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Accounting Of Dynamics Of Standing Waves In A Rotating Ring With Supports

AleksandrIvanovichPolunin

Belgorod State Technological University named after V.G. Shukhov, Russia, 308012, Belgorod, Kostyukovastreet, 46
Larissa GenadievnaSmyshlyaeva
Belgorod State Technological University named after V.G. Shukhov, Russia, 308012, Belgorod, Kostyukovastreet, 46

TatyanaVladimirovnaBondarenko

Belgorod State Technological University named after V.G. Shukhov, Russia, 308012, Belgorod, Kostyukovastreet, 46

Summary

In article dynamics of standing waves arising in a ring rotating on two supports owing to action of revolting factors has been considered. The theorem of the law of excited standing wave movement in a rotating ring with a constant angular speed on supports has been proved. The ring's dynamics is described by the linear differential equations. The mathematical model of dynamics of a ring rotating on supports with the extensible average line has been received. Upon receipt of the equations used widely used method of reducing the equations of the dynamics of the ring partial to ordinary differential equations by setting the own waveforms.

Keywords: dynamicsofrotatingringonsupports, standingwave, precession resonance in a rotating ring, influence of speed of ring rotation with supports on the frequency of its own fluctuations, effect of influence of angular speed of rotating ring on supports on the frequency of its own fluctuations.

Introduction.

The rings rotating on supportand covers are widely used in various industries, in technological processes. However there are enough scientific works devoted to the analysis of dynamics and the intense deformed condition of such designs in the course of their functioning [1], [2]. Besides, there is a modern technology of processing of large-size rings (with a diameter up to several meters), which covers for the purpose of giving a circular form to them. The body is put on a vertical plane on two basic rollers, given to rotation by the electric drive and processed by the added machine module (fig. 1). In process of such processing there are fluctuations of a body influencing quality of shaping, that is necessary to consider at a choice of technological modes of processing. It is possible to carry out it only using mathematical model of dynamics of a ring.

Main part.

We will receive the dependences describing behavior of a ring of constant radius r, rotating with a constant speed at two basic rollers.

There are two approaches for obtaining equations of the dynamics of the ring. First, most accurate, is to obtain partial differential equations and solving them. Its flaw is the difficulty of obtaining the solution of partial differential equations especially in the presence of derivatives and non-linear relationships. So often takes a different approach, which consists in setting waveforms studied body using a Fourier series. Its coefficients are functions of time, which is necessary to determine. Analysis of the system is reduced to solving a system of ordinary differential equations. The flaw of this approach is less accurate; advantage is the ability to take into account the communication and nonlinearity. Thearticleusesthesecondaccess.

At a conclusion of the equations we will accept the following assumptions. Basic rollers have a form of the cylinder of constant radius. Axes of rollers are parallel, their position in process of rotation changes. The axis of rotation of a ring in the indignant and not indignant movement coincides.

We will be define the behavior of rings, using the hypothesis of flat sections.

For the description of behavior of a ring we will set the system of coordinates of X_1, Y_1, Z_1 (fig. 1). The axis $O_1Y_1 is$an axis of rotation of a ring, axis O_1Z_1 is sent vertically, the axis O_1X_1 lies orthogonally. For a task of provision of points of the average line of the rotating ring we will use the polar system of coordinates (teta) θ, r, where (teta) θ is the corner which is setting the provision of a point on the average line of a ring concerning theO_1X_Caxis, connected with a rotating ring, r is the ring radius. The beginning of the system of coordinates is in the center of the ring rotating with an angular speed (omega) Ω. Width of a ring of$a$, h is thickness. Using this system of coordinates, movement of any point of the ring in the process of deformation we will set projections of its movement to the axes n_1, n_2. Their beginning is on the average line of a ring in the point which has been set by a corner (teta) θ. The $n_1$axisison r vector, the n_2 axis is tangent to the average line and orthogonally to n_1.Thecornerbetweensupportsis (2 alfa) 2α.

One of features of a problem of dynamics modeling of the ring rotating on two support is the account arising from rotation, owing to the action of Koryolis forces, the precession movement of the standing waves excited by action of indignations. The analysis of this phenomenon is given in works [3] - [9] where it is shown that in a case of free rotated rings the angular speed of a precession depends on number of the excited harmonica and angular speed of rotation of the ring.Existence of supports complicates a dynamic picture due to restrictions of area of the precession movement of standing waves and overlaying of rigid restrictions on movement of points of a ring in places of its contact with support. The available works [10] on dynamics of the flexible tape rotating on a support are devoted to the research of its established state and don't consider emergence of dynamics of the excited standing waves.

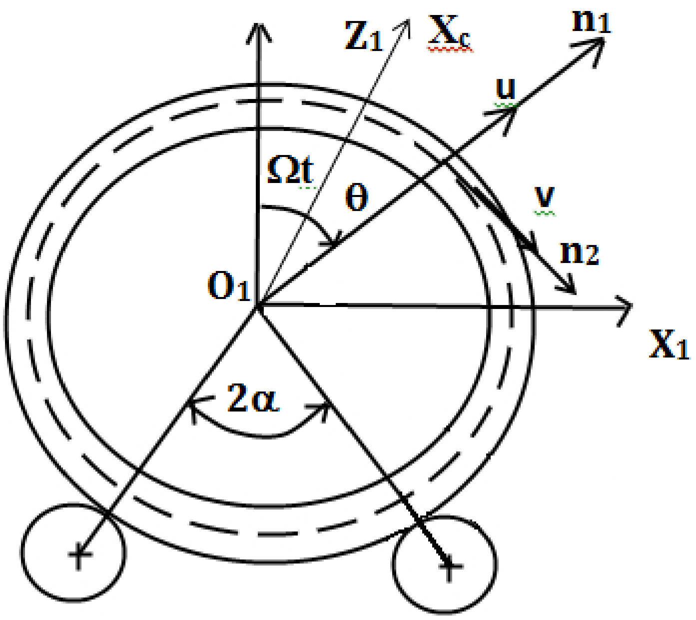

Figure 1. System of coordinates

We will designate projections of movement of a point of the average line of a ring, at its deformation, on a vector n_1, $n_2$is respectively$U(teta\theta)$, $V(teta\theta)$.

For simplification of receiving the equations and their decision, the account in the equations of conditions of communication imposed on removal of U owing to existence of support, it is better to present initial system in the form of system concentrated, having used decomposition of coordinates of movements of a body in a row Fourier when receiving the equations, with the distributed parameters. To receive dynamics equations we can use Lagrange's equation of the second sort with uncertain multipliers.

