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Delay- Dependent Stability Analysis of Load Frequency Control System Using Rekasius's Substitution and Resultant Theory

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Abstract

The performance of power system is deteriorated due to the delay in the transmission of control signals from control center to generating unit in Load Frequency Control (LFC) scheme. In this paper, a new flexible methodology for stability analysis of time delayed LFC is presented. The existing stability analysis methods utilize time domain approach based on Lyapunov- theory and Linear Matrix Inequalities. The proposed method is an alternative approach based on frequency domain for analyzing the stability of the system. With the help of Rekasius's Substitution and Resultant Theory, the characteristic equation of the power system is investigated to compute the delay margin. Simulation studies were carried out to demonstrate the effectiveness of the proposed method in estimating the delay margin. The proposed method offers larger delay margin and takes less computation time as compared to the existing methods.

Keywords: Delay Margin, Load Frequency Control system, Power System Stability, Rekasius Substitution, Resultant Theory, Time Delay.

Introduction

A well planned power system must uphold both voltage and frequency within acceptable limits even during the times when there are sudden variations in the load. The Load Frequency Control (LFC) [1] plays a vital role in maintaining the frequency deviation and tie-line power exchange to zero. The time delay occurring during the transmission of control signals in LFC scheme weakens the performance of power system [2, 3] and may even paves way to instability.

Understanding the importance of this issue, there exist continuous interests from many researchers in this area. Generally, the stability analysis of any time delay

system can be performed by two approaches, namely frequency domain approach and time domain approach. The frequency domain approach which is the direct method of finding delay margin includes tracing of critical eigen value [4, 5] and cluster treatment of characteristics roots [6-8]. The time domain approach is the indirect method based on Lyapunov stability theory and LMI [9-11].

Several papers have been published on LFC with delays. Most of the works reported in literature is based on time domain approach. Specifically, Lyapunov-Krasovskii functionals and linear matrix inequalities are well known for time domain approach. X.Yu et.al [12] recommended a Linear Matrix Inequalities (LMI) technique for LFC system with communication delays. Hassa Bevarani et.al [13] suggested a method based on H_2/H_∞ control technique to design a robust decentralized PI controller for three area interconnected power system with communication delay. L.Jiang et.al [14] examined the delay dependent stability of multi area LFC scheme with PI controllers using Lyapunov-theory and LMI techniques. Dey R,Ghosh S et.al[15] proposed the delay dependent / independent design of H_∞ controller for LFC of two area system. Chuan-ke Zhang et.al [16] analyzed the delay dependent stability of multi- area of LFC system by determining the delay margin using LMI technique. Chuan-ke Zhang et.al [17] proposed a method to design a robust PID controller for delay dependent load frequency control .

In frequency domain approach, the Routh-Hurwitz criterion was exploited for stability analysis of single area LFC with communication delay [18]. In this method all the roots of the polynomial formulated from Routh's array have to be determined to estimate the delay margin which is computationally tough and takes more time.

In this paper, a unique method based on frequency domain approach is proposed to compute the maximum tolerable delay time (delay margin) with less computation efforts. The proposed method applies two concepts namely Rekasius substitution and Resultant Theory. First, the transcendental characteristic equation is transformed into an algebraic polynomial equation. Then the minimum positive real roots of the polynomial Z (ω) is derived which gives the exact positive delay margin (τ_k). Thus the drawback of Routh's criterion is eliminated. The delay margin obtained using the proposed method is highly accurate compared to existing time domain approaches. Simulation studies are carried out to illustrate the accuracy of the proposed method in computing the time delay margin.

Single Area LFC Model With Time Delay

This section explains the dynamic model of single area LFC system with communication delays. Figure 1 shows the block diagram of single area LFC system including a time delay. Each block of the system, namely, governor, turbine and generator is modeled by a first-order transfer function [1]. The transfer function of each block is given below:

Transfer Function of governor- $\frac{1}{1+ST_g}$

Transfer Function of non- reheat turbine- $\frac{1}{1+ST_T}$

Transfer Function of generator – $\frac{1}{MS+D}$

where T_g and T_t are time constants of governor and turbine respectively. M and D are moment of inertia and damping co-efficient respectively.

