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A Method of Ranking Generalized Trapezoidal Intuitionistic Fuzzy Numbers

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Abstract

This paper describes a method of rankinggeneralized trapezoidal intuitionistic fuzzy numbers based on area of both membership and non-membership parts of the numbers. The area used in this method is obtained from the generalizedtrapezoidalintuitionistic fuzzy number, first by splitting the membership part of the number into three plane figures and then calculating the centroids of each plane figure followed by the centroid of these centroids and then finding the area of this centroid from origin. The same area is calculated for the non-membership part also. This method is simple in evaluation and can rank various types of generalized intuitionistic fuzzy numbers and also crisp numbers which are considered to be a special case of fuzzy numbers.

2010 Mathematics Subject Classification: 03E72, 03F55

Key words: Generalized Trapezoidal fuzzy number, Intuitionistic fuzzy set, Ranking function.

Introduction

The intuitionistic fuzzy sets were first introduced by Atanassov [2, 3] which is a generalization of the concept of fuzzy sets. Ranking fuzzy numbers is one of the fundamental problems of fuzzy arithmetic and fuzzy decision making. Fuzzy numbers must be ranked before an action is taken by a decision maker. Real numbers are linearly ordered by the relations ≤or ≥, however this type of inequality does not exist in fuzzy numbers. Since fuzzy numbers are represented by possibility distribution, they can overlap with each other and it is difficult to determine clearly whether one fuzzy number is larger or smaller than other. An efficient method for ordering the fuzzy numbers is by the use of a ranking function, which maps each fuzzy number into the real line, where a natural order exists. Lingling, Shen, Hai Wang and XiangqianFeng [8] introduced a ranking method for intuitionistic fuzzy numbers in general based on probabilities and hesitations. Hassan MishmastNehi[7], SalimRezvani[12], Amit Kumar and ManjotKaur[1], and S. SagayaRoseline and E.C. Henry Amirtharaj[11] introduced methods for ordering two trapezoidal intuitionistic fuzzy numbers. ShashiAggarwal and Chavi Gupta [14],Satyajit Das and DebashreeGuha[13],and A.NagoorGani and V.N.Mohamed[l0]introduced methods for ordering two generalized trapezoidal intuitionistic fuzzy numbers. P.K.De and DebarotiDas [4] and Stephen Dinagar and Thiripurasundari[15] introduced how to defuzzify a generalized trapezoidal intuitionistic fuzzy number. Here we will propose a new method ofdefuzzifyinggeneralized trapezoidal intuitionistic fuzzy numbers.

This paper is organized as follows. First, in section 2, we will give the basic definitions of fuzzy numbers and intuitionistic fuzzy sets. In sections 3 and 4, we will introduce the concepts of intuitionistic fuzzy numbers (IFN), generalized trapezoidal intuitionistic fuzzy numbers (GTIFN) and some operations on GTIFNs. In section 5, we will propose a method of ordering generalized trapezoidal intuitionistic fuzzy numbers.In section 6, the significance of the proposed method over the existing methods is given. Numerical examples are presented in section 7, and finally the conclusion is given in section 8.

Preliminaries on Fuzzy Numbers and Intuitionistic Fuzzy Sets

In this section we will review the basic concepts of fuzzy numbers and intuitionistic fuzzy sets.

Fuzzy Numbers

The most important subfamily of all fuzzy sets is fuzzy numbers. It is not surprising since the predominant carrier of information is numbers. The notion of a fuzzy number was introduced by Dubois and Prade [5, 6].

Definition:A fuzzy set$\tilde{A}$, defined on the universal set of real numbers $\mathbb{R}$, is said to be a generalized fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}}: \mathbb{R} \to [0, \omega]$is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$.
3. $\mu_{\tilde{A}}(x)$is strictly increasing on $[a, b]$ and strictly decreasing on [c, d].
4. $\mu_{\tilde{A}}(x) = \omega$ for all $x \in [b, c]$, where $a < b < c < d$and $0 < \omega \leq 1$

Definition: A generalized fuzzy number $\tilde{A}$=$(a, b, c, d; \omega)$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \omega\left(\frac{x-a}{b-a}\right), \qquad a \leq x \leq b$$

$$= \omega, \qquad b \leq x \leq c$$

$$= \omega\left(\frac{x-d}{c-d}\right), \qquad c \leq x \leq d$$

$= 0,$ otherwise

Intuitionistic Fuzzy Sets

Definition [2, 3]: Let X be the universal set. An intuitionistic fuzzy set (IFS) A in X is given by

$$A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$$

where the functions $\mu_A(x), \nu_A(x)$ define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Obviously, every fuzzy set has the form

$$A = \{(x, \mu_A(x), \mu_{A^c}(x)) : x \in X\}.$$

For each intuitionistic fuzzy set $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ in X,

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$$

is called the hesitancy degree of x to lie in A. If A is a fuzzy set, then $\pi_A(x) = 0$ for all $x \in X$.