Sizes of radial U and tangential the V of points' movements of the average line of a ring, taking into account its tensile properties, we will set in the form of Fourier's ranks

$$U = a_0 + \sum_{i=1}^{N} a_{ui} \cos\big(i(\theta + \varphi_{uj})\big) + \sum_{i=1}^{N} b_{ui} \sin\big(i(\theta + \varphi_{uj})\big), \qquad (1)$$

$$V=\sum_{i=1}^{N}a_{vi}\cos\bigl(i(\theta+\varphi_{vj})\bigr)+\sum_{i=1}^{N}b_{vi}\sin\bigl(i(\theta+\varphi_{vj})\bigr)\ (2)$$

Here$a_0, a_{ui}, b_{ui}, a_{vi}, b_{vi}$– arethegenericcoordinates–unknownfunctionsoftime*t*, whicharetobedefined; (fi$_{uj}$)φ_{uj}, (fi_{vj}) φ_{vj} — unknownfunctionsoftime, definingthe precession of standing waves, which are to be defined; N – number of the considered summand of Furier's rank.

We determine kinetic energy of a ring rotating on support by dependence [4]

$$T=\frac{}{2}\int_{0}^{2\pi}\left[\left(\dot V+\Omega r+\Omega U\right)^2+\left(\dot U-\Omega V\right)^2\right]d\theta, \qquad (3)$$

Where(HI) $=r\rho F, r, \rho\ (ro)\ , F$ – isrespectivelyradiusofthering'sline, specific density of a material, area of cross section.

Potential energy of a bend and stretching of a ring it is calculated on dependences [4]

$$\Pi_1=\frac{\mu}{2}\int_{0}^{2\pi}\left(\frac{\partial V}{\partial\theta}-\frac{\partial^2 U}{\partial\theta^2}\right)^2 d\theta\,,\ \Pi_2=\frac{\nu}{2}\int_{0}^{2\pi}\left(\frac{\partial V}{\partial\theta}+U\right)^2 d\theta, \qquad (4)$$

Whereμ = EIr^{-2}, ν = EFr^{-1}, E – modulus of elasticity of a material, I–inertial moment of the ring's section.

Communications are imposed on the generalized coordinates of a ring owing to existence of points of support in which radial movement of U is equal to zero.

$$a_0+\sum_{i=1}^{N}a_{ui}\,cos\bigl(i(\pi-\propto-\Omega t+\varphi_{uj})\bigr)+\sum_{i=1}^{N}b_{ui}\,sin\bigl(i(\pi-\propto-\Omega t+\varphi_{uj})\bigr)=0, \qquad (5)$$

$$a_0+\sum_{i=1}^{N}a_{ui}\,cos\bigl(i(\pi+\propto-\Omega t+\varphi_{uj})\bigr)+\sum_{i=1}^{N}b_{ui}\,sin\bigl(i(\pi+\propto-\Omega t+\varphi_{uj})\bigr)=0. \qquad (6)$$

Formulas of kinetic and potential energy of a ring, condition of communications are used for receiving the differential equations of its behavior. However, inourcasewecannotusethemcausethe rear unknownfunctionsoftime(FI$_{uj}$)φ_{uj}, (FI$_{vj}$)$\varphi_{vj}, j=(1,2,\dots,N)$, setting a procession of harmonicas of standing waves in a ring, the behavior of which is unknown. For clarification of nature of their movement we will prove the following THEOREM.

At emergence of periodic fluctuations in rotating with a constant angular speed at two parallel support a ring, a precession of harmonicas (FI$_{qi}$)$\varphi_{qi}(t)$ (*i=1,2,...,N*), (*q=U,V*) occurswithangularspeedoftheringrotation(omega)**Ω**, andprecessionofastandingwave – withangularspeedofaring(omega)Ω at$a_{qi}(t) = k_{qi}b_{qi}(t)$or when one coordinate is equal to zeroa_{qi}, b_{qi}, where$a_{qi}(t), b_{qi}(t)$- are generic coordinates on i – harmonic, k_{qi} –accidental coefficient for every harmonic, or, at$a_{qi}(t) \neq k_{qi}b_{qi}(t)$procession of a standing wave occurs with a periodical angular speed $\Omega + f_{qi}(t)$, where $f_{qi}(t)$-is a periodic function.

Proving

The functions appearing in formulas of kinetic and potential energy (FI$_{uj}$)φ_{uj},(FI$_{VJ}$)$\varphi_{vj}(j = 1, 2, \ldots, N)$influence behavior of a ring and they can be considered as one more generalized coordinates which behavior submits to mechanics laws. From this it follows that for determination of nature of behavior of harmonicas of standing waves it is necessary to find the differential equations of their behavior. For this purpose we use also Lagrange's equation of the second sort.

Let'sdefinethereceptionofequationsa_{uj}, b_{uj}, (FI$_{uj}$)$\varphi_{uj}, j = (1, 2, \ldots, N)$in general case using formalism of Lagrange.

Let'sdefinekineticandpotentialenergy of the system in a general way

$$T = \int_0^{2\pi} f_T(\dot{U}, U, \dot{V}, V)d\theta\,, \Pi = \int_0^{2\pi} f_\Pi\,(U, U'', V, V')d\theta.$$

We can write down

$$\frac{dT}{d\dot{a}_{uj}} = \int_0^{2\pi} \frac{\partial f_T}{\partial \dot{U}}\frac{\partial \dot{U}}{\partial \dot{a}_{uj}}d\theta,$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{a}_{uj}}\right) = \int_0^{2\pi}\left[\frac{d}{dt}\left(\frac{\partial f_T}{\partial \dot{U}}\right)\frac{\partial \dot{U}}{\partial \dot{a}_{uj}} + \frac{\partial f_T}{\partial \dot{U}}\frac{d}{dt}\left(\frac{\partial \dot{U}}{\partial \dot{a}_{uj}}\right)\right]d\theta,$$

$$\frac{\partial T}{\partial a_{uj}} = \int_0^{2\pi}\left[\frac{\partial f_T}{\partial \dot{U}}\frac{\partial \dot{U}}{\partial a_{uj}} + \frac{\partial f_T}{\partial U}\frac{\partial U}{\partial a_{uj}}\right]d\theta, \frac{\partial \Pi}{\partial a_{uj}} = \int_0^{2\pi}\left[\frac{\partial f_\Pi}{\partial U}\frac{\partial U}{\partial a_{uj}} + \frac{\partial f_\Pi}{\partial U''}\frac{\partial U''}{\partial a_{uj}}\right]d\theta.$$

We will substitute these dependences in Lagrange's equation of the second sort

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial \Pi}{\partial q_j} = \sum_{i=1}^{k}\lambda_i\ e_{ij}\ , \qquad (7)$$

Where*T*,*Π*- arekineticandpotential energy; q_j–j -generic coordinate; t – time; k - number of the communications imposed on coordinates q_j; (lambda$_i$)λ_i - Lagrange's uncertain multiplier; e_{ij}–derivative ofi- communication equations on *j* - coordinate.

