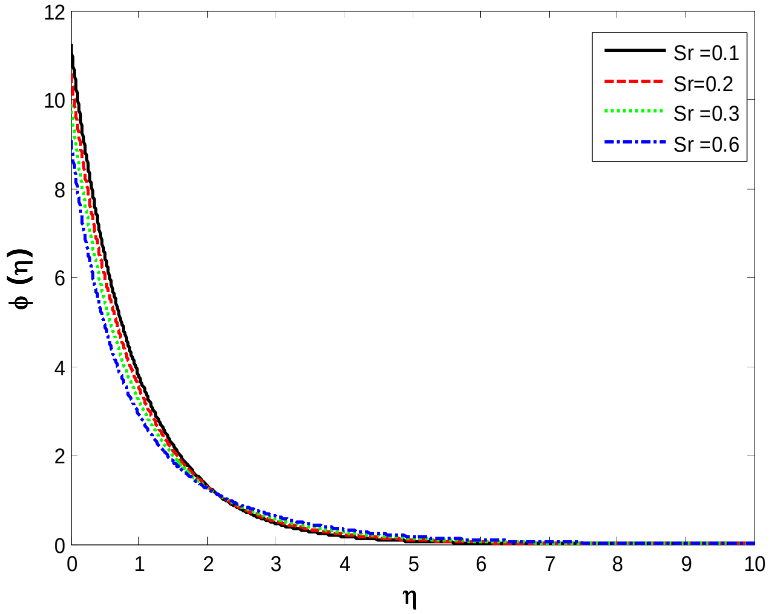

Fig.20: Variation of the concentration for different Soret number.

Conclusion

The present chapter analyzes the combined effect of the free convective heat and mass transfer on the unsteady two-dimensional boundary layer flow over a stretching vertical plate by taking heat generation/absorption, and soret and dufour effects into account. The governing equations are approximated to a system of non-linear ordinary differential equations by similarity transformation. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. The results are presented graphically and the conclusion is drawn that the flow field and other quantities of physical interest are significantly influenced by these parameters. It is noticed that Skin-friction coefficient, local Nusselt and Sherwood number increases with an increase in the Schmidt number. It is observed that local heat and mass transfer rate at the plate decreases with increase in the unsteadiness parameter whereas an opposite phenomena observed for skin-friction coefficient.

Table 1. Comparison table of skin-friction with different values of parameters β,Gr, Pr, M,K_0 with A=4.35,Gc=0.0,$D_f = 0.2$,Sc=Sr=0.1,

Pr	β	M	K_0	Khalid et al. [12]	Present result
0.3				1.02992	1.027746
0.71				1.29166	1.146599
	1.0			0.856995	0.548847
		1.0		1.08472	1.196518
			1.0	0.670207	0.662593

Table.2.Numerical values of skin friction-coefficient $f''(0)$,Local nussalt number$-\theta'(0)$, local Sherwood number $-\varphi'(0)$.

β	A	Gr	Gc	M	Pr	Df	Sc	Sr	K_0	Bi	$f''(0)$	$-\theta'(0)$	$-\varphi'(0)$
0.5	0.8	0.2	0.5	0.5	1.0	0.5	1.0	0.5	0.5	1.0	0.547585	13.751328	-4.950800
1.0											0.247816	9.167552	-4.950800
2.0											0 .098389	6.875664	-4.950800
	0.5										0.201094	16.193311	-4.863694
	0.9										0.633211	13.211464	-4.967874
	1.0										0.709393	12.750669	-4.981274
		0.5									0.301956	13.751328	-4.950800
		1.0									-0.107426	13.751328	-4.950800
		2.0									-0.926191	13.751328	-4.950800
			1.0								-0.375450	13.751328	-4.950800
			2.0								-2.221521	13.751328	-4.950800
			3.0								-4.067593	13.751328	-4.950800
				1.0							0.759829	13.751328	-4.950800
				2.0							1.107818	13.751328	-4.950800
				3.0							1.382798	13.751328	-4.950800
					0.5						0.030966	20.510114	-7.508862
					1.3						0.699866	11.910119	-4.195094
					1.5						0.777240	11.029647	-3.809435
						1.0					1.114909	5.823395	-2.036165
						1.3					1.251760	3.976200	-1.329713
						1.5					1.314805	3.150577	-1.006805
							0.2				-1.822901	67.920238	-11.945710
							0.6				0.007890	23.374232	-6.770318
							0.7				0.186952	19.997166	-6.194174
								0.2			0.435311	14.955497	-5.397231
								0.3			0.471599	14.559161	-5.254305
								1.0			0.762477	11.646790	-4.072330
									3.0		0.231833	13.751328	-4.950800
									5.0		0.158649	13.751328	-4.950800
									10		0.088670	13.751328	-4.950800
										0.6	0.430358	13.751328	-4.950800
										0.8	0.430358	13.751328	-4.950800
										1.0	0.430358	13.751328	-4.950800

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