Intuitionistic Fuzzy Numbers

Here we will introduce the intuitionistic fuzzy number (IFN) [9] and generalized trapezoidal intuitionistic fuzzy number (GTIFN) [15] and some properties of them.

Definition: An IFS $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ of the real line $\mathbb{R}$ is called an intuitionistic fuzzy number (IFN) if

a) A is convex for the membership function $\mu_A(x)$, i.e., if $\mu_A(\lambda x_1 + (1 - \lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2)$ for all $x_1, x_2 \in \mathbb{R}, \lambda \in [0, 1]$.
b) A is concave for the non-membership function $\nu_A(x)$, i.e., if $\nu_A(\lambda x_1 + (1 - \lambda)x_2) \leq \nu_A(x_1) \vee \nu_A(x_2)$ for all $x_1, x_2 \in \mathbb{R}, \lambda \in [0, 1]$.
c) A is normal, that is, there is some $x_0 \in \mathbb{R}$ such that $\mu_A(x_0) = 1$ and $\nu_A(x_0) = 0$.

Definition: An intuitionistic fuzzy number A is said to be a generalized trapezoidal intuitionistic fuzzy number (GTIFN) with parameters

$$b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4,$$

and denoted by

$$A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4; \omega_A, u_A) \text{ or } A = ((a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4); \omega_A, u_A)$$

if its membership function and non-membership functions areas follows:

$\mu_A(x) = 0$ if $x < a_1$

$= \omega_A \left(\frac{x - a_1}{a_2 - a_1}\right)$ if $a_1 \leq x \leq a_2$

$= \omega_A$ if $a_2 \leq x \leq a_3$

$$= \omega_A \left(\frac{a_4 - x}{a_4 - a_3}\right) \text{ if } a_3 \leq x \leq a_4$$

$= 0$ if $x > a_4$

and

$\nu_A(x) = 1$ if $x < b_1$

$$= \frac{(b_2 - x) + u_A(x - b_1)}{b_2 - b_1} \text{ if } b_1 \le x \le b_2$$

$= u_A$ if $b_2 \le x \le b_3$

$= \frac{(x-b_3)+u_A(b_4-x)}{b_4-b_3}$ if $b_3 \le x \le b_4$

$= 1$ if $x > b_4$.

where $0 < \omega_A \le 1, 0 \le u_A \le 1$ and $0 < \omega_A + u_A \le 1$.

If $b_1 \ge 0$ and one of the eight values $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4$ is not equal to 0, then the TIFN $A = ((a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4); \omega_A, u_A)$ is called a positive GTIFN, denoted by $A > 0$. Likewise, if $b_4 \le 0$ and one of the eight values $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4$ is not equal to 0, then the GTIFN A is called a negativeTIFN, denoted by $A < 0$.

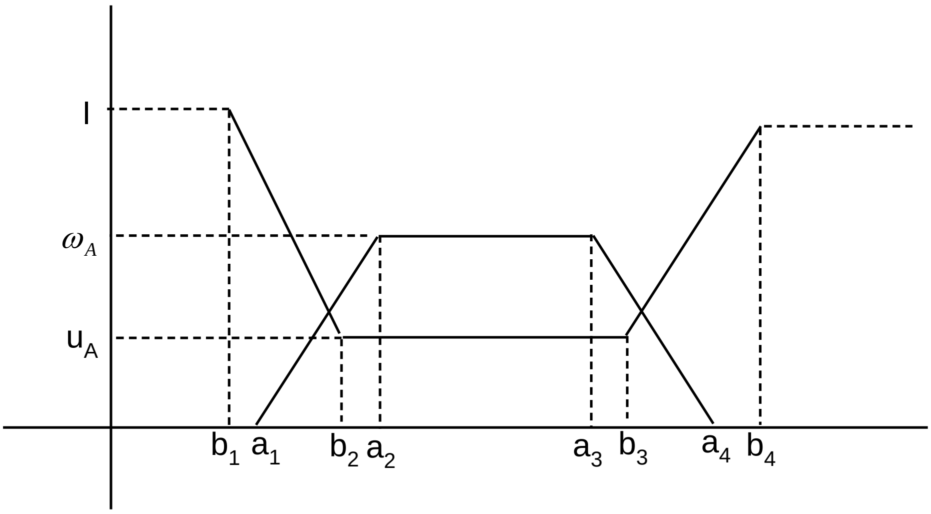

Figure 1: Generalized Trapezoidal Intuitionistic Fuzzy Number

Properties of Generalized Trapezoidal Intuitionistic Fuzzy Numbers

Let

$$A = ((a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4); \omega_A, u_A)$$

