

Efficacy of A Machine Learning Approach To Pricing Options In The Indian Context

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Abstract

Pricing an option is a mathematically and conceptually challenging problem. The most significant model to do this was proposed by Black and Scholes. However, the proponents have themselves accepted that Black-Scholes model makes assumptions about the input parameters of the model which do not hold ground in the real world. A lot of non-parametric approaches have been proposed in the recent past to price an option. This paper studies the predictive machine learning approach to option pricing. It compares the predictive machine learning approach with the Black Scholes model and calculates the error in estimation of both approaches. The study finds that when the same inputs as in Black Scholes are used to train the machine learning algorithm, the Black-Scholes model continues to perform better in pricing an option.

Keywords: Machine Learning, Black Scholes, Option Pricing, Non-parametric option pricing

Introduction

Pricing an option on a stock (or index of stocks, bonds etc. in general) is a mathematically and conceptually challenging problem. The most significant model to do this was proposed by Black and Scholes. This model became the most popular model in valuing options. However, the proponents have themselves accepted that Black-Scholes model makes assumptions about the input parameters of the model which do not hold ground in the real world.

Recently, with the improvement in algorithms and processing power, several machine learning techniques have come up predictive capabilities.

About Black-Scholes Model

The Black-Scholes formula offered a ground breaking model to value options. This model was a first of its kind in the sense that its inputs were all measurable parameters. The model is based on the assumption that the underlying stock follows a geometric Brownian motion;

$$dS = \mu dt + \sigma S dW$$

Here, W is Brownian and dW is the uncertainty in the stock price.

The second order Black-Scholes partial differential equation can be obtained using Ito's Lemma and the no arbitrage condition;

$$\partial V / \partial t + 1/2 (\sigma^2 S^2 \partial^2 V / \partial S^2) + rS(\partial V / \partial S) - rV = 0$$

The Black-Scholes formula is obtained by solving the above partial differential equation. Accordingly, the call option value is:

$$C(S, t) = SN(d_1) - Xe^{-r(T-t)}N(d_2)$$

$$\text{where } d_1 = [\ln(S/K) + (r + \sigma^2/2)(T-t)] / \sigma \sqrt{(T-t)}$$

$$\text{and } d_2 = d_1 - \sigma \sqrt{(T-t)}$$

Here, $C(S,t)$ is the call premium and S is the spot price of the underlying stock. Also X is the strike price and r is the risk free interest rate. $(T-t)$ denotes time to expiration whereas σ^2 is the annual variance of return on underlying stock.

The BS formula is based on assumptions which are not realistic. For instance, a key assumption is that markets are efficient. It is assumed that stocks move randomly, or in a random walk. At any given point of time, there is equal probability of the price going up or down and the price in time t is independent of the price at $t-1$. Further, another key assumption of the Black Scholes model is that interest rates are known and remain constant.

In order to circumvent the problem with these unrealistic assumptions and to achieve better congruence with observed prices, non-parametric models are being explored. Recently, with the improvement in algorithms and processing power, several machine learning techniques have come up predictive capabilities.

Samur and Temur (2009) consider the ability of ANN (Artificial Neural Network) in option valuation using the S&P 100 index. Whereas, prior studies primarily consider European options, this study covered American options as well. They find that Artificial Neural Network performs better for the valuation of call options. Further, they did not find any significant improvement in performance by using volatility as an input.

Amilon (20003) studied the success of the neural network in valuing call options and compared it with the traditional BS model. The study discovered that neural networks performed better than the Black-Scholes model though at times the results were inconclusive at 5% level of significance.

Daglish (2003) studied the degree to which various option valuation methods can explain the price of an index option. Daglish found that the parametric methods demonstrate a greater capability to describe future prices and show better hedging performance as well.

Charalambous, Panayiotis and Martzoukos (2008) examined the Black and Scholes model and ANN (Artificial Neural Networks) for valuation of European call options. They find that hybrid ANN model built on Black-Scholes model performs better than vanilla ANN models. They incorporate this into trading strategies and identify profitable opportunities.

Carelli, Silani and Stella (2000) studied the ability to use Neural Networks to model implied volatility. The paper elaborate how in case of lack of significant volume of data, that is critical to train the model, the modeling of the neural network model becomes numerically challenging.

