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Modeling & Simulation For Transformation Matrix, Rotation Matrix & Remaining Energy

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Abstract

The aim of this work is to describes simulation of the innovative model for transformation and rotation motion. When the trajectory is simulated using six degree of freedom to estimate the location point of projectile motion, this simulation helps in prediction of trajectory. Apart from this kinematic energy is also modeled; simulation is done on the basis of experimental data. This model is derived for six degree of freedom (6DOF) to predict position coordinate (x,y,z) as well as angular rotation (φ, θ, ϕ) with respect to time. This technique is based on iterative process of up gradation of transformation matrix as well as rotation matrix with time. A complete set of six-degrees-of freedom to define rotation and transformation in terms of angular velocity components as well as in term of Euler angles. Matlab based simulation results are shown in form of figures.
This part describes the characteristics and effect of roll, pitch and yaw on motion. The developed model is able to describe the direct effect of angular velocity and angular rotation technically. Finally, the method derived to predict motion of launched weapon with uncontrolled roll and drift. It is used as a tool for an independent validation of the performance and stability for controlled flight.

Index Terms: Aerodynamic (AD),) Six degree of freedom (6DOF), Kinetic Energy (KE), Transformation (TS), Rotation (RT)

Introduction

The classical problem of optimizing a trajectory model with respect to time, which is fully kinematic and dynamic problem. In basic physics, the ballistic trajectory of projectile is the path of thrown object will take under the action of gravity neglecting other forces, but in real scenario the projectile is moving under gravity and drag. Hence, it follows translation as well as rotation both during its flight. This off-line trajectory design is useful in obtaining trajectory arcs. A 6DOF trajectory modeling deals with determination for the accurate prediction of future position at any time of instant.

In designing this kind of model for motion control system[1], the short period dynamics of the launch vehicle is used for expressing the rigid-body and flexible-body motion. It is assumed that the motion of the launch vehicle consists of small deviations from a reference trajectory[2].

Another important assumption is that time varying mass, inertial, and other physical properties are changing slowly during the flight. As a result, all parameters of the launch vehicle[3] can be “frozen” over a short period of time. In this way, analysis and design techniques can be exploited most fully.

Problem Description

The proposed method is developed to predict the arc of trajectory to achieve the desired location, by providing different location points in limited time period. This model gives performance 6-DOF simulation, which was developed to generate a trajectory along the state. Using initial state parameters the orientation, position and remaining velocity are computed using this simulation. This simulation is sufficient for converging solution within some time limit. This simulation is verified on the basis of experimental data. For additional accuracy, simulation can be verified by practical trials, and accordingly model can be modified. Proposed method can be developed to find the following.

- Change in Body Axis System
- Total Angular Moment
- Change in Remaining Kinetic energy
- Validation of Model on basis of experimental data

The trajectory generation algorithm determine the path which is modeled and implemented by simulation. The Six Degree Of Freedom (DOF)[4] model describes the motion of the projectile in terms of six-tuples. Three among six are expressed as (x',y',z') in terms of body motion. The three angular rotations (φ,θ,ϕ) are denoted by yaw, pitch and roll of the plane respectively. The derivation results in six coupled differential equations. The spatial coordinates x, y, z describe the motion of the projectile mass center. The angular coordinates (φ,θ,ϕ) describe the motion of the projectile about its mass center. The motion of the body in three directions is specified by 3x3= 9 constraints.

So, degree of freedom = 9-3(No. of independent variables)= 6

Simulation Model

The developed model for translation and rotation is derived by Newton’s laws of motion[5], for body undergoing transformation and rotation.

