International Journal of Applied Engineering Research
ISSN 0973-4562 Volume 10, Number 10 (2015) pp. 26573-26580

http://www.ripublication.com

Solution to Graph Coloring Problem using Evolutionary Optimization through Symmetry-Breaking Approach

Raja Marappan,
Department of Computer Applications,
SASTRA University,
Thanjavur - 613401, India
e-mail: raja_csmath@cse.sastra.edu
Gopalakrishnan Sethumadhavan,
Department of Computer Applications,
SASTRA University,
Thanjavur - 613401, India
e-mail: sgk@mca.sastra.edu

Abstract

The evolutionary algorithms performance on NP-hard optimization problems are strongly depend on how the evolutionary population breaks the symmetry of the solution search space by avoiding the local optimum. To reduce the computational complexity of crossovers and to achieve more suboptimal solutions in the population, the symmetry of the population should be eliminated using right strategy. Graph Coloring is a NP-hard combinatorial optimization problem which takes high symmetric population in the solution space and it is applied in engineering domains. This paper exhibits the strength of genetic algorithm in solving graph coloring through symmetry-breaking approach in evolutionary optimization. In general the population-based methods are not yielding fine offspring during the recombination of fine offspring chosen from different parts in the search space. The proposed genetic algorithm employs an n-point crossover and mutation operators. Since the population based approaches allow large jumps in the search space it is necessary to break the symmetries in the population. This paper focuses on breaking the symmetries in the population by applying the cyclic permutations based on the continuous probability distribution in the gene sequences so that solution search space is minimized to obtain the near optimal convergence of graph coloring problem. The performance of this approximation method is simulated on some of the benchmark graphs and the results are presented.

Keywords: graph coloring, genetic algorithm, NP-hard, chromatic number, symmetry elimination

Introduction

The undirected simple graph G = (V, E) has V(G) = {$v_1, v_2, \ldots, v_n$} which is a vertex set and E(G) = {$e_1, e_2, \ldots, e_m$} represents an edge set such that every edge is incident with an unique end vertices, say (v_i, v_j). G is represented using the adjacency matrix, denoted as A(G), which is an n × n symmetric binary matrix where A_{ij} = 1 if there is an edge between the end vertices v_i and v_j and also called adjacent vertices; and A_{ij} = 0, otherwise. The smallest number of integers or colors, $\chi(G)$ required to color V(G) with no conflicts in its adjacent vertices is a Graph Coloring Problem [2]. For every simple graph G with n vertices the minimal color $\chi(G)$ can be obtained in n! solution space. Approximation method which minimizes the search space by allowing large jump in the solution space towards reaching near optimal solution is expected. Most of the real world problems such as scheduling, register allocation, frequency assignment [3] etc requires the applications of coloring. Since it is a NP-hard problem, until now there is no method devised to solve it in a polynomial time [1]. Because of its NP-completeness, a fast and effective algorithm to generate near optimal coloring is required. Solutions of GCP are also found using methods such as branch-and-bound, branch-and-cut, and backtracking algorithms [7], [8], [9], [11]. The different genetic operators are proposed to solve graph coloring [4], [5], [6], [12], [13], [14].

This paper presents a new genetic method to solve graph coloring using evolutionary optimization through the elimination of symmetry in the population. The n-point crossover and mutations are devised to reduce the search space and also to minimize the generations. This procedure has been tested with some of the benchmark graphs such as queen, Mycielski, huck.col, jean.col, games120.col, miles250.col, david.col, anna.col and the results are presented. Sections II focuses on the need of symmetry-breaking in evolutionary optimization; Section III focuses on the proposed symmetry-breaking method to solve graph coloring. Simulation on some of the benchmark graphs are shown in Section IV and conclusions are drawn in Section V.

Symmetry-Breaking in Evolutionary Optimization

To obtain the near-optimal solution with the minimized computational complexity it is necessary to break the symmetries in the parent population during the generations of offspring. Suppose the color assignments of the gene sequences are represented using the integer elements in the color set S = {1, 2, 3, … c}. For G with n=6, let the gene sequence '112341' assumes color[v_1]=color[v_2]=color[v_6]=1; color[v_3]=2; color[v_4]=3; color[v_5]=4; Clearly the gene sequence '443214' also denotes the same solution since v_1, v_2, v_4 are taken together without considering its color value assigned. In general, the degree of symmetrical population results in the exponential growth with number of elements in S. Thus it is necessary to speedup search by eliminating the symmetries in the population.

The population based techniques, for example, the genetic algorithms will be a good choice for graph coloring if the symmetries are eliminated during the crossover operation; otherwise the relevant information of the parent gene sequences will be lost and it will further increase the computational complexity of the searching the near-optimal solution.

