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2-D Digital Waveguide and Finite Difference Modeling of A Sitar

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Abstract

The paper focuses on implementing the acoustic properties of the Sitar, with the help of physical modeling techniques, mainly of bridge and main strings. The effect of bridge geometry on string vibration is of special importance. The model is simulated using Finite Difference Model (FDM) and digital waveguide method using simulation tool LabVIEW. The instrument is modeled using the two-dimensional digital waveguide, a new approach to Sitar modeling and simulation.

Keywords: Sitar FDM, bridge, digital waveguide, spectrogram, physical model.

Introduction

Musical instruments have unique structures whose designs result in each having a unique sound. The past few decades have seen the increasing interest of many acousticians and physicists in using different techniques to model and simulate musical instruments. Modeling a musical instrument involves establishing a mathematical form of that instrument structure and implementing it using appropriate simulation tools for their computer realization. Musical instruments are of different types, namely percussion instruments, wind instruments, stringed instruments, keyboard instruments, etc. The sound emanating from a musical instrument is characterized by three fundamental factors- rhythm, timbre and pitch. These along with certain secondary factors vary the sound that we perceive. Stringed instruments are a category of instruments which have gained the interest of many researchers. Each stringed instrument has some peculiar properties which helps it to distinguish it from the rest. Generalizing all stringed instruments, for the purpose of modeling and simulation would be incorrect as it is their different structural styles which provide them their identity.

Extensive work has been done on the modeling of western stringed instruments such as the guitar. Little has gone into development of Asian stringed instruments such as the Veena and Sitar, even though having unique characteristics. Sitar is one of the popular traditional Indian plucked stringed instruments. It has a very distinctive timbre. The distinctive sound of Sitar differentiating it from other stringed instruments is due to the presence of a unique bridge structure and sympathetic strings [1].

This paper deals with the various aspects related to the modeling of the playable Sitar string. Different string instrument sounds mainly differ due to the number of strings present, bridge and resonator structure. The physical modeling of musical instruments using digital waveguide is an booming topic in acoustic science since last two decades. It is an effective way to model acoustic instrument models [2]. Ensuring desired sound production by the musical instruments so designed using simplified laws of physics, is done by these models. The algorithms and models can be manipulated according to the need of user. This provides great freedom as well as assistance to the designer, to develop an instrument that provides the desired quality of sound.

Sitar

Sitar is a classical stringed instrument having 6-7 main strings which are struck manually and the sympathetic strings, which vibrate as the effect of vibration of the main strings. The flow of sound wave vibration starts with striking of the main string. The vibrations are carried forward to the sympathetic string, then to the bridge, and finally to the resonator which amplifies the vibration so that the tone is audible for the listener. Hence only the vibration of the main string alone does not produce audible sound; bridge and resonator also play an important part in the final sound that emerges from the instrument. The shape of the two bridges that of the resonator affect amplification of sound and hence for the sound that emanates from the instrument when the main string is struck [1].

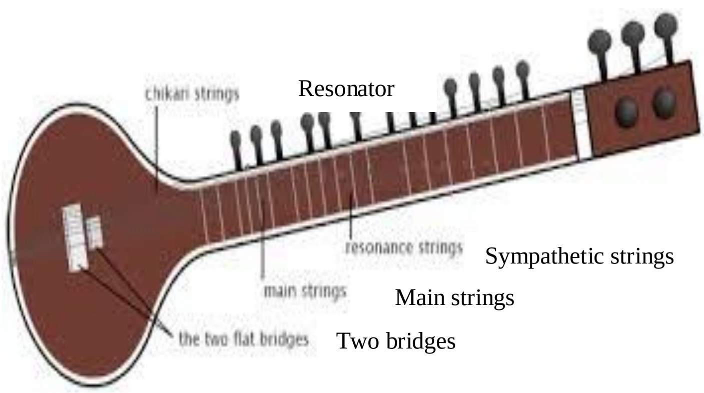

Figure 1: Sitar (www.talaash.at)

The bridge disturbs the free vibration of string resting on it. This varies the apparent length of the string, producing overtones and giving the Sitar its unique buzzing sound. Thus the vibrating string has two phases-initially it vibrates in nonlinear fashion due to bridge and than in periodic form before coming to a standstill [4].

Raman suggested in [1] that the bridge (also called Javari) of Sitar is the main factor for distinctive sound in the instrument. Indian classical stringed instruments have similar resonator shapes but different bridge structures. These are highlighted in the pictures of different stringed instruments below.