Lajbcygier and Connor (1997) deployed a hybrid neural network to forecast the difference in the market price for an option and the price suggested by the pricing model. They find that though models based on a hybrid neural network option may better the forecast, it incorporates bias which may be reduced with bootstrapping. The altered method, based on bootstrapping, performed better than the hybrid model.

Yao, Yili and Chew (2000) employed back propagation neural networks to predict price of options. They find that in markets with high volatility, neural networks perform better than the BS formula which though continues to perform well in certain scenarios. The authors suggest that choosing a valuation model depends on the risk-return outlook of the investor.

Bennell and Sutcliffe (2004) study the Artificial Neural Network and draw a comparison with the traditional BS model for the valuation of an European call option. They find that the Artificial Neural Network performs better for options out of money. Further, when options in the money are considered, for a given set of conditions, the artificial neural networks based model is comparable to the BS formula.

In this paper, we aim to compare the Black-Scholes model with a predictive machine learning algorithm.

Data and Methodology

The data used in this study was for NIFTY Call Options. The date range was Sep 02, 2014 to 30 Apr, 2015 for the Call option expiring on Dec 31 2015 with a strike price of INR 10000. This yielded 160 end of day closing prices. These end of day closing prices were compared with the Black Scholes price estimate and the machine learning price estimate.

Data For The Black Scholespricing Model:

In order to find the risk free rate to be used in the Black Scholes model, the 3-month MIBOR (Mumbai Interbank Offer Rate) was used. It is the Indian counterpart of LIBOR and is a good substitute for the risk free rate.

The time to expiry was calculated assuming 260 trading days per year. The approach was number of working days between current date and expiry date divided by 260.

In order to calculate volatility, the standard deviation of the last 20 trading days was used. Specifically, the daily log returns were first calculated. The standard

deviation of the daily log returns was then taken for the last 200 trading days. The standard deviation was then multiplied by square root of 260 to annualize the data. This was then converted to a percentage number for direct use on the Black Scholes model by multiplying 100. Since, the standard deviation was based on last 20 days data, we started the Black Scholes estimation from record number 21 which meant that there are 140 estimates for 160 records.

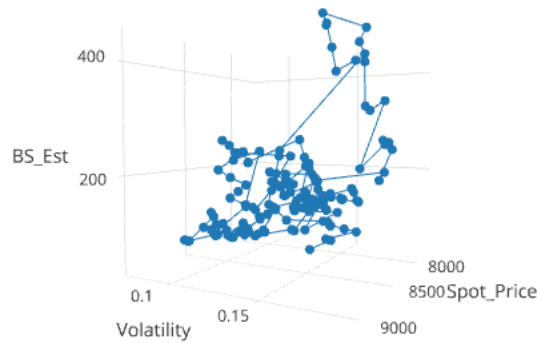


Figure 1: BS Estimate with Volatility and Spot Price

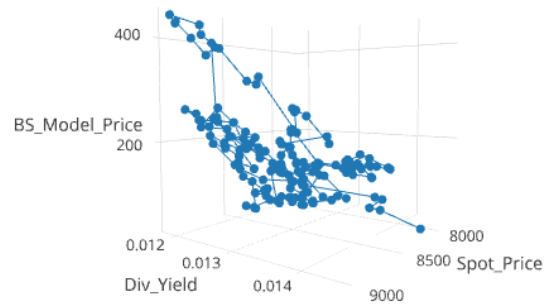


Figure 2: BS Estimate with Div Yield and Spot Price

After the Black Scholes estimated prices were obtained, the mean squared errors were computed. In order to do that, the square of the error (difference between the Black Scholes model price and the closing price in the market) was calculated. Also, the standard deviation of the errors was calculated.

For the dividend yield, the data was directly used as-is from the National Stock Exchange.

Data for the Machine Learning Algorithm:

The parametric data used in the Black-Scholes model above were retained and used as *predictors* of the machine learning algorithm. This included sport price, Risk free rate

(MIBOR), time to expiry (in number of years), volatility (based on standard deviation, annualized and in percentage) and Dividend Yield.

In order to use any machine learning methodology, the model first needs to be *trained*. What this means is that the model *consumes* data before it can start *predicting* answers. In our study, we used 130 records of data available to *train* the model. This was then used in the prediction of the rest 30 records.

