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Modeling And Estimation Of Casual Components Errors Of Polygon Measurements Without Attraction Reference Measurements

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Abstract

On the basis of the analysis of dependence of errors of smoothing of polygon measurements the polynom of the first and second degree received a way which allows to create simple algorithm of modeling and estimation of a methodical error of processing and dispersion of the casual making errors of measurements without attraction of reference measurements. Thus it is necessary that the accuracy characteristics of standard measuring devices are much higher accuracy of the tested vehicles. The model of reference coordinates of a trajectory of flight of air object is set in the form of a polynom. The universal rule allowing to calculate at the same time values of estimates of both a mean square error, and a population mean of a square of a mathematical error is received. It is important that the received rule doesn't depend on a type of reference function.

Keywords: error of measurements, reference measurements.

I. Introduction

When determining precision characteristics of the tested means it is necessary that the error of measurements is considered a difference between the measured value of coordinate of location of the air object (AO) and reference value of the same coordinate. For definiteness we will assume that radar station (RLS) is subjected to tests, and reference values of coordinates turn out as a result of application of some reference measuring means with the subsequent mathematical processing of the registered reference measurements.

By default it is necessary that precision characteristics of standard measuring devices are much higher accuracy characteristics of the tested means. However in practice this condition isn't always satisfied, and then the technique of verification of characteristics of RLS distorts them towards deterioration of the estimated characteristics. Thereby artificially the risk of receiving a fallacy about discrepancy of precision characteristics of RLS to the set requirements while these requirements actually are fulfilled increases. At last, reference measurements are expensive and therefore not always available in the necessary volume. In this regard techniques of an assessment of precision characteristics of measurements in the conditions of lack of reference measurements began to appear.

The work purpose – estimation of dispersion of casual components of errors of polygon measurements in the conditions of lack of reference measurements with simultaneous estimation of a methodical error of the applied method of mathematical data processing.

2. Statement of the Problem

Model reference coordinate values of a trajectory of flight AO we will set in the form of a polynom $f^{RF}(t)=\sum_{r=0}^{R}\frac{a_r}{r!}t^r$, on a supervision interval $t\in[0,T]$. Legitimacy of such task follows from Weierstrass's theorem of completeness of polynoms in space of continuous functions on a piece $[0,T]$. Obviously, $a_i=\frac{d^i f^{RF}}{dt}\Big|_{t=0}$, (i=*0, 1, ... ,* R*)*. We will present the measured values of coordinates in the form of the tabular function $f^{MS}(t_i)=f^{RF}(t_i)+\Delta_i,\ \ (i=0,1,\dots,M)$ set in equidistant knots $t_i=i\,\Delta t$,, where $\Delta t=\frac{T}{M}$ – a step of measurements, and Δ_i – errors of measurements of which a priori it is known that they uncorrelated $(k(\Delta_i,\Delta_j)=0,\ if\ i\neq j)$, and have the identical dispersions equal σ^2. We will apply procedure of smoothing to the measured values with a width of an interval of smoothing $N=2k+1$ of knots when the knot $t_i,\ \ k+1\le i\le M-k$,, is symmetrically spread with an interval so that at the left and to the right of knot t_i in an interval it appears exactly to κ knots. Then on an interval approximation of the measured function on a method of the smallest squares (MMS) by n=0 degree polynom, 1, 2 is carried out. After that value $f^{MS}(t_i)$ is replaced with value of the approximating polynom $P^{MMS}_{n,N}(t_i)$.

More conveniently to work with the approximating polynom, on each interval it is expedient to apply linear transformation of argument $u_s=s=\frac{t_j-t_i}{\Delta t}=j-i$, transferring time reference mark to a point and doing all knots whole: $u_s\in\mathbf{U}=\{0,\pm1,\pm2,\dots,\pm k\}$. In this case narrowing of a reference polynom on an interval will assume an air

$$f^{RF}(t_j)=\sum_{r=0}^{N-1}\frac{a'_{r,N}}{r!}u_s^r$$

where

$$a'_{r,N} = \Delta^r t \sum_{m=r}^{N-1} \frac{a_m}{(m-r)!} t_i^{m-r} .$$

We will enter for narrowings on intervals with the center in knot the following designations: for reference function – $f^{RF}(i,s)$, for the measured function – $f^{MS}(i,s) = f^{RF}(i,s) + \Delta_s$, for the approximating polynom $-P^{MMS}_{n,N,i}(s)$. Applying polynoms, orthogonal on a smoothing interval $Q_0 = 1, \quad Q_1 = \frac{1}{k}u, \quad Q_2 = \frac{3u^2 - k(k+1)}{k(2k-1)}$, we will receive:

$$P^{MMS}_{0,N,i}(s) = c_0 Q_0, \quad P^{MMS}_{1,N,i}(s) = c_0 Q_0 + c_1 Q_1, \quad P^{MMS}_{2,N,i}(s) = c_0 Q_0 + c_1 Q_1 + c_2 Q_2 ,$$