We will have

$$\int_0^{2\pi}\left[\frac{d}{dt}\left(\frac{\partial f_T}{\partial \dot{U}}\right)\frac{\partial \dot{U}}{\partial \dot{a}_{uj}}+\frac{\partial f_T}{\partial \dot{U}}\frac{d}{dt}\left(\frac{\partial \dot{U}}{\partial \dot{a}_{uj}}\right)\right]d\theta-\int_0^{2\pi}\left[\frac{\partial f_T}{\partial \dot{U}}\frac{\partial \dot{U}}{\partial a_{uj}}+\frac{\partial f_T}{\partial U}\frac{\partial U}{\partial a_{uj}}\right]d\theta+$$

$$+\int_0^{2\pi}\left[\frac{\partial f_\Pi}{\partial U}\frac{\partial U}{\partial a_{uj}}+\frac{\partial f_\Pi}{\partial U"}\frac{\partial U"}{\partial a_{uj}}\right]d\theta=\frac{\partial F_{u1}}{\partial a_{uj}}\lambda_1+\frac{\partial F_{u2}}{\partial a_{uj}}\lambda_2.$$

Here $\frac{\partial F_{u1}}{\partial a_{uj}}, \frac{\partial F_{u2}}{\partial a_{uj}}$ – derivatives of communications′conditions (5), (6).

Carrying out transformation of this expression, we will have the differential equation received by differentiation of Lagrange on a variable $a_{uj}(j=1,2,\dots,n)$.

$$\int_0^{2\pi}\frac{d}{dt}\left(\frac{\partial f_T}{\partial \dot{U}}\right)\frac{\partial \dot{U}}{\partial \dot{a}_{uj}}d\theta-\int_0^{2\pi}\frac{\partial f_T}{\partial U}\frac{\partial U}{\partial a_{uj}}d\theta+$$

$$+\int_0^{2\pi}\left(\frac{\partial f_\Pi}{\partial U}\frac{\partial U}{\partial a_{uj}}+\frac{\partial f_\Pi}{\partial U"}\frac{\partial U"}{\partial a_{uj}}\right)d\theta=\frac{\partial F_{u1}}{\partial a_{uj}}\lambda_1+\frac{\partial F_{u2}}{\partial a_{uj}}\lambda_2. \quad (8)$$

Using the transformations similar used when receiving (8), we will have at differentiation on variables b_{uj}, (FI_{UJ}) $\varphi_{uj}(j=1,2,\dots,N)$

$$\int_0^{2\pi}\frac{d}{dt}\left(\frac{\partial f_T}{\partial \dot{U}}\right)\frac{\partial \dot{U}}{\partial \dot{b}_{uj}}d\theta-\int_0^{2\pi}\frac{\partial f_T}{\partial U}\frac{\partial U}{\partial b_{uj}}d\theta+\int_0^{2\pi}(\frac{\partial f_\Pi}{\partial U}\frac{\partial U}{\partial b_{uj}}+\frac{\partial f_\Pi}{\partial U''}\frac{\partial U''}{\partial b_{uj}})d\theta=$$

$$=\frac{\partial F_{u1}}{\partial b_{uj}}\lambda_1+\frac{\partial F_{u2}}{\partial b_{uj}}\lambda_2, \quad (9)$$

$$\int_0^{2\pi}\frac{d}{dt}\left(\frac{\partial f_T}{\partial \dot{U}}\right)\frac{\partial \dot{U}}{\partial \dot{\varphi}_{uj}}d\theta-\int_0^{2\pi}\frac{\partial f_T}{\partial U}\frac{\partial U}{\partial \varphi_{uj}}d\theta+\int_0^{2\pi}(\frac{\partial f_\Pi}{\partial U}\frac{\partial U}{\partial \varphi_{uj}}+\frac{\partial f_\Pi}{\partial U''}\frac{\partial U''}{\partial \varphi_{uj}})d\theta=$$

$$=\frac{\partial F_{u1}}{\partial \varphi_{uj}}\lambda_1+\frac{\partial F_{u2}}{\partial \varphi_{uj}}\lambda_2. \quad (10)$$

$$\frac{\partial F_{u1}}{\partial \varphi_{uj}}=-ja_{uj}\,sin\left(j\left(\pi-\alpha-\Omega t+\varphi_{uj}\right)\right)+jb_{uj}\,cos\left(j\left(\pi-\alpha-\Omega t+\varphi_{uj}\right)\right),$$

$$\frac{\partial F_{u2}}{\partial \varphi_{uj}} = -ja_{uj}\,sin\left(j\left(\pi + \alpha - \Omega t + \varphi_{uj}\right)\right) + jb_{uj}\,cos\left(j\left(\pi + \alpha - \Omega t + \varphi_{uj}\right)\right).$$

Substituting expressions for derivatives, using (1), (2), in dependence (8), (9), (10) wewillhavethesystemofdifferentiatedequationsin a general view, receivedbydifferentiationofLagrangeon variables $a_{uj}, b_{uj}, (FI_{Uj})$ φ_{uj}(*j=1,...,N*).

$$\int_0^{2\pi} \frac{d}{dt}\left(\frac{\partial f_T}{\partial \dot{U}}\right) cos\left(j\left(\theta + \varphi_{uj}\right)\right) d\theta - \int_0^{2\pi} \frac{\partial f_T}{\partial U} cos\left(j\left(\theta + \varphi_{uj}\right)\right) d\theta +$$

$$+\int_0^{2\pi} \frac{\partial f_\Pi}{\partial U} cos\left(j\left(\theta + \varphi_{uj}\right)\right) d\theta + \int_0^{2\pi} \frac{\partial f_\Pi}{\partial U''}\left(-j^2 cos\left(j\left(\theta + \varphi_{uj}\right)\right)\right) d\theta =$$

$$= cos\left(j\left(\pi - \alpha - \Omega t + \varphi_{uj}\right)\right)\lambda_1 + cos\left(j\left(\pi + \alpha - \Omega t + \varphi_{uj}\right)\right)\lambda_2, \qquad (11)$$

$$\int_0^{2\pi} \frac{d}{dt}\left(\frac{\partial f_T}{\partial \dot{U}}\right) sin\left(j\left(\theta + \varphi_{uj}\right)\right) d\theta - \int_0^{2\pi} \frac{\partial f_T}{\partial U} sin\left(j\left(\theta + \varphi_{uj}\right)\right) d\theta +$$