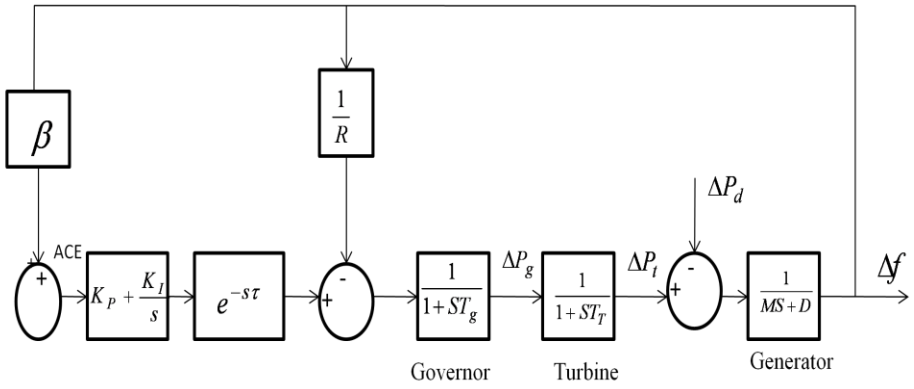


Figure 1: Dynamic model of single area Load Frequency Control scheme

An exponential term $e^{-s\tau}$ is introduced in to the control loop for representing the delay in transmitting the control signals. The transfer function of the PI controller is given as

$$K_P + \frac{K_I}{s}$$

where K_P and K_I are the proportional and integral controller gains, respectively. The linear open loop model of LFC can be expressed in state equation as

$$\dot{x}(t) = Ax(t) + Bu(t-\tau) + Fw(t) \quad (1)$$

$$y(t) = Cx(t) \quad (2)$$

where

x(t) is a state vector, y(t) is output vector, w (t) is disturbance vector and u(t) is control input.

$x(t) = [\Delta f, \Delta P_t, \Delta P_g, \int ACE]$;

$y(t) = [ACE, \int ACE]$;

$w(t) = [\Delta P_d]$

A=[■((−D)/M&1/*M*&0&0@0&(−1)/T_t &1/*T_t* &0@(−1)/(RT_g)&0&(−1)/ T_g &0@β&0&0&0)] ; B=[0 0 $\frac{1}{T_g}$ 0] ; F=[$-\frac{1}{M}$ 0 0 0]

C=[■(β&0&0&0@0&0&0&1)]

τ is a constant delay. $\Delta f, \Delta P_t$ and ΔP_g are the deviation in frequency, the turbine power output and governor output respectively. ΔP_d is the change in load. As there is

no tie-line power exchange for a single area LFC system, the area control error ACE is expressed as

$$ACE=\beta\Delta f \quad (3)$$

where β is frequency bias factor. The PI type load frequency controller is represented by equation as

$$u(t) = [K_p ACE + K_I \int ACE\ ] \quad (4)$$

The state space model of closed loop system can be obtained by altering the PI control problem in to state output feedback control problem. The closed loop model of single area LFC system in state space form can be expressed as

$$\dot{x}(t) = Ax(t) + A_d x(t-\tau) + Fw(t) \quad (5)$$

$$y(t) = Cx(t) \quad (6)$$

where
A_d=[B*K*C] ;
K=[K_p K_I]

In the next section, the stability analysis of single area LFC system with respect to the time delay is discussed by investigating the characteristic equation of the LFC system.