and

$$B = ((c_1, c_2, c_3, c_4), (d_1, d_2, d_3, d_4); \omega_B, u_B)$$

be two GTIFNs and λ be a real number. Then

1. $A + B = ((a_1 + c_1, a_2 + c_2, a_3 + c_3, a_4 + c_4), ((b_1 + d_1, b_2 + d_2, b_3 + d_3, b_4 + d_4); \omega, u)$
   where $\omega = \min(\omega_A, \omega_B)$ and $u = \max(u_A, u_B)$.
2. $A - B = ((a_1 - c_4, a_2 - c_3, a_3 - c_2, a_4 - c_1), (b_1 - d_4, b_2 - d_3, b_3 - d_2, b_4 - d_1); \omega, u)$

where$\omega = \min(\omega_A, \omega_B)$ and $u = \max(u_A, u_B)$.

3. $\lambda A = ((\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4), (\lambda b_1, \lambda b_2, \lambda b_3, \lambda b_4); \omega_A, u_A)if\lambda > 0$
$= ((\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1), (\lambda b_4, \lambda b_3, \lambda b_2, b a_1); \omega_A, u_A)$ if$\lambda < 0$.

4. $A \otimes B = ((a_1c_1, a_2c_2, a_3c_3, a_4c_4), (b_1d_1, b_2d_2, b_3d_3, b_4d_4);\ \omega, u)ifA > 0, B > 0$
$= ((a_1c_4, a_2c_3, a_3c_2, a_4c_1),(b_1d_4, b_2d_3, b_3d_2, b_4d_1);\ \omega, u)$if $A < 0, B > 0$
$= ((a_4c_4, a_3c_3, a_2c_2, a_1c_1), (b_4d_4, b_3d_3, b_2d_2, b_1d_1);\ \omega, u)ifA < 0, B < 0$

where$\omega = \min(\omega_A, \omega_B)$and $u = \max(u_A, u_B)$.

5. $A \div B = \left(\left(\frac{a_1}{c_4}, \frac{a_2}{c_3}, \frac{a_3}{c_2}, \frac{a_4}{c_1}\right), \left(\frac{b_1}{d_4}, \frac{b_2}{d_3}, \frac{b_3}{d_2}, \frac{b_4}{d_1}\right), \omega = \min(\omega_A, \omega_B) \text{ and } u = \max(u_A, u_B)\right)$, if $d_1 > 0$.

The Proposed Method for Ranking Generalized Trapezoidal Intuitionistic Fuzzy Numbers

Consider the GTIFN $A = ((a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4); \omega_A, u_A)$. The centroid point of a trapezoid is considered to be the balancing point of the trapezoid. Divide the trapezoid corresponding to the membership function $\mu_A(x)$into three plane figures. These three plane figures are a triangle APB, a rectangle BPQC and again a triangle CQD respectively. Let the centroids of the three plane figures be G_1, G_2 and G_3respectively.