The mathematical formulas are derived for moving coordinates of body axis system. This model is driven to express rate of change (position and orientation) of body w.r.t to inertial (earth-fixed) coordinate system[6]. Where, x-axis represents longitude direction while y-axis represents latitude in motion. The TS and RT models are developed with the help of newton's laws.

$$\sum f = \frac{d}{dt}(mV)$$

$$\sum I\omega = \frac{d}{dt}(P)$$

Position equation $F_x = \frac{d}{dt}(mV_x)$

$$F_y = \frac{d}{dt}(mV_y)$$

$$F_z = \frac{d}{dt}(mV_z)$$

Moment equation $(I\omega)_x = \frac{d}{dt}(L)$

$$(I\omega)_y = \frac{d}{dt}(M)$$

$$(I\omega)_z = \frac{d}{dt}(N)$$

A. Transformation(TS) Matrix

This is derived for finding model to determine range, altitude and azimuth[7]. Three among six are expressed as (x',y',z') in terms of body motion. The three angular rotations (ψ, θ, ϕ) are denoted by yaw, pitch and roll of the plane respectively. y is a vertical plane containing initial velocity vector. x is altitude i.e. horizontal plane & z is rotation i.e diversion in horizontal plane. Center of mass is located at origin of axis (x,y,z) at t(time)=0. The rate of change in directions are specified by directional cosines (l,m,n) and (i,j,k) are unit vector in unprimed axes. The coordinate of a point in a given frame are components of the position vectors of this point along the axis of system.The position vectors(x',y',z') are derived as:

$$x' = xl_1 + ym_1 + zn_1$$
$$y' = xl_2 + ym_2 + zn_2$$
$$z' = xl_3 + ym_3 + zn_3$$

Similarly, general component of velocity vectors are derived

$$V_x' = V_x l_1 + V_y m_1 + V_z n_1$$
$$V_y' = V_x l_2 + V_y m_2 + V_z n_2$$
$$V_z' = V_x l_3 + V_y m_3 + V_z n_3$$

The velocity vector (Vx,Vy,Vz) is converted to body axis components by Transformation matrix K1.

$$\begin{bmatrix} dx/dt \\ dy/dt \\ dz/dt \end{bmatrix} = k1 \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}$$

Where, $$k1 = \begin{bmatrix} \cos\theta\cos\varphi & \sin\theta\sin\phi\cos\varphi - \cos\phi\sin\varphi & \sin\theta\cos\phi\cos\varphi + \sin\phi\sin\varphi \\ \cos\theta\sin\varphi & \sin\theta\sin\phi\sin\varphi + \cos\phi\cos\varphi & \sin\theta\cos\phi\sin\varphi + \sin\phi\cos\varphi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix}$$

$$\begin{bmatrix} Range \\ Altitude \\ Azimuth \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\varphi & \sin\theta\sin\phi\cos\varphi - \cos\phi\sin\varphi & \sin\theta\cos\phi\cos\varphi + \sin\phi\sin\varphi \\ \cos\theta\sin\varphi & \sin\theta\sin\phi\sin\varphi + \cos\phi\cos\varphi & \sin\theta\cos\phi\sin\varphi + \sin\phi\cos\varphi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}$$

B. Rotation (RT)Matrix

Remaining three equations are expressed as $\{\varphi,\theta,\phi\}$ in term of angular velocities $(\omega_x,\omega_y,\omega_z)$ [7] which are dependent upon external aerodynamic forces[8] about plane of center of mass. The angular rotations $\{\varphi,\theta,\phi\}$ are called Euler angles denoted by yaw, pitch and roll of the plane respectively shown in fig 1.

The angular momentum of rigid body about axis is given by

$$P = \sum_{i=1}^{N} m_i \{r_i \times (\varpi \times r_i)\}$$

$$P = (I_{xx}\omega_x + I_{xy}\omega_y + I_{xz}\omega_z) + (I_{yx}\omega_x + I_{yy}\omega_y + I_{yz}\omega_z) + (I_{zx}\omega_x + I_{zy}\omega_y + I_{zz}\omega_z)$$

where,

$$I_{xx} = \sum_{i=1}^{N} m_i (y_i^2 + z_i^2)$$

$$I_{yy} = \sum_{i=1}^{N} m_i (x_i^2 + z_i^2)$$

$$I_{zz} = \sum_{i=1}^{N} m_i (x_i^2 + z_i^2)$$

$$I_{xy} = \sum_{i=1}^{N} m_i x_i y_i = I_{yx}$$

$$I_{yz} = \sum_{i=1}^{N} m_i y_i z_i = I_{yz}$$

$$I_{zx} = \sum_{i=1}^{N} m_i z_i x_i = I_{xz}$$

$P = L + M + N$ = yaw moment + pitch moment + roll moment

Total Angular moment = yaw moment + pitch moment +roll moment.