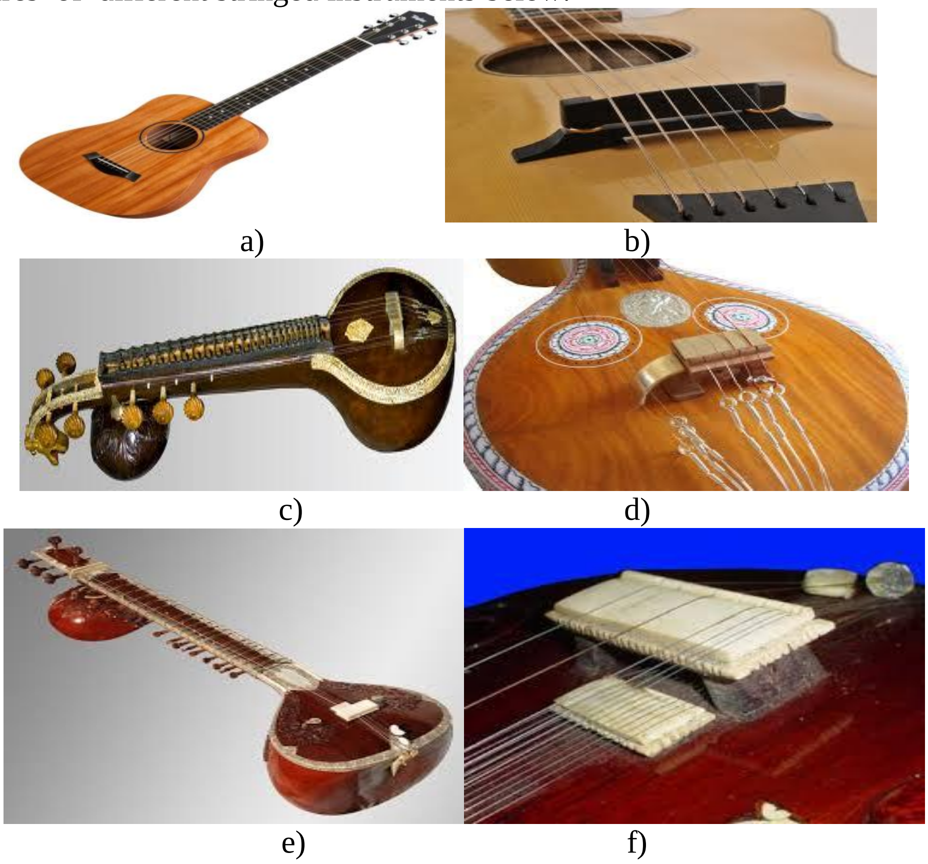

Figure 2: a) Guitar (www.datsons.co.uk) b) Guitar bridge (www.koentoppgguitars.com) c) Veena (www.orgs.usd.edu) d) Veena bridge (www.superbrass.com) e) Sitar (www.schneiderguitars.com) f) Sitar bridge (www.chandrakantha.com)

Spectrogram is a time-frequency representation of sound showing the variation of a signal with both frequency and time. A spectrogram is plot of magnitude of Short Term Fourier Transform (STFT) of a signal. Therefore it helps to analyze and observe variations of sound amplitude for both frequency and time components simultaneously, thus being one of the best analysis tools for music where there is large variation of frequency content at every instance of time. The spectrogram is two dimensional representation of a three dimensional plot with the two axes having time and frequency variation and the magnitude represented as color spectra or intensity range of a color on the graph. The horizontal axis represents time and the vertical axis represents frequency. The frequency components of naturally occurring sound signal observed in everyday life change over time and the sound is therefore a mixture of multiple frequencies. To analyze them under one window, either time or frequency is not feasible. To analyze such varying component signal would require a technique which can manage and take into consideration time dependent frequency variations in the signal. STFT is most extensively used technique for analyzing such signals. The basic idea of STFT is to break the signal into small time frames and Fourier analyze each time frame to obtain the frequencies that exist in that frame. Each frame provides a different spectrum depicting different harmonic content and the concatenation of all these time frames will provide a time-frequency distribution of the signal. Hence this spectrogram is used to demonstrate the comparison of the outputs generated by the model with the actual instrument output. The software used to plot the spectrogram is Overtone Analyzer (www.sygyt.com). The actual Sitar tone used for analysis is downloaded from http://freewavesamples.com/Sitar-c5. Figure 3 shows the spectrogram of actual Sitar tone.

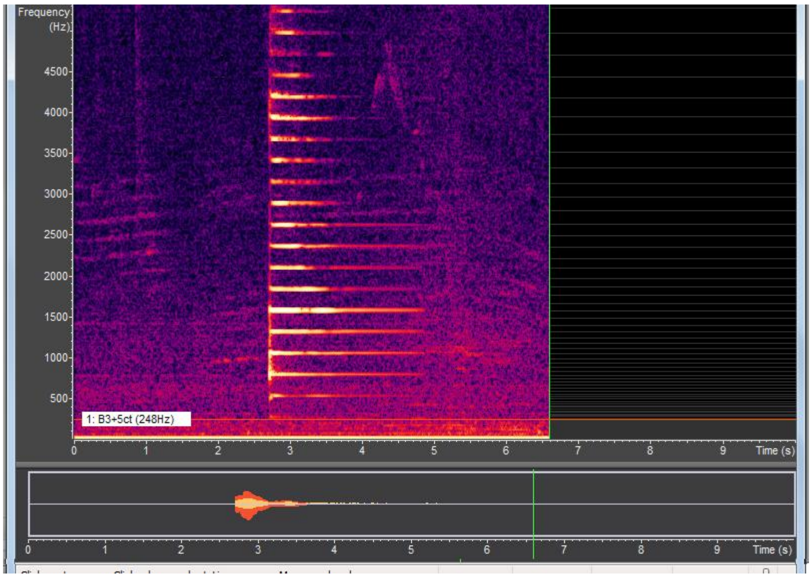

Figure 3: Actual Sitar tone spectrogram

The horizontal axis describes time in seconds and vertical axis plots the frequency in Hertz. We can observe the distribution of harmonics along the vertical axis explained in details in succeeding section. The change in intensity shows the distribution of energy (amplitude) in the individual harmonic.

Nature of Sound

Sound is the thing that we perceive when something vibrates. Some sounds are pleasant and some are of disturbing. Music is one type of sound which gives a pleasant feeling but the sound of a ringing bell causes some distraction.

Music is defined basically by three main factors- rhythm, timbre and pitch. Rhythm relates to the timing, timbre relates to the nuance and pitch relates to the height in a musical tone. There are few secondary factors too, whose occurrence is based on the different structures of instrument. Even though these factors define a musical tone there exist one common base for them i.e. harmonic series, also called overtones. Harmonic series is the integer multiple of the fundamental frequency (i.e. the basic frequency).

Harmonic series is responsible for origination of musical tones. These harmonics are related to the standing wave patterns. A combination of all these multiple tones in a peculiar fashion results in the generation of music.

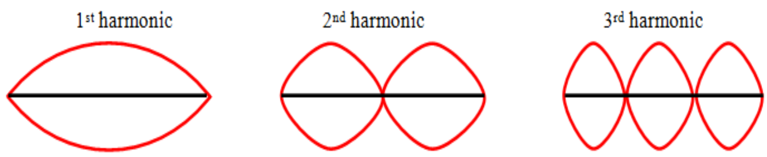

Figure 4: Harmonic series (up to 3rd harmonic)

String

The distinguishing tone and pitch in the stringed instrument occurs due to a change some specific factors, one of them being the length of the string. Its effect has been explained further. Other factors like string tension, the material and the linear mass density of the string also affect the tone but to a lesser degree. The tension of the string is altered by adjusting the peg present on the instrument and the length of the string is altered by pressing the string with one’s fingers at particular points known as frets. Varying the length of the string varies the fundamental frequency of vibration, as the length determines the fundamental frequency of vibration.