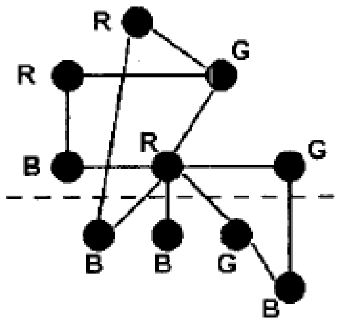

Figure 1: Selected Parent 1

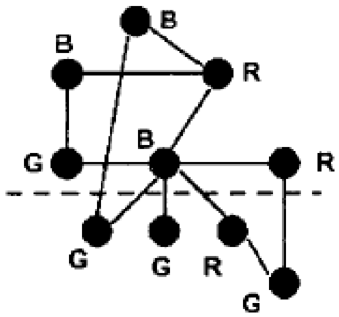

Figure 2: Selected Parent 2

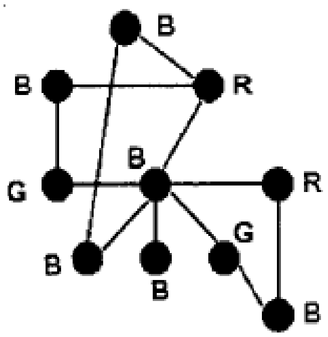

Figure 3: Crossover of the selected parents

Figure 1 and 2 shows that the parents 1 and 2 are selected for crossover operation. Assume the color assignment of parent 1 is RRGBRGBBGB and the color assignment of parent 2 is BBRGBRGGRG as represented in the Fig 1 and 2. Assume the crossover is performed at 6th point.

Parent 1: **RRGBRG***BBGB*
Parent 2: *BBRGBR***GGRG**
Crossover at 6th point: *BBRGBRBBGB*

Figure 4: Crossover of the selected parents 1 and 2

Figures 3 and 4 show the crossover operation performed at 6th point. The crossover result is the gene sequence *BBRGBRBBGB*, which is the disruption of two better gene sequences. The generated offspring *BBRGBRBBGB* becomes worse than the corresponding selected parents. This occurred because of the improper inheritance of the composition of the color classes and reflecting a difference in color assignments. This kind of crossovers produces average population with a high cost and maintaining the population with high diversity. The good solution should always be retained by applying elitism during the genetic generation; hence the symmetry should be broken during crossover operation. Hence this kind of disruption should be avoided to minimize the computational complexity of the searching process.

Genetic Algorithm with Symmetry-Breaking Method

A genetic algorithm to solve graph coloring problem with the cyclic permutation based symmetry breaking approach is presented in this section. The algorithm performs symmetry breaking when the selected parents are not equal.

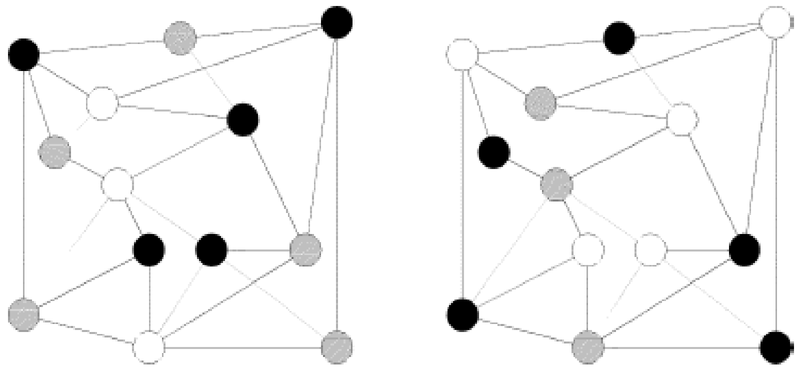

Figure 5: Permutation of the coloring for the graph G with n=13 vertices

Figure 5 shows that the graph G with n=13 vertices differ by the color permutations. Hence the crossover operator can produce poor offspring with many conflicting edges and fitness function will get increased. The permutation symmetry size grows very rapidly with $\chi(G)$ [11]. Hence to avoid the poor quality offspring in the population, the selected parents should be fine tuned through the proposed "stochastic permutation operator" before applying crossover operation. Random samples in the particular interval, say [a, b] from specific probability distribution are generated. If the color value is greater than the random sample at some of the selected points, then subpopulation can cyclically be permuted for both of the selected parents. The proposed stochastic based cyclic permutation for graph coloring is given below.