Delay-Dependent Stability Analysis

Consider a time-delay LFC system represented in equation (5)

$$\dot{x}(t) = Ax(t) + A_d x(t-\tau) \quad (7)$$

where x $\in R_n$ is the state variable, $\tau \geq 0$ is the time delay, and A, $A_d \in R_{n\times n}$ are given constant matrices. The characteristic polynomial of the system represented in equation (5) can be written as

$$h(s,\tau) = \det\,(sI - A - A_d e^{-s\tau}) \quad (8)$$

$$= P(s) + Q(s)\, e^{-s\tau} \quad (9)$$

where

$$P(s) = p_4 s^4 + p_3 s^3 + p_2 s^2 + p_1 s$$

$$Q(s) = q_1 s + q_0$$

$P(s)$ and $Q(s)$ are polynomials of s having real coefficients. The coefficients of these polynomials are given in Appendix-I. The time delayed LFC system is asymptotically stable if and only if all the roots of the characteristic equation lie in the left half complex plane. The exponential term in equation (9) makes the characteristic equation transcendental and therefore the characteristic equation has infinite number of roots. It is very difficult to determine all the roots. However, to analyze the stability of LFC system, it is sufficient to determine the variation of roots of characteristic equation with respect to time delay.

The basic idea of the proposed approach is to study the location of the roots of the characteristic equation and to determine the delay margin for stability using the Rekasius substitution [18, 19] and Resultant theory [20]. The first step in the proposed approach is Rekasius substitution [19], [20] for

$$e^{-s\tau} = \frac{1-sT}{1+sT} \qquad T \geq 0, \tag{10}$$

which is defined only on the imaginary axis in the complex plane, s = jω. However, equation (10) is exact if and only if

$$\tau = \frac{2}{\omega} tan^{-1}(\omega T) + \frac{k\pi}{2} \tag{11}$$

k = 0, 1, 2, .Note that this substitution transforms the transcendental characteristic equation (9) into an algebraic equation. Applying Rekasius's substitution to the equation (9), the quasi polynomial characteristic equation is given by

$$h(j\omega, T) = \left(h(j\omega, \tau)\middle| e^{-s\tau} = \frac{1-j\omega T}{1+j\omega T}, T \geq 0\right)(1 + j\omega T) \tag{12}$$

The transformed equation (12) is complex which has real and imaginary parts in terms of T and ω and it can be decomposed as

$$h(j\omega, T) = h_R(j\omega, T) + jh_I(j\omega, T) \tag{13}$$

where $h_R = \Re(h)$; $h_I = \Im(h)$ .With the help of Resultant theory [20], the resultant of h_R and h_I can be obtained eliminating T. The resultant Z(ω) is a polynomial in terms of ω.

$$Z(\omega) = R_t(h_R, h_I, T) = 0 \tag{14}$$

The minimum positive real roots of Z (ω) gives the crossing frequency ω_0 corresponding to T. The relation between τ_k and T is given by (11).Then the time delay margin (τ_k) is computed using the equation

$$\tau_k = \frac{2}{\omega_0} \tan^{-1}(\omega_0 T) \tag{15}$$

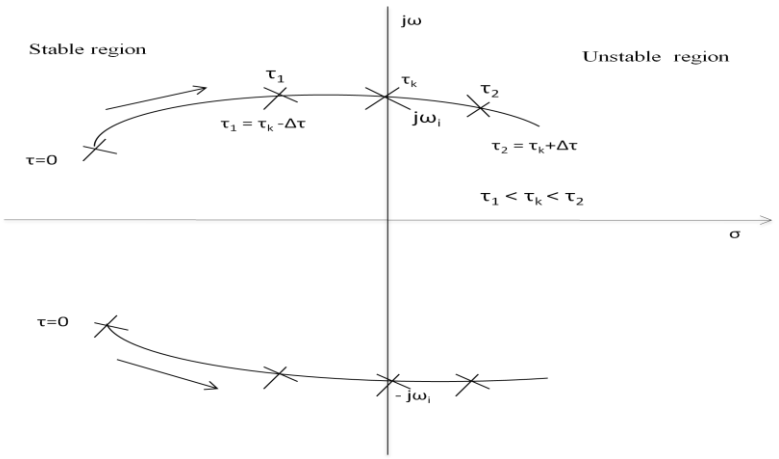


Figure 2: Illustration of movement of the roots of characteristic equation with respect to time delay

Figure.2 shows the movement of roots as time delay τ varies. The system is assumed to be asymptotically stable for τ= 0. As τ increases, the roots start moving from LHP to RHP. The time delay τ_k at which the roots cross imaginary axis by moving from LHP to RHP is defined as the time delay margin. Simulation is carried out for various values of time delay (τ) to find delay margin which is explained in next section.