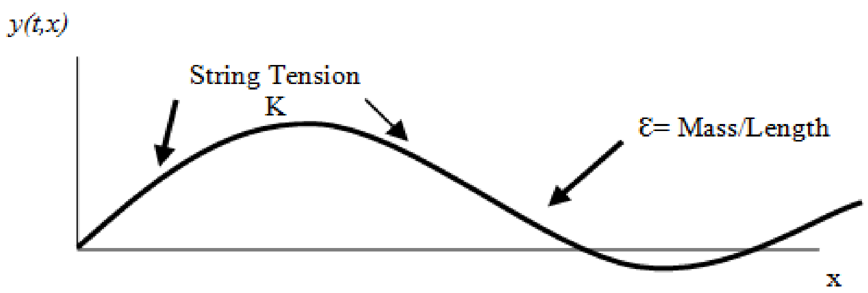

Figure 5: Ideal vibrating string

A string naturally vibrates with number of frequencies known as the harmonics of the string. The first harmonic of the string, called as the fundamental frequency (and also natural frequency) of the string depends basically on the length of string. The length of the string and the wavelength, (thus frequencies) of the harmonics are related by a mathematical relation as displayed in the Fig. 6 and Fig. 7.

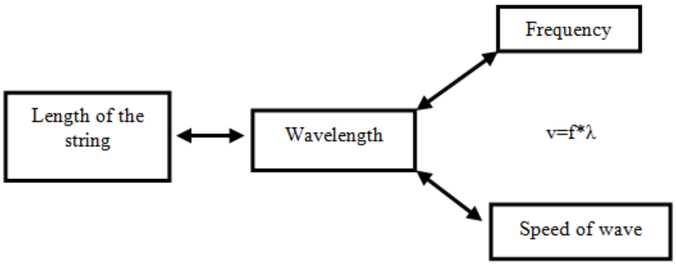

Figure 6: Relation between length, wavelength, frequency and speed of wave

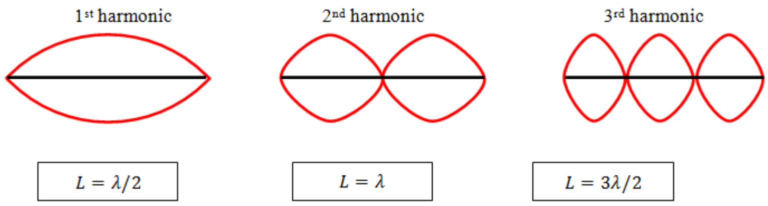

Figure 7: First three harmonics with their length and wavelength relation

Physical Modeling Using Digital Waveguide

Delay lines

Delay lines are the basic building block of digital waveguide. They are used to model acoustic propagation delays where its function is to introduce a time delay between its input and output as shown in Fig. 8.

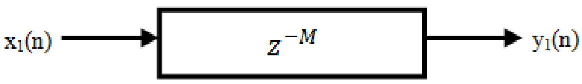

Figure 8: M-sample delay line

where,
$x_1(n)$: input signal,
M: samples of the delay line,
$y_1(n)$: output signal.

$$y_1(n) = x_1(n - M) \quad (1)$$

Thus the delay line of length M provides a delay to the input by the factor of M.

Digital Waveguide

The string vibration can be modeled using traveling waves on a string, in waveguide modeling. The string when plucked results in its vibration and these vibrations cause the generation of traveling waves. Hence the behavior of a vibrating string can be analyzed by describing the dynamic motion of the string using the ideal string. We consider a second order, lossless, one dimensional wave equation for the string as follows (Assumption of wave equation derivation: the slope of the string is much less than one at all positions [3]):

$$(\frac{\partial^2 y}{\partial t^2}) \equiv (c^2) * (\frac{\partial^2 y}{\partial x^2}) \quad (2)$$

Where:
y: vertical transverse displacement of the string,
x: any point along the length of the string,
c: speed of wave.

The above wave equation is solved by d'Alembert's method, and the solution so obtained is:

$$y(x,t) = y_r(x,t) + y_l(x,t) \quad (3)$$

The relation of length of the string and frequency of vibration is explained in the Section IV. In terms of modeling, there exists one more mathematical form which deals with length of string and fundamental frequency in terms of sampling frequency. The length of waveguide L is determined by using the formula-

$$L = \frac{f_s}{2 * f_o} \quad (4)$$

Where f_s is the sampling frequency and f_0 is the fundamental frequency.

The equation (3) gives the string displacement y at instant t when plucked at point x on the string. Thus the solution suggests that a traveling wave on a string can be considered to be the sum of a right traveling wave and a left traveling wave [3]. These traveling waves when sampled using equation 5 give the resultant equation as-

$$y(x,t) = y_r(x - ct) + y_l(x + ct) \quad (5)$$

The string after plucking at a particular point separates into two components, a right traveling wave and a left traveling wave. The waves shift by one sample step in their respective directions. Thus the resultant shape of the string at a particular instance is the addition of the two components on the string at that particular instant. The amplitude of the components of the traveling wave at any instant is exactly half that of the amplitude at the previous instant at the pluck point [3].

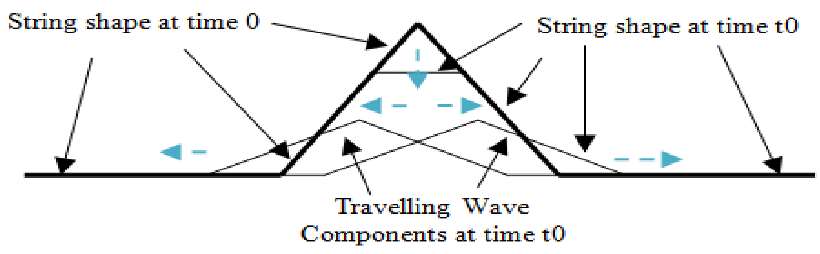

Figure 9: String behavior

The string can be defined in acoustic terms as a one-dimensional linear acoustic system. A single delay line is used to model an acoustic plane wave and thus a bidirectional delay line can be used to model any one-dimensional linear acoustic system. Thus a string can be modeled using bidirectional delay line. A bidirectional delay line along with filters is called digital waveguide. The computation of the displacement of a physical string (y) at any sample point along the length of the string (x) is given by adding the upper and lower delay line at that sample point along the bidirectional delay line.