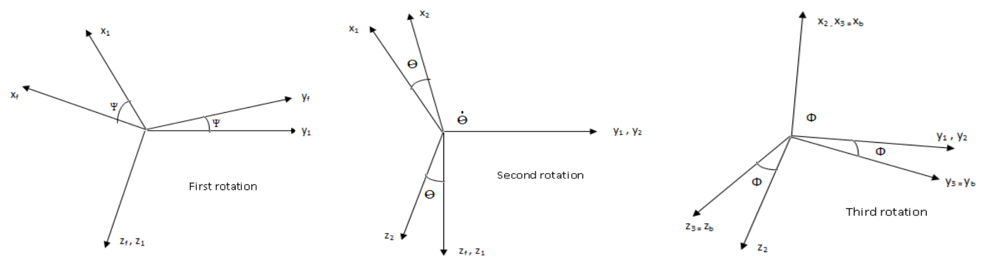

Figure 1: Rotation/ Euler Angles

The equations of motion also give Euler rate determined in term of angular velocities $(\omega_x, \omega_y, \omega_z)$ and Rotation matrix K2.

$$\begin{bmatrix} d\varphi / dt \\ d\theta / dt \\ d\phi / dt \end{bmatrix} = k2 \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

Where,

$$k2 = \begin{bmatrix} 0 & \sec\theta \sin\phi & \sec\theta \cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 1 & \tan\theta \sin\phi & \tan\theta \cos\phi \end{bmatrix}$$

$$\begin{bmatrix} Yaw \\ Pitch \\ Roll \end{bmatrix} == \begin{bmatrix} 0 & \sec\theta \sin\phi & \sec\theta \cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 1 & \tan\theta \sin\phi & \tan\theta \cos\phi \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

Euler rates [9] in terms of angular velocities are calculated considering aerodynamic forces and moments. These are assumed to be the functions of Mach number(M)[10] and nonlinear with flow incidence angle with aerodynamic coefficients[11].

C. Remaining KE :

The kinetic energy is gradually reduced throughout the trajectory. So, it is very important to predict reaming energy in calculation of trajectory arc[12]. It gives clear idea about how much remaining distance will be covered by projectile at any point.

$$T = \frac{1}{2} \sum_{i=1}^{N} m_i v_i^{\,2}$$

$$T = \frac{1}{2} \sum_{i=1}^{N} m_i \, (\varpi \times r_i) \times (\varpi \times r_i)$$

$$T = \frac{1}{2} \sum_{i=1}^{N} m_i \, (\varpi \times r_i) \times (\varpi \times r_i)$$

$$T = \frac{1}{2} (I_{xx} \, \omega_x^{\,2} + I_{yy} \omega_y^{\,2} + I_{zz} \omega_z^{\,2} + 2 I_{xy} \omega_x \omega_y + 2 I_{yz} \omega_y \omega_z + 2 I_{zx} \omega_z \omega_x)$$

$$T = \frac{1}{2} k3 \varpi$$

which is derived as :

Remaining kinetic energy

$$= \frac{1}{2} \left[I_x (\phi^{\bullet} \sin\theta \sin\varphi + \theta^{\bullet} \cos\varphi)^2 + I_y (\phi^{\bullet} \sin\theta \cos\varphi - \theta^{\bullet} c \sin\varphi)^2 + I_z (\phi^{\bullet} \cos\theta + \varphi^{\bullet})^2 \right]$$

Results

The proposed simulation [13,14] was evaluated regarding the estimation of all required parameters. The experimental data has been generated and the corresponding analyzed results are shown. As a typical case the model has been evaluated under following conditions:

Projectile diameter: 5 ft
Initial velocity: 900 ft/s
Time step:20 sec
Total time taken: 120 sec.