$$+\int_0^{2\pi} \frac{\partial f_\Pi}{\partial U} sin\left(j\left(\theta + \varphi_{uj}\right)\right) d\theta + \int_0^{2\pi} \frac{\partial f_\Pi}{\partial U''}\left(-j^2 sin\left(j\left(\theta + \varphi_{uj}\right)\right)\right) d\theta =$$

$$= sin\left(j\left(\pi - \alpha - \Omega t + \varphi_{uj}\right)\right)\lambda_1 + sin\left(j\left(\pi + \alpha - \Omega t + \varphi_{uj}\right)\right)\lambda_2, \qquad (12)$$

$$\int_0^{2\pi} \frac{d}{dt}\left(\frac{\partial f_T}{\partial \dot{U}}\right)\left[-a_{uj} j\,sin\left(j\left(\theta + \varphi_{uj}\right)\right) + b_{uj} j\,cos\left(j\left(\theta + \varphi_{uj}\right)\right)\right] d\theta -$$

$$-\int_0^{2\pi} \frac{\partial f_T}{\partial U}\left[-a_{uj} j\,sin\left(j\left(\theta + \varphi_{uj}\right)\right) + b_{uj} j\,cos\left(j\left(\theta + \varphi_{uj}\right)\right)\right] d\theta +$$

$$+\int_0^{2\pi} \frac{\partial f_\Pi}{\partial U}\left[-a_{uj} j\,sin\left(j\left(\theta + \varphi_{uj}\right)\right) + b_{uj} j\,cos\left(j\left(\theta + \varphi_{uj}\right)\right)\right] d\theta +$$

$$+\int_0^{2\pi} \frac{\partial f_\Pi}{\partial U''}\left[j^3 a_{uj}\,sin\left(j\left(\theta + \varphi_{uj}\right)\right) - j^3 b_{uj}\,cos\left(j\left(\theta + \varphi_{uj}\right)\right)\right] d\theta =$$

$$= \left[-j\, a_{uj} sin\left(j\left(\pi - \alpha - \Omega t + \varphi_{uj}\right)\right) + j\, b_{uj} cos\left(j\left(\pi - \alpha - \Omega t + \varphi_{uj}\right)\right)\right] \lambda_1 +$$

$$+ \left[-j\, a_{uj} sin\left(j\left(\pi + \alpha - \Omega t + \varphi_{uj}\right)\right) + j\, b_{uj} cos\left(j\left(\pi + \alpha - \Omega t + \varphi_{uj}\right)\right)\right] \lambda_2. \quad (13)$$

The analysis of the equations (11), (12), (13) shows that the differential equation (13) received by differentiation on (FI_{uj})φ_{uj}, it can be received by combination of equations (11), (12), received by differentiation on a_{uj}, b_{uj}. Forthisweneedtheequation(11) to multiply by $jb_{uj}$and to deduct equation (12), multiplied by ja_{uj}. Buttheequations(11), (12) havethefunctions (FI_{uj}) $\varphi_{uj}(t)$*(j=1,...,N).*So, they can be different.

The similar proof can be also carried out for the equations describing behavior of a ring and received by differentiation on variables $a_{vj}, b_{vj},\ (FI_{vj})\ \varphi_{vj}$.

Todefinethecharacterofbehavioroffunctions(FI_{uj}) $\varphi_{uj}, (FI_{VJ})\ \varphi_{vj}$inourcasetransform(11), (12)togetequationsinthestandardformusingunder integralequations for kinetic and potential energy (3), (4).

To ease them we will introduce

$$S_{uj}^{+} = sin\left(j\left(\theta + \varphi_{uj}\right)\right),\ \ S_{uj}^{-} = -\,sin\left(j\left(\theta + \varphi_{uj}\right)\right),\ \ C_{uj}^{+} = cos\left(j\left(\theta + \varphi_{uj}\right)\right),$$

$$C_{uj}^{-} = -\,cos\left(j\left(\theta + \varphi_{uj}\right)\right),\ S_{vj}^{+} = sin\left(j\left(\theta + \varphi_{vj}\right)\right),\ S_{vj}^{-} = -\,sin\left(j\left(\theta + \varphi_{vj}\right)\right),$$

$$C_{vj}^{+} = cos\left(j\left(\theta + \varphi_{vj}\right)\right),\ C_{vj}^{-} = -\,cos\left(j\left(\theta + \varphi_{vj}\right)\right),$$

Using them we will put down

$$\dot{V} = \sum_{i=1}^{N} \dot{a}_{vi}\, C_{vi}^{+} + \sum_{i=1}^{N} \dot{b}_{vi}\, S_{vi}^{+} + \sum_{i=1}^{N} a_{vi}\, i\dot{\varphi}_{vi} S_{vi}^{-} + \sum_{i=1}^{N} b_{vi}\, i\dot{\varphi}_{vi} C_{vi}^{+},$$

$$\dot{U} = \dot{a}_0 + \sum_{i=1}^{N} \dot{a}_{ui}\, C_{ui}^{+} + \sum_{i=1}^{N} \dot{b}_{ui}\, S_{Ui}^{+} + \sum_{i=1}^{N} a_{ui}\, i\dot{\varphi}_{ui} S_{ui}^{-} + \sum_{i=1}^{N} b_{ui}\, i\dot{\varphi}_{ui} C_{ui}^{+},$$

$$\frac{\partial V}{\partial \theta} = \sum_{i=1}^{N} a_{vi}\, iS_{vi}^{-} + \sum_{i=1}^{N} b_{vi}\, iC_{vi}^{+},\ \ \frac{\partial^2 U}{\partial \theta^2} = \sum_{i=1}^{N} a_{ui}\, i^2 C_{ui}^{-} + \sum_{i=1}^{N} b_{ui}\, i^2 S_{ui}^{-},$$

$$\ddot{U} = \ddot{a}_0 + \sum_{i=1}^{N} \ddot{a}_{ui}\, C_{ui}^{+} + \sum_{i=1}^{N} \dot{a}_{ui}\, i\dot{\varphi}_{ui} S_{ui}^{-} + \sum_{i=1}^{N} \ddot{b}_{ui}\, S_{ui}^{+} + \sum_{i=1}^{N} \dot{b}_{ui}\, i\dot{\varphi}_{ui} C_{ui}^{+} +$$

$$+\sum_{i=1}^{N} \dot{a}_{ui}\, i\dot{\varphi}_{ui} S_{ui}^{-} + \sum_{i=1}^{N} a_{ui}\, i\ddot{\varphi}_{ui} S_{ui}^{-} + \sum_{i=1}^{N} a_{ui}\, i^2 \dot{\varphi}_{ui}^2 C_{ui}^{-} + \sum_{i=1}^{N} \dot{b}_{ui}\, i\dot{\varphi}_{ui} C_{ui}^{+} +$$

$$+\sum_{i=1}^{N} b_{ui}\, i\ddot{\varphi}_{ui} C_{ui}^{+} + \sum_{i=1}^{N} b_{ui}\, i^2 \dot{\varphi}_{ui}^2 S_{ui}^{-}.$$