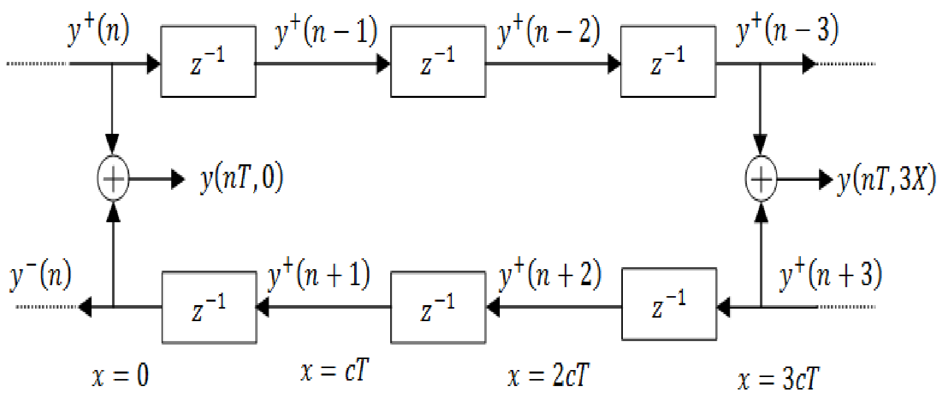

Figure 10: Digital representation of waveguide having ideal, lossless characteristics with three sample bidirectional delay line (z^{-1} represents one sample delay)

Lumping of all the delay elements along one rail results in the simplified version displayed in Fig. 11 representing digital waveguide of a 2M delay line and two terminal ends.

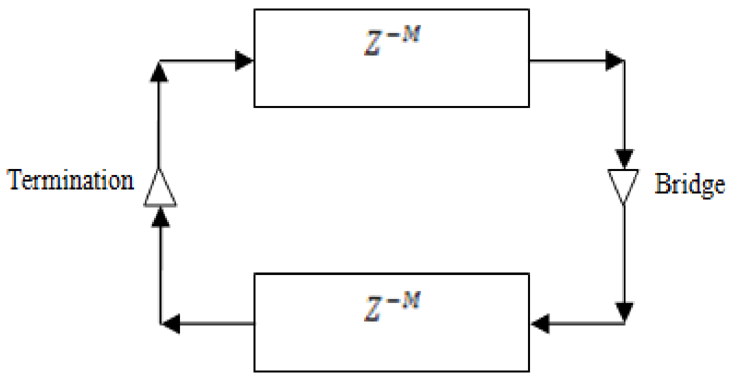

Figure 11: Digital representation of digital waveguide with 2M-sample delay line

The string has fixed ends called as terminations. The vibration of the string is obstructed by these terminations and hence the displacement of the string is nil at the terminations. The modeling of string at the rigid terminations is done by appending the sample shifted out from a particular traveling wave model to the opposite traveling wave model with an inverted sign. The traveling wave nature at different instances along the string length and also at the terminals is explained in the Fig. 12.

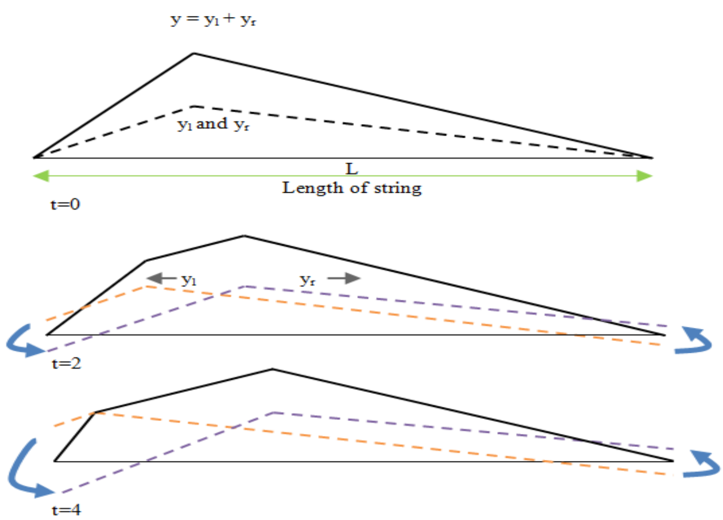

Figure 12: String shape determined by two traveling waves

In the Sitar one termination is peg which is used to alter the tension in the string and other termination is the bridge which adds the non-linearity into the string [4]. The bridge is responsible for the distinctive sound of Sitar [1]. The bridge structure is one of the main factors to affect the Sitar output tone. It can be flat or curved; its length can be altered [5]. The bridge restricts the motion of string at the terminal end and thus the normal string vibration is obstructed at the bridge. This behavior can be seen in [5].

Physical Modeling Using Finite Difference Model

The wave equation specified in (2) is approximated using finite differences using finite difference modeling. The approximation of first derivation of any general function f(x) is given as-

$$f'(x) = \frac{f(x+\frac{\Delta x}{2}) - f(x-\frac{\Delta x}{2})}{\Delta x} \quad (6)$$

Similarly second derivation approximation is done as-

$$f''(x) = \frac{f(x+\Delta x) - 2f(x) + f(x-\Delta x)}{\Delta x^2} \quad (7)$$

Substituting the approximations into wave equation gives the following solution-

$$\frac{y(x,t+\Delta t) - 2y(x,t) + y(x,t-\Delta t)}{\Delta t^2} = c^2 \frac{y(x+\Delta x,t) - 2y(x,t) + y(x-\Delta x,t)}{\Delta x^2} \quad (8)$$

where y(x, t): vertical displacement of the string,
x: position along the string,
t: time instance.

By obeying the speed-distance relation i.e. distance=speed*time, we can say that $\Delta x = c*\Delta t$. By substituting the value of Δx in (8), we get the following simplified equation-

$$y(x,t+\Delta t) = y(x+\Delta x,t) + y(x-\Delta x,t) - y(x,t-\Delta t) \quad (9)$$

The above equation gives the vertical displacement y at sample point x along length of string and at sampled time interval $t+\Delta t$ using vertical displacement at different sample points and time intervals. This relation is schematically presented in Fig. 13.