Algorithm: Graph coloring with stochastic cyclic permutation to remove symmetry

```
Initialize the population;
Evaluate the fitness function of individuals using fitness proportionate method;
While the termination conditions are not satisfied
        Select two parents P1 and P2;
If P1 and P2 are not equal then
        P1 = stochastic cyclic permutation (P1);
        P2 = stochastic cyclic permutation (P2);
        If P1 and P2 are not equal then
                Perform the n-point crossover operation;
        Perform the mutation operation;
        Evaluate the individuals and update the population;
Print the performance measurements;
```

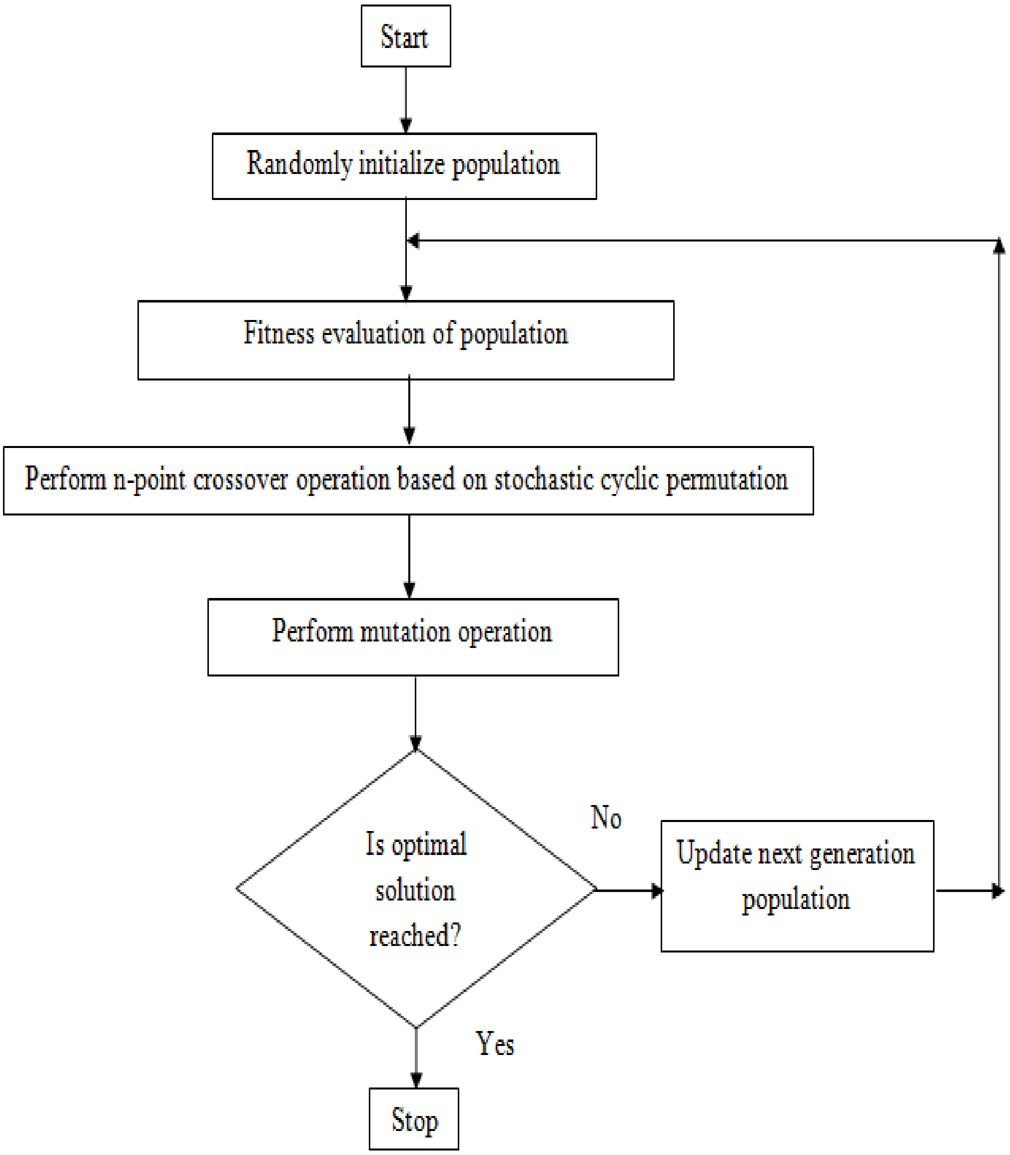

Figure 6: Flowchart of the proposed graph coloring with symmetry-breaking

Simulation on Benchmark Graphs

The experiments are conducted on some of the benchmarking graphs using Intel Core i5-2450M 2.5GHz with Turbo boost up to 3.1GHz system in Java JDK 1.7 environment. Algorithm has been implemented using JAVA language and the results are tabulated.