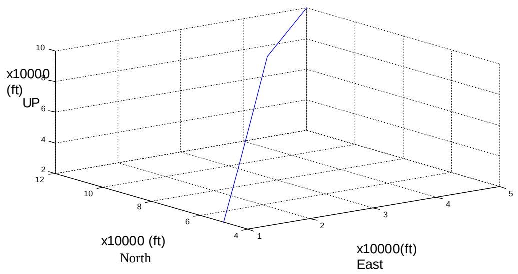

Figure 2: Predicted Trajectory in Earth Fixed Launching Pad

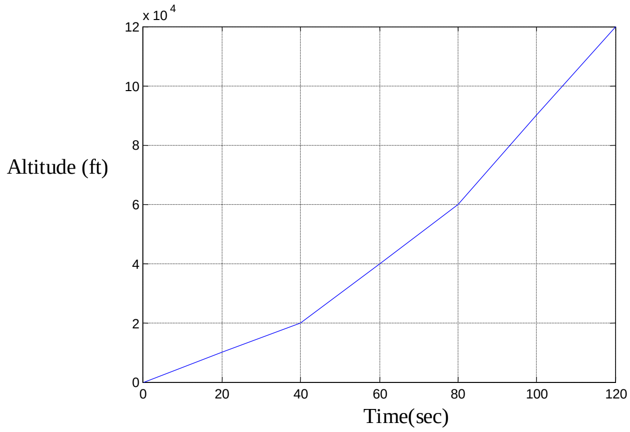

Figure 3: Altitude history with time

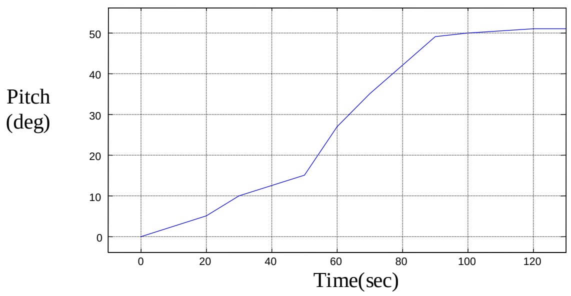

Figure 4: Pitch angle history with time

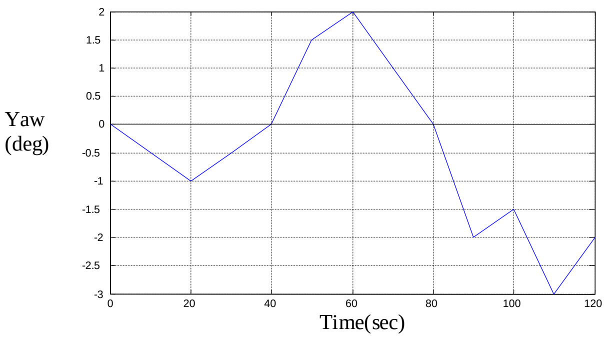

Figure 5: Yaw angle history with time

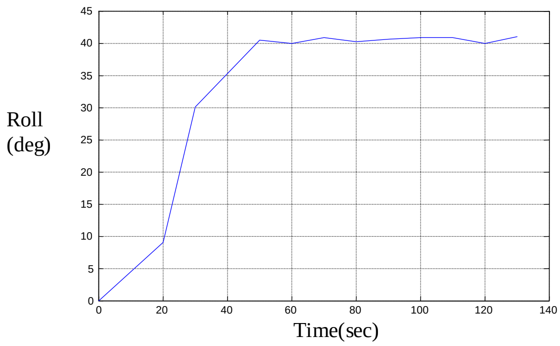

Figure 6: Roll angle history with time

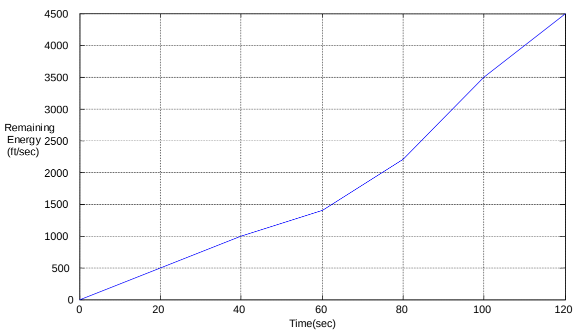

Figure 7: Remaining Energy with time

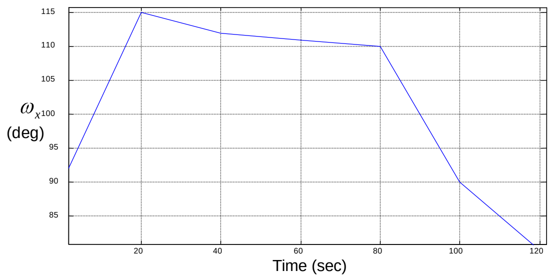

Figure 8: Euler angle ω_x with time

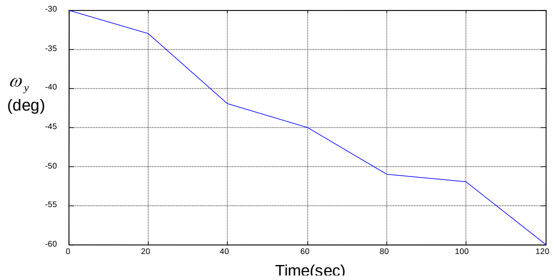

Figure 9: Euler angle ω_y with time

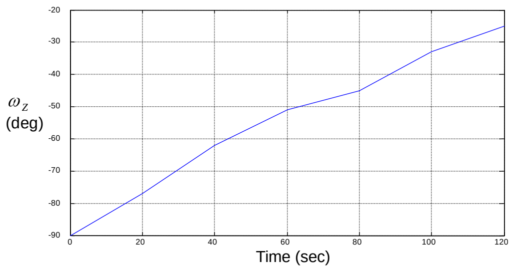

Figure 10 : Euler angle ω_z with time

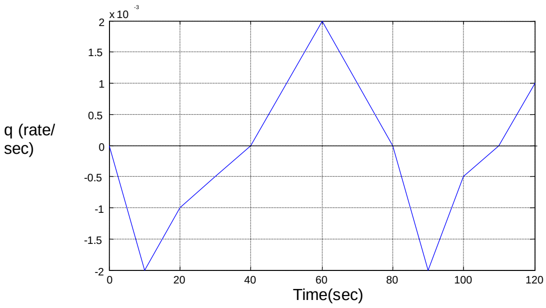