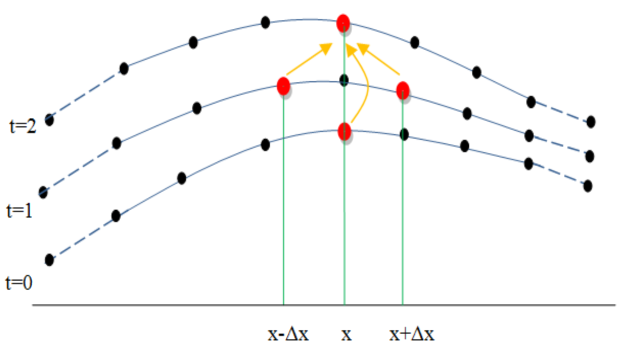

Figure 13: Computation of one point in FDM

Bridge

As discussed in Section II, every stringed instrument has a peculiar structure which differentiates it from other instruments. Sitar instrument has sympathetic strings and bridge as differentiating factor in the design. Bridge of Sitar is curved in nature. But the paper has considered the effects of flat bridge along with the curved bridge to properly demonstrate exact effects of bridge shape in output sound produced. The equation for flat bridge is given as-

$$\text{if} \quad f(x) = 0 \qquad x < x_c \tag{10}$$

f(x) represents profile of flat bridge and x_c defines length of bridge. In this form the string touches the entire bridge length when it is at rest position.

Similarly equation for curved bridge for Sitar instrument is defined as follows-

$$\text{if} \quad f(x) = kx \qquad x < x_c \tag{11}$$

where k is the slope of the bridge surface of Sitar instrument.

$$k = \tan\theta \tag{12}$$

θ is assumed to be 0.008 radian and hence tan θ becomes 0.008 [5]. The string touches the bridge tangentially in this form of bridge. The curved nature of Sitar bridge can be seen in Fig. 14.

Figure 14: Curved bridge of Sitar instrument [4]

Two Dimensional Digital Waveguide

The digital waveguide discussed in previous sections describes one dimensional state of the string vibration. It considers the vibration of string only in vertical plane. But actually the string also vibrates in horizontal plane. The string vibration for horizontal plane can be modeled using the same one dimensional digital waveguide model. No string in the instruments is rigidly terminated. If that was the case the vibration would have not been transmitted to other parts of instrument, which would mean that there is no effect of other parts of the instrument on the sound generated when the string is struck or pulled. The coupling of planes, the horizontal and the vertical plane occurs mostly at the bridge in the world of acoustic stringed instruments [13]. Figure 15 describes how transfer of energy between two planes at the bridge can be modeled. This coupling causes the string to rotate around its rest position. The general coupling relation equation is given as-

$$\begin{bmatrix} F_v^-(z) \\ F_h^-(z) \end{bmatrix} = \begin{bmatrix} H_{vv}(z) & H_{vh}(z) \\ H_{hv}(z) & H_{hh}(z) \end{bmatrix} * \begin{bmatrix} F_v^+(z) \\ F_h^+(z) \end{bmatrix} \quad (13)$$

$F_v^-(z)$ and $F_h^-(z)$ are the modified displacement waves which are reflected back or the outgoing waves from the bridge for vertical and horizontal plane respectively computed using $F_v^+(z)$ and $F_h^+(z)$ which are the incoming waves. $H_{vv}(z)$ and $H_{hh}(z)$ are self-coupling and $H_{vh}(z)$ and $H_{hv}(z)$ are cross-coupling transfer functions.

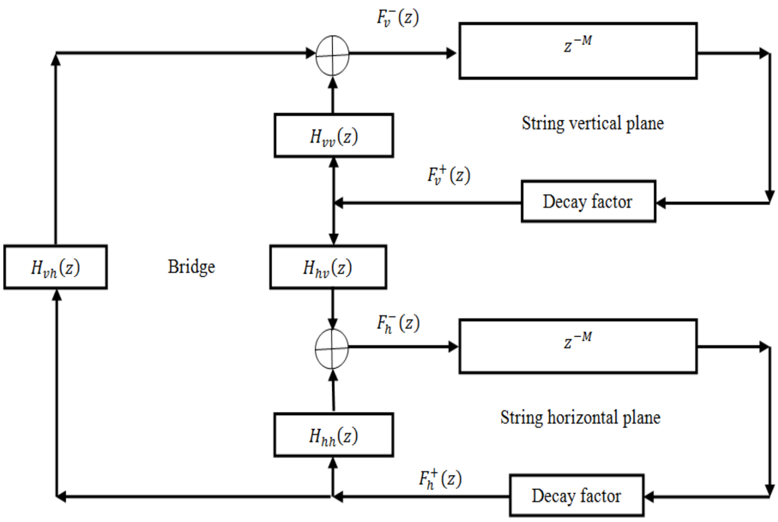

Figure 15: Coupling of the two vibration planes at the bridge

By using the rotation relation, the above equation can be simplified as-

$$\begin{bmatrix} F_v^-(z) \\ F_h^-(z) \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} * \begin{bmatrix} F_v^+(z) \\ F_h^+(z) \end{bmatrix} \quad (14)$$

In the above equation, θ gives angle of rotation of the string which is small in value and frequency independent [13]. This equation is simplified to be used for implementations with low computational cost.

Simulation

Simulation of all models is done in LabVIEW simulation tool. All simulated models have sound outputs. Spectrogram is used to demonstrate the comparison of the simulated module output sounds with the recording of actual Sitar tone.

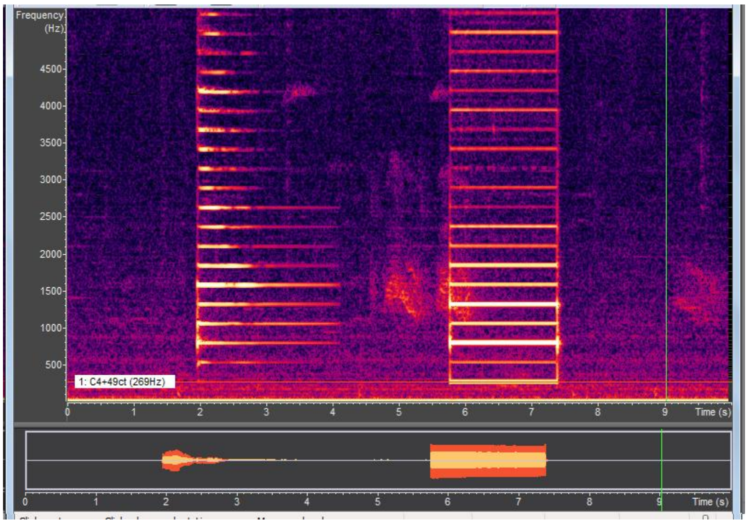

Figure 16: Actual Sitar tone plot along with FDM implementation of ideal string

Ideal string does not have any kind of losses and thus the tone is continuously sustained and does not decay out. But the tone should have decay otherwise it would sound like a boring noise tone. Thus the FDM model is integrated with loss using decay factor of 0.991. We have arrived at this factor by different trial and error.