Where - $c_m = \left(f^{MS}(s), Q_m(s)\right) = \sum_{s=-k}^{k} f^{MS}(s) Q_m(s)$ - corresponding coefficients of Fourier. In [1] it is shown that smoothing in a middle part of the table of knots $i \in [k+1, M-k]$ are obtained the most effective results when using a symmetric interval. In the same place it is proved that if degree of a reference polynom $R > n+1$, exists an interval of optimum width N^{opt} which gives the minimum error of smoothing.

Thus, the problem definition is reduced to that when smoothing to find the sliding interval value N^{opt} for a priori unknown reference function and to define the rule, following to which it is possible to specify value σ.

3. Decision technique

Applying to a problem of smoothing of a formula, removed for sizes $\eta^{RF}_{n,N}, \quad \eta^{MS}_{n,N}$ by which mean square deviations from zero differences $f^{RF}(s) - P^{MMS}_{n,N,i}(s)$ and $f^{MS}(s) - P^{MMS}_{n,N,i}(s)$ are respectively designated and, and considering that methodical, especially a casual component of errors of approximation practically don't depend on number i. [2,3], we will have:

$$\left(\eta^{RF}_{n,N}\right)^2 = A_{n,N} + \sigma^2 B_{n,N}, \quad \left(\eta^{MS}_{n,N}\right)^2 = A_{n,N} + \sigma^2 (1 - B_{n,N}) \text{ where}$$

$$A_{n,N} = \left[\sum_{r=n+1}^{N-1} c_r Q_r(0)\right]^2, \quad B_{n,N} = \sum_{r=0}^{n} P_r^2(0), \quad P_r(s) = \frac{Q_r(s)}{\|Q_r\|}$$

$$B_{n,N} = \begin{cases} \frac{1}{N}, & n = 0,1; \\ \frac{1}{N} + \frac{5(N^2 - 1)}{4N(N^2 - 4)}, & n = 2,3 \end{cases}$$

From a condition that in a point N^{opt} equality takes place $(\sigma^{RF}_{n,N^{opt}})^2 = \min_N (\sigma^{RF}_{n,N})^2$, it is possible to write down $\frac{dA_{n,N}}{dN} = -\sigma^2 \frac{dB_{n,N}}{dN}$, as, therefore, in the vicinity of a point of N = N^{opt}

$$A_{n,N} = C_n - \sigma^2 B_{n,N} \qquad (1)$$

where C_n – the unknown parameter which isn't depending on N. On the other hand, we can always receive functional dependence of selective dispersion on size N

$$F(N) = \frac{1}{M - 2k} \sum_{i=k}^{M-k} \left[f^{MS}(t_i) - P^{MMS}_{n,N,i}(0) \right]^2 \qquad (2)$$

which is in the neighborhood of N = N^{opt} is an estimate of the value of

$$(\sigma^{MS}_{n,N})^2 = C_n + \sigma^2 (1 - 2B_{n,N}) \approx F(N) \qquad (3)$$

Therefore, the required rule for search of value σ^2 can be formulated as follows: it is necessary to find such point of $F(N)$ in range of definition N = N^{opt}, in which local neighborhood (points - N^{opt}- 2, N^{opt}, N^{opt}+ 2), dependence $F(N)$ in the best way comes nearer to a curve from parametrical family (3) with parameters C_n and σ^2. The received rule is universal as doesn't depend on a type of reference function. It allows to receive at the same time both value of an assessment of a mean square error σ, and value of an assessment of a population mean of a square of a methodical error $A_{n,N^{opt}}$ (1).

However, in practice the task of search of an extremum can be ambiguous. If errors of measurements have autocorrelated function or volume of measurements is small, selective dependence $F(N)$, as a rule, has some extrema and some inflection points.