Atcalculationofintegralswewilluse the dependences

$$\int_0^{2\pi} cos\left(i\left(\theta + \varphi_{pi}\right)\right) cos\left(i\left(\theta + \varphi_{qi}\right)\right) d\theta = \pi\, cos\left(i\left(\varphi_{pi} - \varphi_{qi}\right)\right) = \pi C_i^{pq},$$

$$\int_0^{2\pi} sin\left(i\left(\theta + \varphi_{pi}\right)\right) sin\left(i\left(\theta + \varphi_{qi}\right)\right) d\theta = \pi\, cos\left(i\left(\varphi_{pi} - \varphi_{qi}\right)\right) = \pi C_i^{pq},$$

$$\int_0^{2\pi} sin\left(i\left(\theta + \varphi_{pi}\right)\right) cos\left(i\left(\theta + \varphi_{qi}\right)\right) d\theta = \pi\, sin\left(i\left(\varphi_{pi} - \varphi_{qi}\right)\right) = \pi K_i^{pq},$$

$$\int_0^{2\pi} C_{pj}^{+}\, C_{qi}^{+} d\theta = \int_0^{2\pi} C_{pj}^{+}\, S_{qi}^{+} d\theta = \int_0^{2\pi} S_{pj}^{+}\, S_{qi}^{+} d\theta = 0, \ \ (i \neq j).$$

Usingthedependenceswewillhavefortheequations(11), obtained by differentiating on a_{uj}.

$$\frac{\partial f_T}{\partial \dot{U}} = \left(\dot{U} - \Omega V\right), \ \ \frac{d}{dt}\left(\frac{\partial f_T}{\partial \dot{U}}\right) = \left(\ddot{U} - \Omega \dot{V}\right),$$

$$\int_0^{2\pi} \frac{d}{dt}\left(\frac{\partial f_T}{\partial \dot{U}}\right) cos\left(i(\theta + \varphi_{ui})\right) d\theta = \pi(\ddot{a}_{uj} + 2\dot{b}_{uj} j\dot{\varphi}_{uj} - a_{uj} j^2 \dot{\varphi}_{uj}^2 +$$

$$+b_{uj} j\ddot{\varphi}_{uj} - \Omega \dot{a}_{vj} C_j^{vu} - \Omega \dot{b}_{vj} K_j^{vu} + \Omega a_{vj} j\dot{\varphi}_{vj} K_j^{vu} - \Omega b_{vj} j\dot{\varphi}_{vj} C_j^{vu}),$$

$$\frac{\partial f_T}{\partial U} = \left(\dot{V} + \Omega r + \Omega U\right)\Omega,$$

$$\int_0^{2\pi} \frac{\partial f_T}{\partial U} cos\left(j(\theta + \varphi_{uj})\right) d\theta = \int_0^{2\pi} \Omega(\dot{V} + \Omega r + \Omega U)\, cos\left(j(\theta + \varphi_{uj})\right) d\theta =$$

$$= \pi\Omega\left(\dot{a}_{vj}C_j^{vu} + \dot{b}_{vj}K_j^{vu} - a_{vj}j\dot{\varphi}_{vj}K_j^{vu} + b_{vj}j\dot{\varphi}_{vj}C_j^{vu} + \Omega a_{uj}\right),$$

$$\frac{\partial f_\Pi}{\partial U} = v\left(\frac{\partial V}{\partial \theta} + U\right),$$

$$\int_0^{2\pi} \frac{\partial f_\Pi}{\partial U} cos\left(j(\theta + \varphi_{uj})\right) d\theta = \int_0^{2\pi} v\left(\frac{\partial V}{\partial \theta} + U\right) cos\left(j(\theta + \varphi_{uj})\right) d\theta =$$

$$= \pi\nu\left(-a_{vj}jK_j^{vu} + b_{vj}jC_j^{vu} + a_{uj}\right),$$

$$\frac{\partial f_\Pi}{\partial U''} = -\mu\left(\frac{\partial V}{\partial \theta} - \frac{\partial^2 U}{\partial \theta^2}\right),$$

$$\int_0^{2\pi} \frac{\partial f_\Pi}{\partial U''}\left(-j^2\, cos\left(j(\theta + \varphi_{uj})\right)\right) d\theta =$$

$$= \mu\int_0^{2\pi} \left(\frac{\partial V}{\partial \theta} - \frac{\partial^2 U}{\partial \theta^2}\right) j^2\, cos\left(j(\theta + \varphi_{uj})\right) d\theta =$$

$$= \pi\mu\left(-a_{vj}j^3K_j^{vu} + j^3 b_{vj}C_j^{vu} + j^4 a_{uj}\right).$$

Forequationsdefinedby differentiation on b_{uj}, we have

$$\int_0^{2\pi} \frac{d}{dt}\left(\frac{\partial f_T}{\partial \dot{U}}\right) sin\left(j(\theta + \varphi_{uj})\right) d\theta = \pi(\ddot{b}_{uj} - 2\dot{a}_{uj}j\dot{\varphi}_{uj} - a_{uj}j\ddot{\varphi}_{uj} -$$

$$-b_{uj}j^2\dot{\varphi}_{uj}^2 - \Omega\dot{a}_{vj}K_j^{vu} - \Omega\dot{b}_{vj}C_j^{vu} + \Omega a_{vj}j\dot{\varphi}_{vj}C_j^{vu} - \Omega b_{vj}j\dot{\varphi}_{vj}K_j^{vu}),$$

$$\int_0^{2\pi} \frac{\partial f_T}{\partial U} sin\left(j(\theta + \varphi_{uj})\right) d\theta =$$

$$= \pi\Omega\left(\dot{a}_{vj}K_j^{vu} + \dot{b}_{vj}C_j^{vu} - a_{vj}j\dot{\varphi}_{vj}C_j^{vu} + b_{vj}j\dot{\varphi}_{vj}K_j^{vu} + \Omega b_{uj}\right),$$

$$\int_0^{2\pi} \frac{\partial f_\Pi}{\partial U} sin\left(j(\theta+\varphi_{uj})\right) d\theta = \pi\nu\left(-a_{vj} j C_j^{vu} + b_{vj} j K_j^{vu} + b_{uj}\right),$$

$$\int_0^{2\pi} \frac{\partial f_\Pi}{\partial U''} (-j^2 \, sin\left(j(\theta+\varphi_{uj})\right)) \, d\theta = \pi\mu\left(-a_{vj} j^3 C_j^{vu} + j^3 b_{vj} K_j^{vu} + j^4 b_{uj}\right).$$