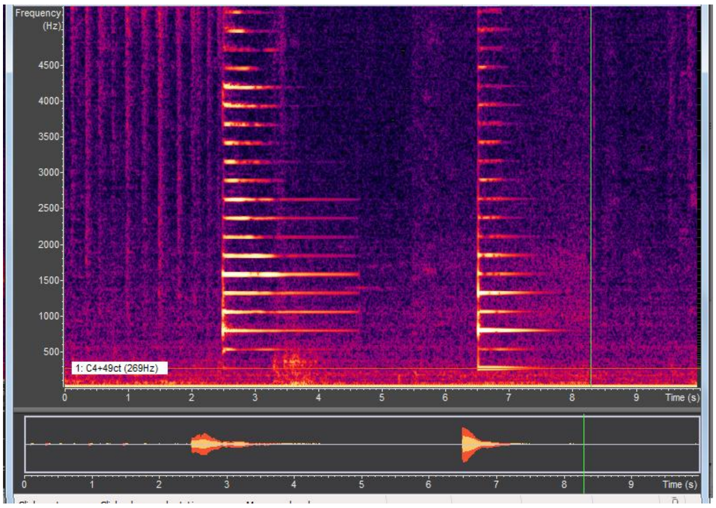

Figure 17: Actual Sitar tone plot along with FDM implementation of string with decay factor of 0.991 with pluck point at center (1/2 length of the string)

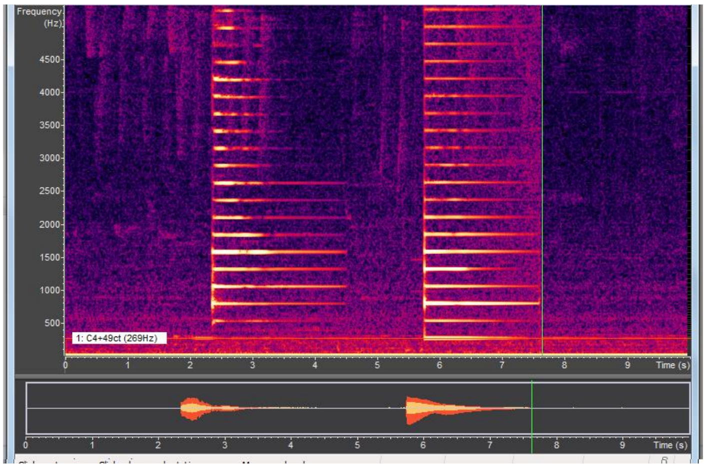

Figure 18: Actual Sitar tone plot along with FDM implementation of string with flat bridge structure and decay factor of 0.991 with pluck point at center (1/2 length of the string)

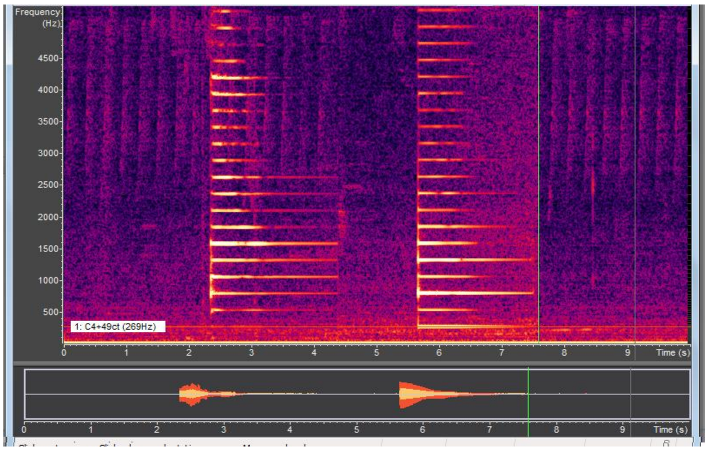

Figure 19: Actual Sitar tone plot along with FDM implementation of string with curved bridge structure and decay factor of 0.991 with pluck point at center (1/2 length of the string)

In Fig. 18 we can observe that by just inclusion of bridge has sustained the early decay of the simulated tone. Figure 19 shows effect of curved bridge, we can observe that the nature of spectrogram of simulated tone approaches the actual one.

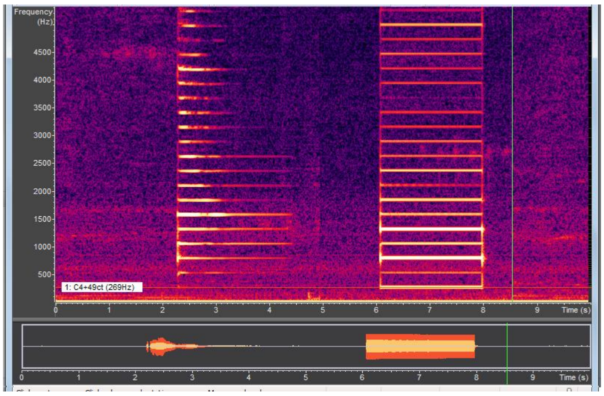

Figure 20: Actual Sitar tone plot along with one dimensional digital waveguide implementation of ideal string with pluck point at center (1/2 length of the string)

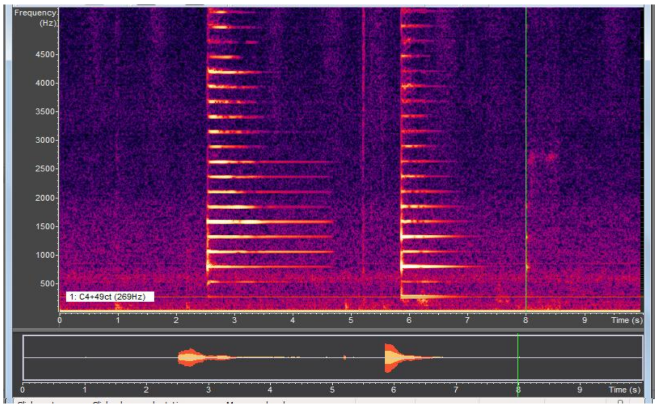

Figure 21: Actual Sitar tone plot along with one dimensional digital waveguide implementation of string with decay factor of 0.991 with pluck point at center (1/2 length of the string)