Simulation Results

The system parameters of single area LFC are tabulated in Appendix-II. The stability of the system is investigated and the delay margin (τ_k) at which the roots of characteristic equation cross jω axis is computed using the proposed method. The results of the delay margin calculated for different sets of PI controller parameters are tabulated in Table.1.

Table 1: Delay margin obtained using the proposed method for various PI controller parameters.

Delay margin τ_k (secs)	Integral controller gain (K_I)						
Proportional controller gain (K_P)	0.05	0.1	0.15	0.2	0.4	0.6	1.00
0	30.915	15.20	9.960	7.335	3.382	2.042	0.9229
0.05	31.875	15.681	10.279	7.575	3.501	2.122	0.9704
0.1	32.751	16.119	10.571	7.794	3.610	2.194	1.0124
0.2	34.226	16.856	11.062	8.162	3.792	2.313	1.0785
0.4	35.834	17.658	11.594	8.558	3.980	2.425	1.1183
0.6	34.922	17.195	11.277	8.312	3.826	2.2811	0.9474
1	0.595	0.586	0.5753	0.564	0.516	0.4634	0.3610

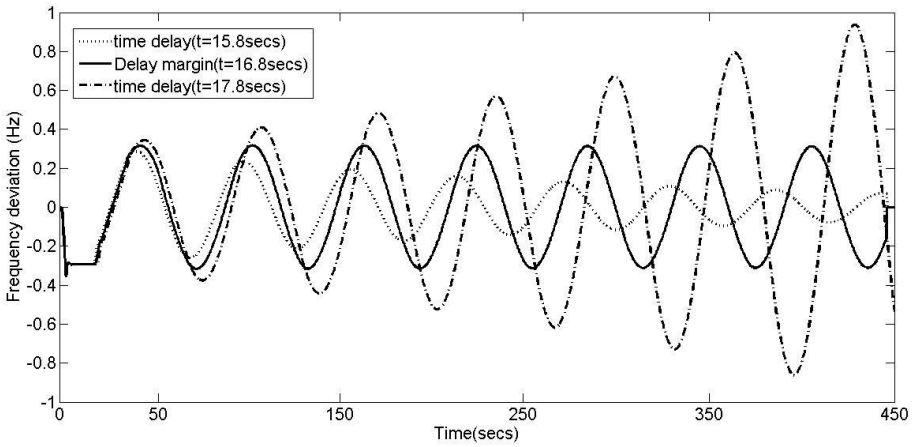
A. Validation of theoretical result

Simulation is performed to assess the accuracy of the theoretical value of delay margin obtained using proposed method. This is implemented by increasing the time delay step by step from 0 secs in simulink model until the system becomes unstable. The time delay at which the system is marginally stable is defined as Delay margin. The system response for PI controller gain for K_P= 0.2 and K_I=0.1 is shown in figure.3.The dashed dotted lines in Figure.3 show the growing oscillations for time delay $\tau = 17.8s$. The dotted lines in Figure.3 show decreasing oscillations for time delay $\tau = 15.8s$ .Thus the system loses its stability somewhere between 15.8 s and 17.8s and delay margin should lie within the range [15.8s, 17.8s]. The solid lines in Figure.3 show the sustained oscillations for delay margin $\tau_k = 16.8s$ which indicates the system is marginally stable. This delay margin value is closer to the theoretically