Carrying out similar transformations for the equations received by differentiation of Lagrange on the variables a_{vj},b_{vj} $(j = 1,2,\dots,N)$, we will receive the system of the equations describing dynamics of a ring in the presence of excited standing waves

$$\pi(\ddot{a}_{uj} + 2\dot{b}_{uj} j\dot{\varphi}_{uj} - a_{uj} j^2 \dot{\varphi}_{uj}^2 + b_{uj} j\ddot{\varphi}_{uj} - \Omega\dot{a}_{vj} C_j^{vu} - \Omega\dot{b}_{vj} K_j^{vu} +$$

$$+\Omega a_{vj} j\dot{\varphi}_{vj} K_j^{vu} - \Omega b_{vj} j\dot{\varphi}_{vj} C_j^{vu}) - \pi\Omega(\dot{a}_{vj} C_j^{vu} + \dot{b}_{vj} K_j^{vu} - a_{vj} j\dot{\varphi}_{vj} K_j^{vu} +$$

$$+b_{Vj} j\dot{\varphi}_{vj} C_j^{vu} + \Omega a_{uj}) + \pi\nu\left(-a_{vj} j K_j^{vu} + b_{vj} j C_j^{vu} + a_{uj}\right) + \pi\mu(-a_{vj} j^3 K_j^{vu} +$$

$$+j^3 b_{vj} C_j^{vu} + j^4 a_{uj}) = \frac{\partial F_{u1}}{\partial a_{uj}}\lambda_1 + \frac{\partial F_{u2}}{\partial a_{uj}}\lambda_2, \qquad (14)$$

$$\pi(\ddot{b}_{uj} - 2\dot{a}_{uj} j\dot{\varphi}_{uj} - b_{uj} j^2 \dot{\varphi}^2{}_{uj} - a_{uj} j\ddot{\varphi}_{uj} - \Omega\dot{a}_{vj} K_j^{vu} - \Omega\dot{b}_{vj} C_j^{vu} +$$

$$+\Omega a_{vj} j\dot{\varphi}_{vj} C_j^{vu} - \Omega b_{vj} j\dot{\varphi}_{vj} K_j^{vu}) - \pi\Omega(\dot{a}_{vj} K_j^{vu} + \dot{b}_{vj} C_j^{vu} - a_{vj} j\dot{\varphi}_{vj} C_j^{vu} +$$

$$+b_{vj} j\dot{\varphi}_{vj} K_j^{vu} + \Omega b_{uj}) + \pi\nu\left(-a_{vj} j C_j^{vu} + b_{vj} j K_j^{vu} + b_{uj}\right) + \pi\mu(-a_{vj} j^3 C_j^{vu} +$$

$$+j^3 b_{vj} K_j^{vu} + j^4 b_{uj}) = \frac{\partial F_{u1}}{\partial b_{uj}}\lambda_1 + \frac{\partial F_{u2}}{\partial b_{uj}}\lambda_2, \qquad (15)$$

$$\pi(\ddot{a}_{vj} + \dot{b}_{vj} j\dot{\varphi}_{vj} - a_{vj} j^2 \dot{\varphi}^2{}_{vj} + \dot{b}_{vj} j\dot{\varphi}_{vj} + b_{vj} j\ddot{\varphi}_{vj} + \Omega\dot{a}_{uj} C_j^{uv} + \Omega\dot{b}_{uj} K_j^{uv} -$$

$$-\Omega a_{uj} j\dot{\varphi}_{uj} K_j^{uv} + \Omega b_{uj} j\dot{\varphi}_{uj} C_j^{uv}) + \pi\Omega(\dot{a}_{uj} C_j^{uv} + \dot{b}_{uj} K_j^{uv} - a_{uj} j\dot{\varphi}_{uj} K_j^{uv} +$$

$$+b_{uj} j\dot{\varphi}_{uj} C_j^{uv} - \Omega a_{vj}) + \pi\mu\left(-a_{uj} j^3 K_j^{uv} - b_{uj} j^3 C_j^{uv} + a_{vj} j^2\right) + \pi\nu(-a_{uj} j K_j^{uv} -$$

$$-j b_{uj} C_j^{uv} + a_{vj} j^2) = 0, \qquad (16)$$

$$\pi(\ddot{b}_{vj} - \dot{a}_{vj} j\dot{\varphi}_{vj} - \dot{a}_{vj} j\dot{\varphi}_{vj} - a_{vj} j\ddot{\varphi}_{vj} - b_{vj} j^2 \dot{\varphi}_{vj}^2 + \Omega\dot{a}_{uj} K_j^{uv} + \Omega\dot{b}_{uj} C_j^{uv} -$$

$$-\Omega a_{uj} j\dot{\varphi}_{uj} C_j^{uv} + \Omega b_{uj} j\dot{\varphi}_{uj} K_j^{uv}) + \pi\Omega(\dot{a}_{uj} K_j^{uv} + \dot{b}_{uj} C_j^{uv} - a_{uj} j\dot{\varphi}_{uj} C_j^{uv} +$$

$$+b_{uj} j\dot{\varphi}_{uj} K_j^{uv} - \Omega b_{vj}) + \pi\mu\left(j^3 a_{uj} C_j^{uv} + j^3 b_{uj} K_j^{uv} + b_{vj} j^2\right) +$$

$$+\pi\nu(a_{uj} j C_j^{uv} + b_{uj} K_j^{uv} + b_{vj} j^2) = 0. \qquad (17)$$

Differentiatedequationsdefinedby differentiation on a_0, has the view

$$2\pi\ddot{a}_0 - 2\pi(\Omega^2 + \nu)a_0 = 2\pi\Omega^2 r + \lambda_1 + \lambda_2, \qquad (18)$$

Let'sanalyzetheequations we got (14) - (17). We are interested in the established fluctuations of a ring. We consider linear system. In this case the differential equations describing behavior of a ring have to be the linear differential equations with constant coefficients. Thiswillbeinthecaseif(FI$_{uj}$with point) $\dot{\varphi}_{uj}$, $(FI_{VJ}$with point) $\dot{\varphi}_{vj}$, $(j = 1, 2, \dots, N)$will be constants. Anotherconditionfordefinition($FI_{uj}$with point)$\dot{\varphi}_{uj}$, ($FI_{vj}$with point) $\dot{\varphi}_{vj}$isthecondition, that precession movement of standing waves is carried out inside the range of angles restricted by supports. Incaseif$(FI_{uj}$with point$)\dot{\varphi}_{uj}$, $(FI_{vj}$with point$)\dot{\varphi}_{vj}$, $(j = 1, 2, \dots, N)$arethe constants that will be at performing conditions $\dot{\varphi}_{uj} = \dot{\varphi}_{vj} = \Omega$, $(j = 1, 2, \dots, N)$. Then we will get $C_j^{vu} = 1, K_j^{vu} = 0, \ddot{\varphi}_{uj} = \ddot{\varphi}_{vj} = 0$.