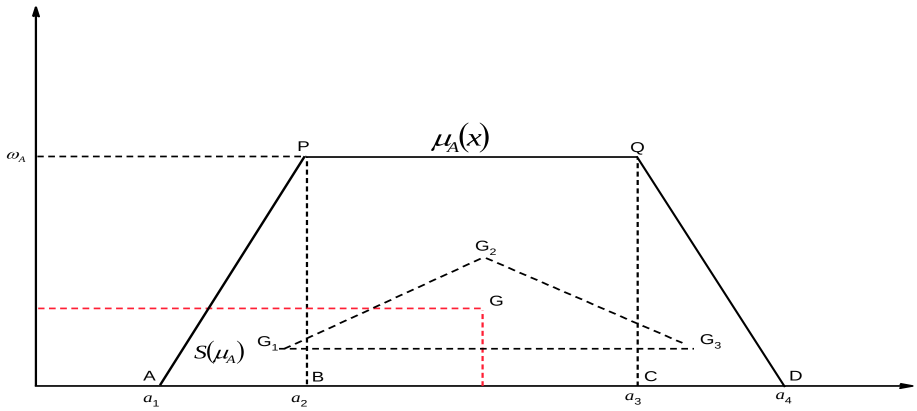

Figure 2: Trpezoid corresponding to the membership part

The centroid of these centroids G_1, G_2 and G_3 is taken as the point of reference to define the ranking of generalized trapezoidal intuitionistic fuzzy numbers. The reason for selecting this point as a point of reference is that each centroid point $G_1$of triangle APB, $G_2$of rectangle BPQC and $G_3$of triangle CQD are balancing points of each individual plane figure and the centroid of these centroid points is a much more balancing point for a generalized trapezoidal intuitionistic fuzzy number.

The centroids of these plane figures are $G_1 = \left(\frac{a_1+2a_2}{3}, \frac{\omega_A}{3}\right)$; $G_2 = \left(\frac{a_2+a_3}{2}, \frac{\omega_A}{2}\right)$ and $G_3 = \left(\frac{2a_3+a_4}{3}, \frac{\omega_A}{3}\right)$ respectively. Equation of the line G_1G_3 is $y = \frac{\omega_A}{3}$ and G_2does not lie on the line G_1G_3. Thus G_1, G_2 and G_3 are non collinear and they form a triangle. Then the centroid $G(\overline{x_0}, \overline{y_0})$ of the triangle with vertices G_1, G_2 and G_3 of the membership part of the generalized trapezoidal intuitionistic fuzzy number

$$A = ((a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4); \omega_A, u_A)$$

is

$$G(\overline{x_0}, \overline{y_0}) = \left(\frac{2a_1+7a_2+7a_3+2a_4}{18}, \frac{7\omega_A}{18}\right).$$

Now define

$$S(\mu_A) = \overline{x_0}.\overline{y_0} = \left(\frac{2a_1+7a_2+7a_3+2a_4}{18}\right)\left(\frac{7\omega_A}{18}\right)$$

This is the area between the centroid of the centroids $G(\overline{x_0}, \overline{y_0})$ and the original point.

Similarly we divide the trapezoid corresponding to the non-membership function $\nu_A(x)$into three plane figures. These three plane figures are a triangle $A'P'B'$, a rectangle $B'P'Q'C'$ and again a triangle$C'Q'D'$ respectively. Let the centroids of the three plane figures be G'_1, G'_2 and G'_3 respectively.

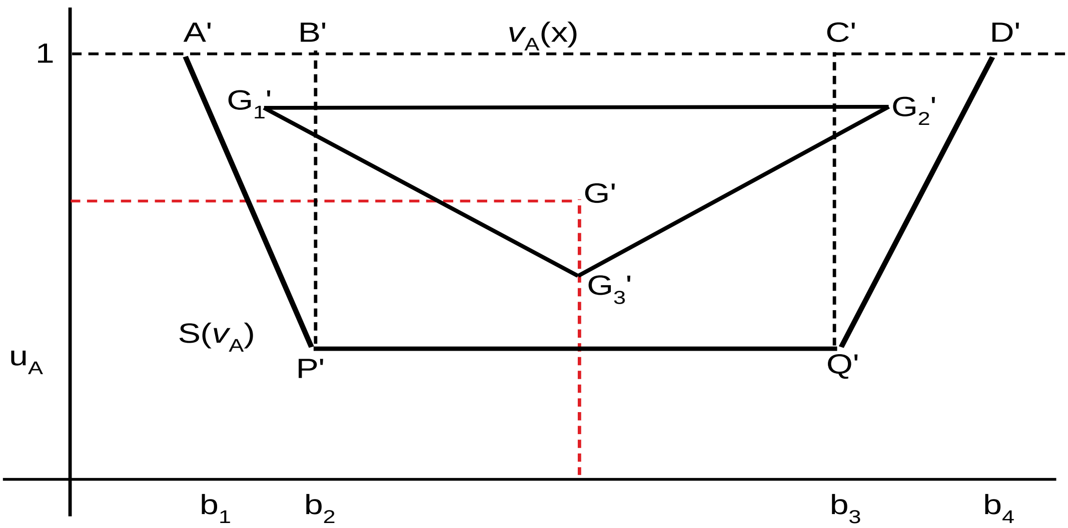

Figure 3: Trapezoid corresponding to the non-membership part

Then the centroid $G'(\overline{x_{0'}}, \overline{y_{0'}})$ of the triangle with verticesG'_1, G'_2 and G'_3 of the non-membership part of the trapezoidal intuitionistic fuzzy number

$$A = ((a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4); \omega_A, u_A)$$

is

$$G'(\overline{x_{0'}}, \overline{y_{0'}}) = \left(\frac{2b_1+7b_2+7b_3+2b_4}{18}, \frac{11+7u_A}{18}\right).$$