Figure 11: Angular velocity q with time

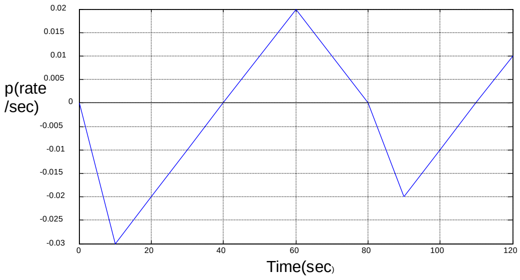

Figure 12: Angular velocity p with time

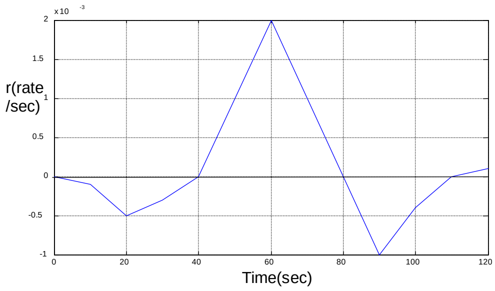

Figure 13: Angular velocity r with time

Conclusion

The developed modeled equations are able to determine Transformation, Rotation and Remaining Energy use to control the commanded system [15,16]. Using initial state parameters the orientation, position and remaining velocity are computed using this simulation. This simulation is sufficient for converging solution within time limit. This simulation is verified on the basis of experimental data. For additional accuracy, simulation can be verified by practical trials and re-engineering can be done accordingly.

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