Let's consider precession of a standing wave. Wehavethe size of radial fluctuations, i.e. of a standing wave

$$U = a_0 + \sum_{i=1}^{N} a_{ui} \cos\left(i(\theta + \varphi_{uj})\right) + \sum_{i=1}^{N} b_{ui} \sin\left(i(\theta + \varphi_{uj})\right),$$

$$\text{or} \quad U = a_0 + \sum_{i=1}^{N} \mathrm{A}_{ui}\, sin\left(i(\theta + \varphi_{uj} + {}_{ui})\right).$$

Here $A_{ui}^2(t) = a_{ui}^2(t) + b_{ui}^2(t)$, 'ϒ$_{ui}$ (t) = β$_{ui}$(t)/i, β$_{ui}$(t) = arc tg$(a_{ui}(t)/b_{ui}(t))$.

At$a_{ui}(t) = k_{ui} b_{ui}(t)$, where$k_{ui}$ – is the constant, we will get$\beta_{ui}(t) = arctg(k_{ui})$.

Consideringthatφ_{uj} = Ωt,wewillget$_{ui}(t) = arctg(k_{ui})/i, \varphi_{ui} + {}_{ui} = \Omega t +$ arctg$(k_{ui})/i, \frac{d}{dt}(\varphi_{ui} + {}_{ui}) = \Omega$, i.e. precessionoccurs with constant angle speed which is equal to the speed of the rotating ring.Besides, the constant speed of a precession of i – harmonicas will be at equality of zero one of the generalized coordinates$a_{ui}(t), b_{ui}(t)$.

At$a_{ui}(t) \neq k_{ui} b_{ui}(t)$ and$a_{ui}(t), b_{ui}(t)$ not equal to zero $\beta_{ui}(t)$ = *arctg(*$a_{ui}(t)/b_{ui}(t)$*)* andis a periodic function, as т.к. the variables $a_{ui}(t), b_{ui}(t)$ describe periodic vibrational process , then (GAMMA$_{ui}$)'ϒ$_{ui}$(t),(GAMMA$_{ui}$ with the point)$\dot{}_{ui}(t)$- are periodic functions.

Thespeedofprecessionofastandingwaveis$\frac{d}{dt}(\varphi_{ui} + {}_{ui}) = \Omega + \dot{}_{ui}(t)$, i.e. consists of the sum of constant and periodic speed of movement. The theorem is proved.

The similar proof can be also provided for the variable *V*.

From the proved theorem as the special case, we can get the speed of precession of the excited standing wave in freely rotating ring, coinciding with the received one in [4].

Using result of the theorem, and also the dependences (14) - (18), we will receive final system of equations of dynamics of a ring rotating on support with the extensible average line in the presence of the excited standing waves. In it Lagrange's uncertain multipliers are excluded and conditions of communications in points of support are considered.

$$\ddot{a}_0 - \frac{1}{2cos(\pi -)}\ddot{a}_{u1} - \frac{\Omega}{cos(\pi -)}\dot{b}_{u1} + \frac{\Omega}{cos(\pi -)}\dot{a}_{v1} +$$

$$+\frac{(\nu - \Omega^2)}{}a_0 - \frac{(\nu + \mu - 2\Omega^2)}{2\,cos(\pi -)}a_{u1} - \frac{(\nu + \mu - 2\Omega^2)}{2\,cos(\pi -)}b_{v1} = \Omega^2 r\,,$$

$$c_j\ddot{a}_{u1} + \ddot{a}_{uj} + 2\Omega c_j\dot{b}_{u1} + 2\Omega j\dot{b}_{uj} - 2\Omega c_j\dot{a}_{v1} - 2\Omega\dot{a}_{vj} +$$

$$+\frac{(\nu + \mu - 2\Omega^2)}{}c_j a_{u1} + \frac{\left(\nu + \mu j^4 - \Omega^2(1 + j^2)\right)}{}a_{uj} +$$

$$+\frac{(\nu + \mu - 2\Omega^2)}{}c_j b_{v1} + j\frac{(\nu + \mu j^2 - 2\Omega^2)}{}b_{vj} = 0,$$

$$s_j\ddot{b}_{u1} + \ddot{b}_{uj} - 2\Omega s_j\dot{a}_{u1} - 2\Omega j\dot{a}_{uj} - 2\Omega s_j\dot{b}_{v1} - 2\Omega\dot{b}_{vj} +$$

$$+\frac{(\nu + \mu - 2\Omega^2)}{}s_j b_{u1} + \frac{\left(\nu + \mu j^4 - \Omega^2(1 + j^2)\right)}{}b_{uj}$$

$$+\frac{(2\Omega^2 - \nu - \mu)}{}s_j a_{v1} + j\frac{(2\Omega^2 - \nu - \mu j^2)}{}a_{vj} = 0,$$

$$(j = 2,3,\dots,N-1),$$

$$c_N\ddot{a}_{u1} + W_1\ddot{p} + 2\Omega c_N\dot{b}_{u1} + 2\Omega N W_2\dot{p} - 2\Omega c_N\dot{a}_{v1} - 2\Omega\dot{a}_{vN} +$$

$$+\frac{(\nu + \mu - 2\Omega^2)}{}c_N a_{u1} + \frac{\left(\nu + \mu N^4 - \Omega^2(1 + N^2)\right)}{}W_1 p +$$

$$+\frac{(\nu + \mu - 2\Omega^2)}{}c_N b_{v1} + N\frac{(\nu + \mu N^2 - 2\Omega^2)}{}b_{vN} = 0,$$

$$s_N\ddot{b}_{u1} + W_2\ddot{p} - 2\Omega s_N\dot{a}_{u1} - 2\Omega N W_1 p - 2\Omega s_N\dot{b}_{v1} - 2\Omega\dot{b}_{vN} +$$