Now define

$$S(\nu_A) = \overline{x_{0'}}.\overline{y_{0'}} = \left(\frac{2b_1+7b_2+7b_3+2b_4}{18}\right)\left(\frac{11+7u_A}{18}\right).$$

Using the above definitions we define the rank of A as follows:

$$\Re(A) = \frac{\omega_A S(\mu_A) + u_A S(\nu_A)}{\omega_A + u_A}.$$

As a ranking method, we compare two generalized intuitionistic trapezoidal fuzzy numbers A and B using the following steps:

Step 1: Find $\Re(A)$and $\Re(B)$.
Step 2:
Case(*i*) If $\Re(A) > R(B)$then $A \succ B$.
Case(*ii*) If $\Re(A) < R(B)$ then $A \prec B$.
Case(*iii*) If $\Re(A) = \Re(B)$then $A \sim B$.

Significance of the Proposed Ranking Method Over Existing Methods

In the existing methods there are some shortcomings. The methods given by Amit Kumar and ManjotKaur [1], Satyajit Das and DebashreeGuha [13], S. SagayaRosleine and E.C. Henry Amirtharaj [11] and SalimRezvani[12] are used only for comparing two generalized intuitionistic trapezoidal fuzzy numbers. They were not giving a single defuzzified value for a GTIFN. The method introduced by ShashiAggarwal and Chavi Gupta [14] is used for comparing two generalized intuitionistic trapezoidal fuzzy numbers A and B with $a_2 = b_2$and $a_3 = b_3$. Hassan Mishmast Nehi [7] is considered only normalized intuitionistic trapezoidal fuzzy numbers. P.K. De and Debaroti Das [4] method is used for numbers with

$$a_1 = b_1, a_2 = b_2, a_3 = b_3 \text{ and } a_4 = b_4.$$

Our method gives a defuzzified value for all types of intuitionistic trapezoidal fuzzy numbers, both normalized and general. This method is also used for comparing two generalized intuitionistic trapezoidal fuzzy numbers. Moreover it is quite easy to see that the above proposed ranking method satisfies the seven axioms namely the Reasonable properties proposed by Wang and Kerre [16].

Numerical Examples

1. Let

$$A=\big((2,7,11,15),(1,6,12,16\,);0.5,0.3\big)$$

and

$$B=((2,5,8,10),(1,3,9,11);0.6,0.1).$$

Then $S(\mu_A)$ =1.7284, $S(\nu_A) = 6.4691$, $S(\mu_B)$=1.4907 and $S(\nu_B) = 3.9$.
So $\Re(A) = 3.5062$ and $\Re(B) = 1.8349$. Since $\Re(A) > \Re(B)$, we have $A \succ B$.

2. Let

$$A=((2,2.5,3,3.5),(1.5,2.2,3.2,4\,);0.7,0.2)$$

and

$$B=((2.5,3.2,3.8,4.2),(2,3,4,4.5);0.6,0.1).$$

Then $S(\mu_A)$ =0.7486,$S(\nu_A) = 1.8677$,$S(\mu_B)$=0.8089 and $S(\nu_B) = 2.2389$.
So $\Re(A) = 0.9973$ and $\Re(B) = 1.0132$.Since $\Re(A) < \Re(B)$, we have $A \prec B$.

3. Let

$$A=\big((2,3.5,4.5,6),(2,3.5,4.5,6\,);0.5,0.2\big)$$

and

$$B=((2,3,5,6),(2,3,5,6);0.5,0.2).$$

Then $S(\mu_A)$ =0.778, $S(\nu_A) = 2.756$, $S(\mu_B)$=0.778 and $S(\nu_B) = 2.756$.
So $\Re(A) = 1.3431$and $\Re(B) = 1.3431$. Since $\Re(A) = \Re(B)$, we have $A \sim B$.

Conclusion

This paper proposes a method that ranks generalized trapezoidal intuitionistic fuzzy numbers which is simple and concrete. This method ranks trapezoidal as well as triangular intuitionistic fuzzy numbers and their images. This method also ranks crisp numbers which are special case of intuitionistic fuzzy numbers where as some methods proposed in literature cannot rank crisp numbers.

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