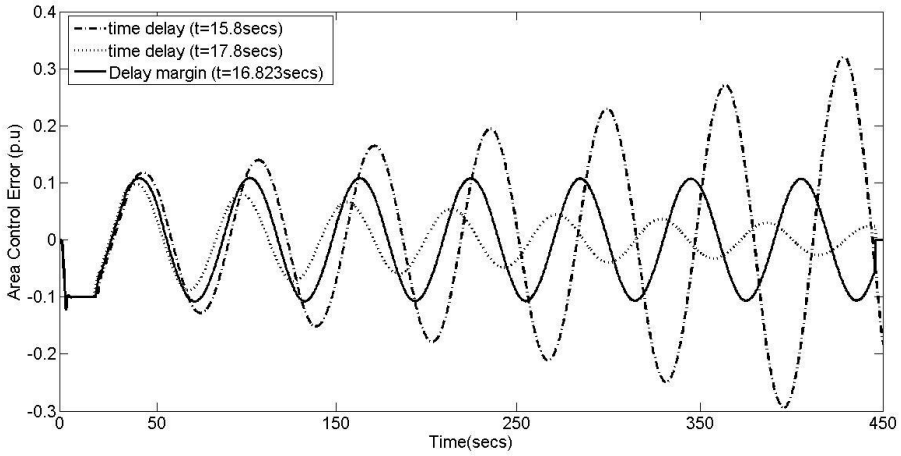
calculated delay margin (16.856s).Simulation results prove that the proposed method is highly accurate in finding delay margin compared to other frequency domain methods [18] and time domain methods [14] . It also reveals that the system is stable for all time delays less than delay margin and it becomes unstable when the time delay exceeds delay margin.

B. Observations and Discussions.

The maximum value of delay margin obtained based on Lyapunov stability criteria [14] is only 27.927s. But in the proposed method, the controller can be tuned to obtain the maximum delay margin as 35.834s. Thus the proposed method is more effective in retaining the stability for longer time period. The controller gain decides the delay margin and damping performance of the system. . For a given value of $K_{I,}$ when K_P is increased the delay margin increases initially and then decreases. For a given K_P, the delay margin decreases with increasing value of $K_{I.}$ These observations provide the relationship between controller gain and delay margin which can be used as a reference for choosing the controller parameters. Its value can be selected in such a way to get larger delay margin and less deterioration in the damping performance of the system. The larger delay margin extends the on service time of LFC scheme in the event of any communication faults and the stability of the system is not disturbed even for longer period of time delay.



(a)



(b)

Figure 3: Time response of the single area LFC with PI controller for K_P= 0.2 and K_I=0.1 (a) Frequency variation (b) ACE

Conclusion

An alternative approach based on frequency domain approach is suggested for the Delay dependent stability analysis of single area LFC scheme. The proposed method exploits two concepts namely Rekasius substitution and Resultant Theory to determine delay margin for stability. First, the transcendental characteristic equation of LFC system is transformed to polynomial equation using Rekasius substitution. Then Resultant theory is applied in the polynomial equation to determine the exact lower bound of crossing frequency .With the help of crossing frequency, the maximal time delay τ_k is derived using back transformation formula (10).Simulation is performed to prove that the system is asymptotically stable for all $\tau \in [0, \tau_k]$. The simulation result also reveals that the proposed method is highly accurate and provides better stability region, less computation time in comparison to other methods based on Lyapunov theory and LMI technique.

Appendix-I

The coefficients of polynomials P(s) and $Q(s)$ in (9) is given in terms of controller gains and time constants of the LFC system.

$p_4 = RT_gT_tM;$

$p_3 = RDT_gT_tM + RT_tM + RT_gM$

$p_2 = MR + RT_tD + RT_gD;$

$p_1 = RD + 1;$

$q_1 = \beta RK_P;$

$q_0 = \beta RK_I$

Appendix-II

Parameters of single area LFC scheme are:

β=21, D=1.0, M=10s, T_g=0.1s, T_t=0.3s, R=0.05

References

[1] P.Kundur, "*Power System Stability and Control*",(McGraw-Hill Inc.,), 1994.

[2] K.Gu,V. L.Kharitonov,J.Chen, "*Stability of Time Delay Systems*", (Birkhauser), 2003

[3] S.I.Niculescu, "*Delay Effects on Stability-A Robust Control Approach*, New York, (Springer- verlag), 2001.