Table 1: Computation of Near Optimal Solution on Benchmark Graphs

Graph No	Graph (G)			Minimum color obtained
	Graph Type	Instances	$\chi(G)$	
1	myciel3.col	n=11; m=20	4	4
2	myciel4.col	n=23; m=71	5	5
3	myciel5.col	n=47; m=236	6	6
4	myciel6.col	n=95; m=755	7	7
5	queen5_5.col	n=25; m=320	5	5
6	queen6_6.col	n=36; m=580	7	8
7	Dodecahedron Schlegel	n=20; m=30	3	3
8	5-connected planar	n=12; m=30	4	4
9	huck.col	n=74; m=301	11	12
10	jean.col	n=80; m=254	10	11

Table I shows the computation of the chromatic number, $\chi(G)$ on some of the benchmark graphs. The following conclusions are drawn from this simulation:

1. The proposed graph coloring obtains the near-optimal solution.
2. The n-point crossover operation with stochastic cyclic permutation breaks the symmetry of the population.
3. For most of the graphs stochastic cyclic permutations are performed many times. This concludes that there is some symmetry in the population.
4. The number of generations is minimized in this stochastic cyclic permutation. Hence the computational complexity is reduced.
5. The symmetry depends on the population size.

Conclusion

The purpose of this paper is to gain the understanding of the performance of evolutionary algorithm in solving graph coloring. Splitting of the search space into regions based on probability distributions is a generic property of most of the real world combinatorial optimization problems. The problems arising from symmetry in the crossover operation is discussed. The genetic algorithm for graph coloring with cyclic stochastic permutation based n-point crossover and mutation operators are designed to find the near optimal solution of $\chi(G)$. The experimental results of proposed algorithm are presented. The proposed approximation genetic algorithm reduces the computational complexity and always produces a near-optimal solution. The new cyclic stochastic permutation breaks the symmetry and hence reduces the search space in minimal computing time.

References

[1] Garey, M. R. and Johnson, D. S., Computers and Intractability: Guide to the Theory of NP-Completeness, Freeman, San Francisco, 1979.

[2] Tommy R. Jensen, Bjarne Toft, Graph Coloring Problems, Wiley-Inter science, 1995.

[3] Noise Reduction in VLSI Circuits using Modified GA Based Graph Coloring - International Journal of Control and Automation, Vol. 3, No. 2, June, 2010.

[4] A. Hertz, and D. E Werra. Using tabu search techniques for graph coloring, Computing Vol. 39, No. 4, pp. 345-351, 1987.

[5] C. Fleurent and J. A. Ferland. “Genetic and hybrid algorithms for graph coloring”, Annals of Operations Research, 63, pp. 437-463, 1995.

[6] C. L. Mumford. “New order-based crossover for the graph coloring problem”, T. P. Runarsson et al. (Eds.): PPSN LX, vol. 4193, pp. 80-88, 2006.

[7] Anuj Mehrotra and Michael A. Trick. A column generation approach for graph coloring. INFORMS Journal on Computing, 8(4):344–354, 1996.

[8] Isabel Méndez-Diaz and Paula Zabala. A branch-and-cut algorithm for graph coloring. Discrete Applied Mathematics, 154(5):826–847, 2006.

[9] R. Monasson. On the analysis of backtrack procedures for the coloring of random graphs. In E. Ben-Naim, H. Frauenfelder, and Z. Toroczkai, editors, Complex Networks, pages 235–254. Springer, 2004.

[10] Back T. “Evolutionary algorithms in theory and practice”, New York Oxford University Press, pp. 21-28, 1996.

[11] Adam Prügel-Bennett, Symmetry Breaking in Population-Based Optimization, IEEE Transactions on Evolutionary Computation, Vol. 8, No. 1, 2004.

[12] Raja Marappan, Gopalakrishnan Sethumadhavan, A new genetic algorithm for graph coloring, In CIMSim 2013, 5th International Conference on Computational Intelligence, Modelling and Simulation, Seoul, South Korea, pages 49-54, 2013.

[13] Gopalakrishnan Sethumadhavan, Raja Marappan, A Genetic Algorithm for Graph Coloring using Single Parent Conflict Gene Crossover and Mutation with Conflict Gene Removal Procedure, 2013 IEEE International Conference on Computational Intelligence and Computing Research, India, pages 350-355, 26-28 December 2013.

[14] Raja Marappan, Gopalakrishnan Sethumadhavan, Solution to Graph Coloring Problem using Heuristics and Recursive Backtracking,

International Journal of Applied Engineering Research, 2015, Research India Publications, ISSN 0973-4562 Volume 10, Number 10 (2015) pp. 25939-25944.