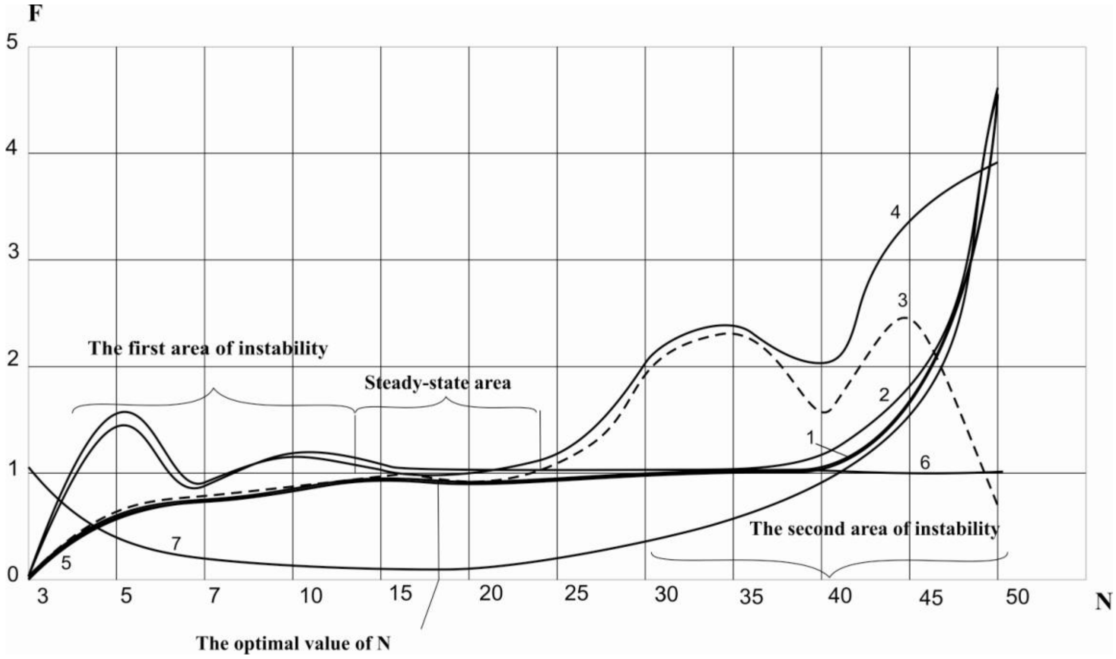

Diagram 1 - Dependency diagrams of measurement uncertainty on the width of the smoothing interval

Schedules of dependences of errors of measurements on smoothing interval width under following conditions are given in figure 1 for an interval between knots $\Delta t = 0{,}02$ and a mean square error of measurements - 1 meter: 1 - are present a methodical and casual component of an error of approximation; 2 - there is a correlation of mistakes; 3 - difficult reference function; 4 - small volume of selection, correlation of mistakes; 5 - the casual component of an error is absent; 6 - the methodical component of an error is absent; 7 - mean square error of definition of reference function.

Thus schedules 1,5,6,7 are received with use of exact dependences, and schedules 2,3,4 represent selective dependences.

In figure 1 it is visible that there are two areas of instability where the behavior of selective dependence can significantly differ from its exact analog. At small N sharply distinguished values of errors of measurements are capable to lead to considerable shifts of coefficients of the approximating polynoms and in case of the small volume of selection it gives to function $F(N)$ oscillatory character in the field of small values N. If there is an autocorrelated communication between errors of measurements, in the field of values N, commensurable with the period of autocorrelated function, we will also have oscillatory nature of function $F(N)$ irrespective of selection volume. The second area of instability is connected with that at the maximum values of width of an interval of smoothing the volume of data for an assessment of an average square of a deviation from zero on a standard formula (2) is sharply reduced. In this area the behavior of function $F(N)$ is unpredictable at great values of a methodical error. Therefore generally at the fixed n the task has no unambiguous decision. In this regard it is necessary to find all solutions for $n = 1$ and $n = 2$, and then to allocate those two decisions in which we will have identical (with a certain accuracy) values σ. These two decisions have to have on so-called "stable sites" on which exact dependences $\eta_1^{MS}(N)$, $\eta_2^{MS}(N)$ have inflection points. As a result we will receive values of the following sizes:

$$\sigma; \quad N_1^{opt}, \Delta_1^M = \sqrt{A_{1,N_1^{OPT}}}, \Delta_1^C = \sigma\sqrt{B_{1,N_1^{OPT}}}, \eta_1^{RF}; \quad N_2^{opt}, \Delta_2^M = \sqrt{A_{2,N_1^{OPT}}}, \Delta_2^C = \sigma\sqrt{B_{2,N_2^{OPT}}}, \eta_2^{RF}$$

These data give a complete idea of with what width of an interval of smoothing the smoothing problem is in the best way solved by a polynom of the first degree, a polynom of the second degree and what thus mean square deviation have errors of measurements and an error of processing. Subtracting a methodical error of processing from smoothed dependence, we will receive not displaced assessment of approximation of reference function and value of dispersion of a casual error of definition of reference function after processing, without having reference measurements.

Conclusion

Thus, the offered way allows to estimate dispersion of the casual making errors of measurements without attraction of reference measurements. The constant systematic, slowly changing systematic and slowly oscillating casual components of errors of measurements, can't be allocated this way essentially as they join the approximating polynoms on an interval of smoothing and it is necessary to apply other methods to their assessment.

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