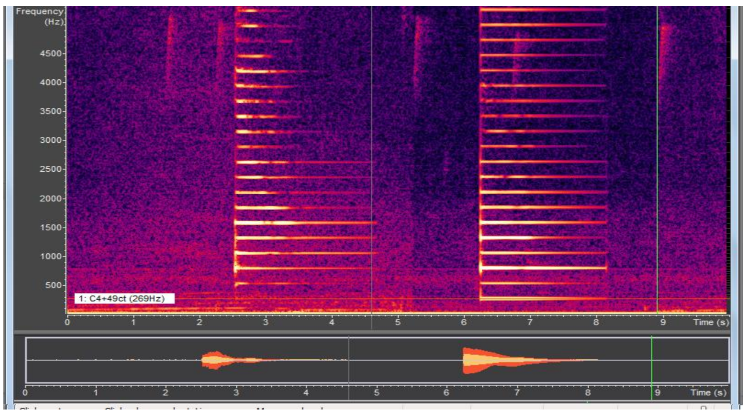

Figure 22: Actual Sitar tone plot along with one dimensional digital waveguide implementation of string with flat bridge and decay factor of 0.991 with pluck point at center (1/2 length of the string)

Fig. 20, 21 and 22 demonstrate ideal string, string with decay and string with decay and flat bridge condition respectively modeled using with digital waveguide technique. We can observe that spectrograms reflect nearly the same nature. But computational cost of FDM is more compared to digital waveguide.

Plucking point affects the amplitude content of harmonics. Changing the pluck point of the string varies the energy content of higher overtones. This effect is visible in Fig. 23 and Fig. 24.

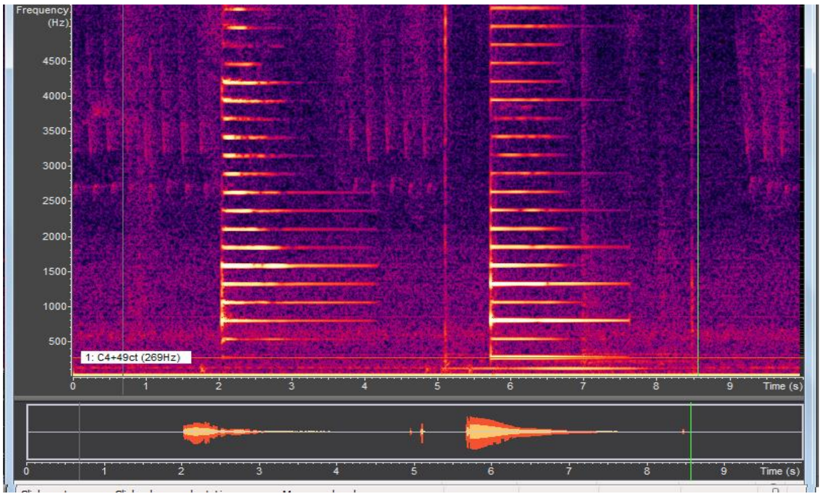

Figure 23: Actual Sitar tone plot along with one dimensional digital waveguide implementation of string with curved bridge and decay factor of 0.991 with pluck point at center (1/2 length of the string)

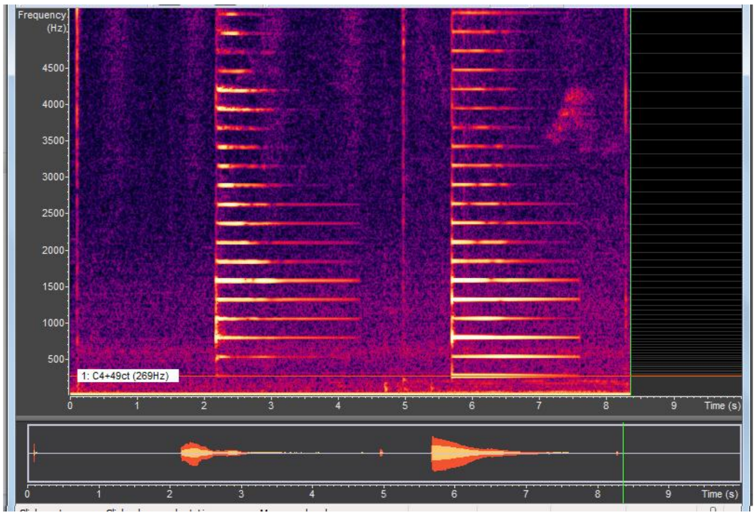

Figure 24: Actual Sitar tone plot along with one dimensional digital waveguide implementation of string with curved bridge and decay factor of 0.991 with pluck point at $1/4^{th}$ length of the string

The preceding digital waveguide implementation considered vibration of string only in vertical plane, plane perpendicular to upper surface of the bridge; hence called as one-dimensional digital waveguide. The succeeding spectrograms are related to simulation of two-dimensional digital waveguide.

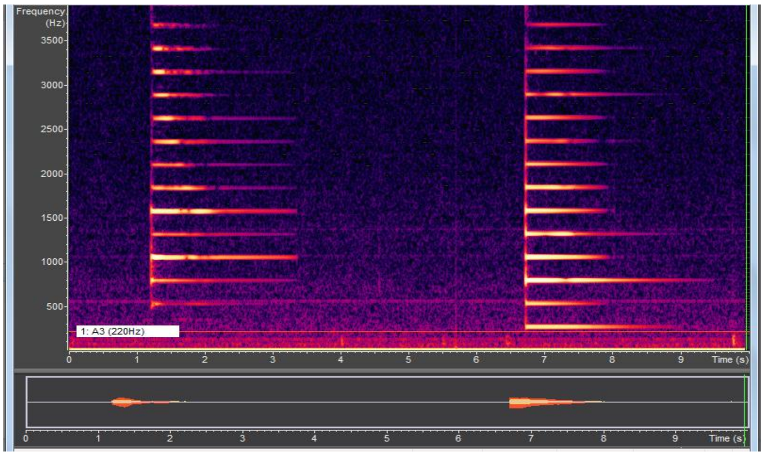

Figure 25: Actual Sitar tone plot along with two dimensional digital waveguide implementation of string with curved bridge and decay factor of 0.991 with pluck point at center (1/2 length of the string)