$$+\frac{(\nu + \mu - 2\Omega^2)}{} s_N b_{u1} + \frac{\left(\nu + \mu N^4 - \Omega^2(1+N^2)\right)}{} W_2 p +$$

$$+\frac{(2\Omega^2 - \nu - \mu)}{} s_N a_{v1} + N\frac{(2\Omega^2 - \nu - \mu N^2)}{} a_{vN} = 0,$$

$$(j = 2, 3, \dots, N-1),$$

$$\ddot{a}_{vj} + 2\Omega\dot{a}_{uj} + 2j\Omega\dot{b}_{vj} + j\frac{\left(2\Omega^2 - (\mu j^2 + \nu)\right)}{} b_{uj} +$$

$$+\frac{\left(j^2(\nu+\mu) - \Omega^2(1+j^2)\right)}{} a_{vj} = 0,$$

$$\ddot{b}_{vj} + 2\Omega\dot{b}_{uj} - 2j\Omega\dot{a}_{vj} + j\frac{(\mu j^2 + \nu - 2\Omega^2)}{} a_{uj} +$$

$$+\frac{\left(j^2(\nu+\mu) - \Omega^2(1+j^2)\right)}{} b_{vj} = 0,$$

$$(j = 1, 2, \dots, N-1),$$

$$\ddot{a}_{vN} + 2\Omega W_1\dot{p} + 2N\Omega\dot{b}_{vN} + N\frac{\left(2æ\Omega^2 - (\mu N^2 + \nu)\right)}{æ} W_2 p +$$

$$+\frac{\left(N^2(\nu+\mu) - æ\Omega^2(1+N^2)\right)}{æ} a_{vN} = 0,$$

$$\ddot{b}_{vN} + 2\Omega W_2\dot{p} - 2N\Omega\dot{a}_{vN} + N\frac{(\mu N^2 + \nu - 2æ\Omega^2)}{æ} W_1 p +$$

$$+\frac{\left(N^2(\nu+\mu) - æ\Omega^2(1+N^2)\right)}{æ} b_{VN} = 0.$$

We will carry out the analysis of the received system. Among coefficients of the equations there are expressions $\frac{(\nu+\mu-2\Omega^2)}{2\cos(\pi-)}$, $\frac{\left(\nu+\mu j^4-\Omega^2(1+j^2)\right)}{}$ *(j=2,…,N)*,and others, depending on strength of characteristics of the ring's material, its geometry (nu)ν, (mu)μand the angular speed of rotation (omega)Ω. These coefficients characterize force, returning a ring in position of balance on which size the frequency of its own fluctuations depends.At (omega) Ω=0theringdoesn't rotate, itsvibrationsatabsenceoffrictionrepresentperiodic vibration process described by the received system of equations. Its frequency depends only on physical and geometrical parameters of a ring. At increase of (omega) Ωfromzero sizes of these forces reduce owing to emergence of the centrifugal force reducing force returning a ring in neutral situation, and the period of own fluctuations of a ring increases. With increaseof(omega) $\Omega$the periods increases and at some speed the size of the returning forces are equal to zero. Value of this critical speed isdifferentfor each harmonica. Atfurtherincreaseof (omega) Ωvalue of the returning force becomes negative and the matrix at coordinates has negative elements that influence the decision. The system of the equations describes now dispersing oscillatory process, i.e. there is a resonance.

Infigures2, 3therearedifferencesofcoefficient (tau$_1$)$\tau_1$for rings with different radiuses, and the first own frequency (omega$_1$)$\omega_1$of the speed of the ring's rotation (omega)Ω in rad/s.

The number of harmonicas of a number of Fourier is equal to 6. Calculation of these characteristics was carried out by the solution of the frequency equation by a numerical method. The coefficient of internal friction is equal to zero, thickness of rings is 0,05m, a corner between supports is 40 degrees. Number 1 designates dependence for a ring of the radius of 5 meters, number 2 – the radius of 3 meters.

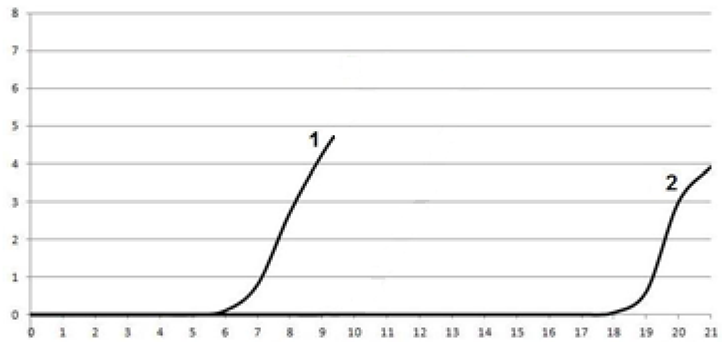

Figure 2. Dependence of an indicator of divergence of the oscillatory process from the speed of rotation of a ring

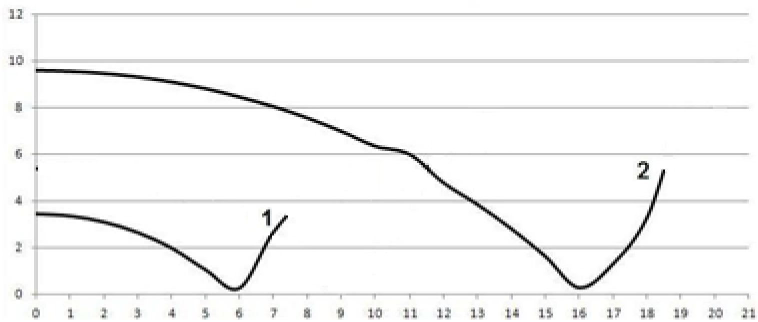

Figure 3. Dependence of the first own frequency of fluctuations of a ring from its speed of rotation

In the figures it is seen that with increase in speed of rotation the own frequency decreases. At some speed of rotation of a ring the coefficient (tau1) $\tau 1$, characterizing divergence of oscillatory process, becomes positive, there is a precession resonance, the frequency of own fluctuations increases.

Conclusion

Wehaveprovedthetheoremofthelawofprocessionalmovementofexcitedstandingwavesin therotatingringontwosupportswithconstantanglespeed. Basedonresultsoftheproofwehavereceivedamathematicalmodelofdynamicsoftherotatin gringwhichhasexcitedstandingwaves.Analysis of the equations and calculations it showed that increasing the speed of rotation of the ring on supports its own oscillation frequency decreases.

Wehavecarriedouttheoreticalbasisforanalysisoftherings' dynamicsrotatingonsupportswithconstantanglespeed. Thereceivedresultscanbeusedformathematicmodelingofqualityofshapingatprocessing of the rings with use of mobile technology.

Findings

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