[4] J.Chen,G.Gu,C.N. Nett,A "New Method for Computing Delay Margins for Stability of Linear Delay Systems", *System and Control Letters*, vol. 26, no.2, (1995), pp. 107-117.

[5] H.J.Jia and X.D.Yu, "A Simple Method for Power System Stability Analysis with Multiple Time Delays", in *proc. IEEE Power and Energy Soc. General Meeting*, (2008),pp. 20-24.

[6] N.Olgac and R. Sippahi, "An Exact Method for Stability Analysis of Time Delayed Linear Time Invariant Systems", *IEEE Trans. Autom. Control*, vol 47,no.5, (2002),pp.793-797.

[7] R.Sippahi and N.Olgac, "Complete Stability Robustness of Third – Order LTI Multiple Time-Delay Systems", *Automatica*, vol.41,no.8,(2005),pp.1413-1422.

[8] Z.Y.Liu, C.Z.Zhu andQ.Y.Jiang, "Stability Analysis of Time Delayed Power System based on Cluster Treatment of Characteristics Roots Method", in *Proc. IEEE Power and Energy Soc. General Meeting*, (2008), pp.20-24.

[9] Y.He, Q.G.Wang,L.H.Xie and C.Lin, "Further Improvement of Free-Weighting Matrices Technique for Systems with Time Varying Delay", *IEEE Trans. Autom. Control*, vol.52,no.2,(2007),pp.293-299.

[10] M.Wu,Y.He,J.H.She and G.P.Liu, "Delay- Dependent Criterion for Robust Stability of Time Varying Delay Systems", *Automatica*, vol 40,no.8, (2004),pp. 1435-1439.

[11] S.Y.Xu and J.La., "On Equivalence and Efficiency of Certain Stability Criteria for Time- Delay Systems", *IEEE Trans. Autom. Control* , vol.52,no.1,(2007),pp.95-101.

[12] X.Yu, K.Tomsovic, "Application of Linear Matrix Inequalities for Load Frequency Control with Communication Delays", *IEEE Trans. Power Syst.*, vol. 19, no. 3, (2004),pp. 1508-1515.

[13] Hassa Bevarani,Takashi Hiyama, " On Load-Frequency Regulation with Time Delays :Design and Real time Implementation" ,*IEEE Trans. on Energy conversions*, vol.24,(2009), pp. 292–300.

[14] L.Jiang ,W.Yao,Q.H .Wu,J.Y.Wen, "Delay Dependent Stability for Load Frequency Control with Constant and Time Varying Delays", *IEEE Trans.. Power Syst.,* vol.27, no.2, (2012), pp.932-941.

[15] R.Dey,S.Ghosh, G.Ray,A.Rakshit, "Load Frequency Control of Interconnected Power Systems with Communication Delays", *Electrical Power and Energy systems*, vol.42, (2012),pp. 672-84.

[16] Chuan-ke Zhang,L.Jiang, "Further Results on Delay Dependent Stability of Multi Area Load Frequency control", *IEEE Trans. Power Syst.*, vol.28, no.4, (2013),pp.4465-4474.

[17] Chuan-ke Zhang,L.Jiang, "Delay Dependent Robust Load frequency control for Time Delay Power System", *IEEE Trans.Power Syst.*, vol.28, no.3 , (2013).

[18] Sahin Sonmez, Saffet Ayasun,Ulas Eminoglu, "Computation of Time delay Margins for Stability of a Single Area Load Frequency Control System with Communication Delays",*Wseas Trans. on Power systems*, vol. 9,(2014),pp67-74.

[19] Christian Ebenbauer and Frank Allgower, *"Stability Analysis for Time-Delay Systems using Rekasius's Substitution and Sum of Squares"*, 45th IEEE Conference on Decision and Control , (2006).

[20] Rifat Sipahi and Ismail Ilker Delice , **"**Advanced Clustering With Frequency Sweeping Methodology for the Stability Analysis of Multiple Time-Delay Systems" ,*IEEE Trans. on Autom Control*,vol.56, issue.2, (2011).