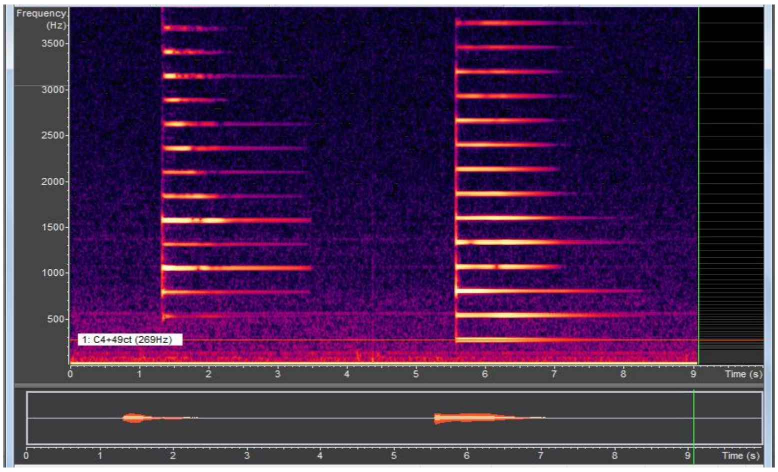

Figure 26: Actual Sitar tone plot along with two dimensional digital waveguide implementation of string with curved bridge and decay factor of 0.991 with pluck point at 1/4th length of the string

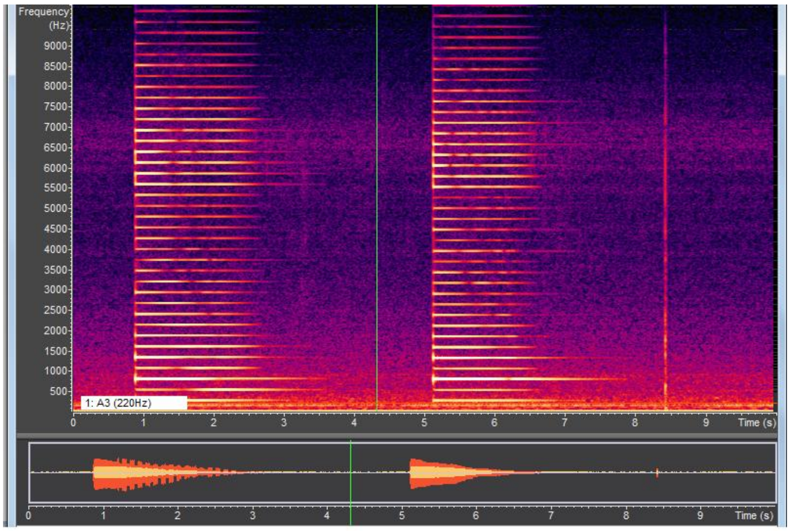

Figure 27: Two dimensional digital waveguide implementation of string with curved bridge and decay factor of 0.991 with pluck point at 1/4th length of the string and two dimensional digital waveguide implementation of string with curved bridge and decay factor of 0.991 with pluck point at center (1/2 length of the string).

Comparing Fig. 23 with Fig. 25 and Fig. 24 with Fig. 26, we can conclude that two dimensional digital waveguide provide comparative higher content of energy (or amplitude) in every harmonic compared to one-dimensional digital waveguide.

Conclusion

Different spectrograms have been plotted using FDM along with one and two-dimensional digital waveguide model. The model's output tone for curved bridge along with decay with pluck point at 1/4th length of the string shows maximum similarity with the actual Sitar tone which reflects real Sitar instrument structure. Computational cost of FDM is high compared to digital waveguide as FDM approximation is direct solution of difference equation and digital waveguide solution involves implementation of simplified wave equation. Two-dimensional digital waveguide displays more energy content in harmonics compared to one-dimensional digital waveguide. The modeled instrument can be designed closer to the Sitar instrument if effects due to sympathetic strings and resonator are also considered.

References

[1] Raman C., 1921, "On some Indian stringed Instruments," Proceedings of the Indian Association for the Cultivation of Science, **7**, pp. 29–33.

[2] Julius O. Smith III, 13 September 2010, "Virtual Acoustic musical Instruments: review and update," Center for Computer Research in Music

and Acoustics (CCRMA)-Department of Music, Stanford University, Stanford, California, USA.
[3] Julius O. Smith III, Winter, 1992, “Physical Modeling using Digital Waveguides,” Computer Music Journal special issue on Physical Modeling of Musical Instruments, Part I, Volume 16, no. 4, pp. 74-91.
[4] Sadjad Siddiq, 2012, “A Physical Model of the Non-linear Sitar String,” Archives Of Acoustics, 37(1), pp. 73–79.
[5] Dmitri Kartofelev, Anatoli Stulov, Heidi-Maria Lehtonen, Vesa Valimaki, 2013, “Modeling A Vibrating String Terminated against a Bridge with Arbitrary Geometry,” Stockholm’s Music Acoustic Conference.
[6] David Ronan, 2010, “Physical Modeling of Sitar,” Thesis report of Master of Science in Music Technology, University of Limerick, pp. 1-23.
[7] Karjalainen M.,Vilimiiki V. and Janosy Z., “Towards high quality Sound Synthesis of the Guitar and String Instruments,” Proceedings of the 1993, International Computer Music Conference, SanFrancisco, California:Computer Music Association,1993, pp. 56-63.
[8] Prof. D. M. Causon, Prof. C. G. Mingham, 2010, “Introductory Finite Difference Methods for PDEs.”
[9] N. Fletcher, T. Rossing, 1998, “The Physics Of Musical Instruments,” Springer, New York, 2nd edition.
[10] K. M.Vilimiiki and Z. Janosy, 1993, “Towards high quality Sound Synthesis of the Guitar and String Instruments,” Proceedings of the 1993, International Computer Music Conference, SanFrancisco, California:Computer Music Association, pp. 56-63.
[11] W. C. Elmore, M. A. Heald, 1969, “Physics of Waves.”
[12] Julius O. Smith, 2010, “Physical Audio Signal Processing,” Stanford University.
[13] Gianpaolo Evangelista, Martin Raspaud, 1-4th September, 2009, “Simplified Guitar Bridge model for the Displacement wave Representation in Digital Waveguides”, Proceedings of the 12th International Conference on Digital Audio Effects, Como, Italy.