When trained with the data, the predictive machine learning algorithm built a model with the following highlights;

- The most significant consideration in forecasting an option price was placed on Spot Price (>82%)
- The next most significant consideration in forecasting was on Dividend Yield (>11%)
- Time to maturity(<1.5%), Volatility (<1%), and MIBOR (<1%) played dormant role in the forecasting

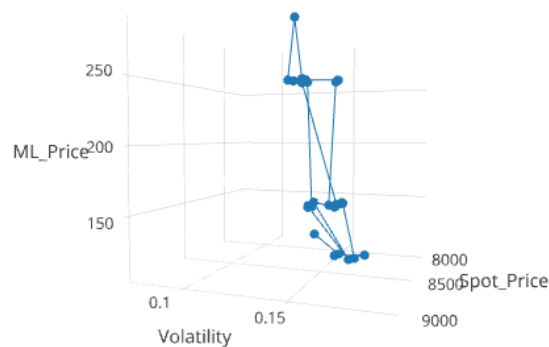


Figure 3: ML Estimate with Volatility and Spot Price

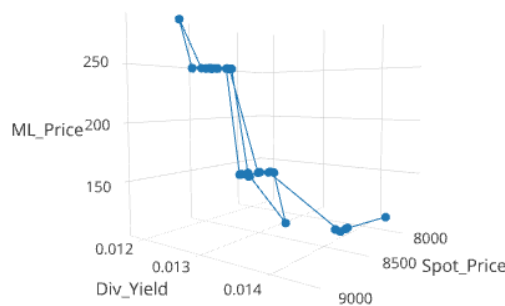


Figure 4: ML Estimate with Div Yield and Spot Price

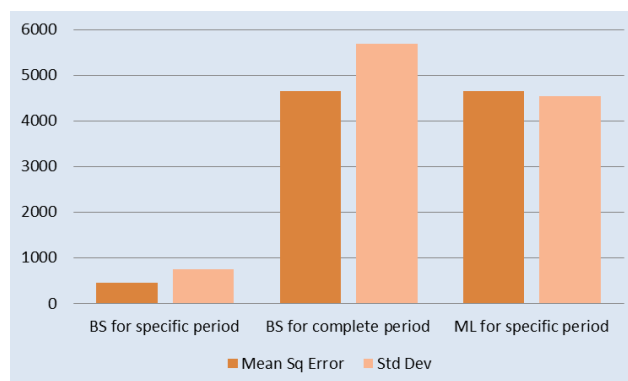
Similar to the Black Scholes approach above, after the machine learning estimated prices were obtained, the mean squared errors were computed. In order to do that, the square of the error (difference between the machine learning estimated prices and the closing price in the market) was calculated. Also, the standard deviation of the errors was calculated.

Results

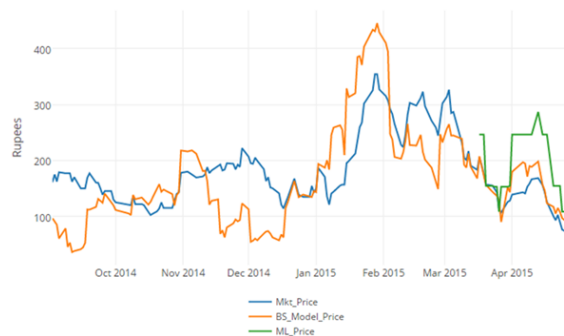
The following observations were made from the study.

Firstly, the Black-Scholes model clearly performs better if we consider the BS forecast only for the specific period during which we have a machine learning forecast. During the period for which we have a forecast, the mean square error and standard deviation of the error is significantly smaller for the Black Scholes model.

Secondly, the Black-Scholes model loses its edge if we consider the BS forecast for the entire period regardless of whether we have a machine learning forecast. In such a comparison, the mean square error and standard deviation of the error of the machine learning model is comparable to that of the Black-Scholes model.



The graph above summarizes the mean squared errors of the two models. A summary of the results is depicted in the below chart.



Conclusion

It is clear from the above study that the parametric Black-Scholes approach still has the potential to outperform a predictive machine learning approach. However, there is a need for additional research to conclude whether the Black-Scholes model because of its theoretical strength can outperform a machine learning approach when enough number of additional data points are fed to the machine learning algorithms. Some of these data points could be market sentiment data using social media such as twitter or foreign market moving data.

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