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Optimization of Fuzzy EOQ Inventory Cost Under Order Crossover

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Abstract

The purpose of this paper is to discuss the EOQ model with the impact of stochastic leadtime reduction on inventory cost under order crossover. For the fixed ordersize inventory models, the economic order quantity (EOQ) model is most wellknown. We use exponential lead times to demonstrate that reducing mean lead time has a secondary reduction of the variance due to order crossover. Here we present a two-stage procedure for reducing the mean and variance for exponentially distributed lead times. The Yager's ranking method (1981) for fuzzy numbers is utilized to find the optimal inventory policies. We assume that the lead time is made of one or several components and is the time between when the need of a replenishment order is determined to the time of receipt. A set of numerical data is employed to analyse the characteristics of proposal models. Finally, for completeness of model, this paper also fuzzies holding cost and shortage cost coefficients.

Keywords: Inventory, Fuzzy, EOQ, backorder, stochastic lead time, lead time reduction, cost effectivenss, inventory optimization, order crossover.

Introduction

We define lead time reduction as the process of decreasing lead time at an increased cost. To data, decreasing lead times has been confined to deterministic instances. We examine the case where lead times are exponential, for when lead times are stochastic, deliveries are subject to order crossover, so that we must consider effective lead times rather than the actual lead times. The result is that the variance of these lead times is less than the variance of the original replenishment lead times. (Jack C. Hayya, Terry P. Harrison, X. James He (2011)).

The order cross over transforms the original lead times into 'effective lead times' whose mean is the same as that of the parent lead time but whose variance is lower

(Hayya et al., 2008). We use the model in Silver et al (1998), where the shortage penalty is applied per unit short and where the demand rate is constant so far, research on lead time reduction has dealt only with deterministic lead times. Lan et al (1999) or Hariga and Ben-Daya (1999), where these authors portray cost as a piecewise linear function of lead time L.

Usually, inventory systems are characterized by several parameters such as cost coefficients, demands etc. Accordingly, most of the inventory problems under fuzzy environment can be addressed by fuzzying these parameters. For instance, Park [1987] discuss the EOQ model with fuzzy cost coefficients. Ishii and Kunno (1998), Petrovic et al. (1996) and Kao and Hsu (2002) investigate the newsboy inventory model with fuzzy cost coefficients and demands respectively. Roy and Maiti (1997), Chang (2009) construct a fuzzy EOQ model with fuzzy defective rate and fuzzy demand. Most of the paper directly supposes the model parameters as a triangular fuzzy number and then finds their optimal solutions. Yao and Chiang (2003), Lee and Yao (1998), Chang and Yao (1998) develops the EOQ model with fuzzy ordering quantities. This paper tries to find the optimal ordering quantities, Q, and time period stock out (Z_0) for the EOQ model with impact of stochastic lead time reduction and fuzzy demands. Finally for completeness of model, the holding cost and shortage cost coefficients will also be fuzzified.

The structure of this paper is organized as follows. In section 2, the preliminaries are given. In Section 3, the total inventory cost of the problem is constructed from the □-cut of the lead time. In Section 4, optimal ordering quantity is derived using Yager's ranking (1981) method. Finally the characteristics of proposed models will be illustrated and some conclusions will be made.

Preliminaries

Definition : Fuzzy Set

A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X, \mu_{\tilde{A}}(x) \in [0, 1]\}$. In the pair $\{(x, \mu_{\tilde{A}}(x))\}$, the first element x belong to the classical set A, the second element $\mu_{\tilde{A}}(x)$, belong to the interval [0, 1], called membership function or grade of membership. The membership function is also a degree of compatibility or a degree of truth of x in $\tilde{A}$.

α - Cut

The set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree α is called the α level set or α - cut. $A(\alpha) = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha\}$.

Generalized Fuzzy Number

Any fuzzy subset of the real line R, whose membership function satisfies the following conditions, is a generalized fuzzy number

1. $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0, 1].
2. $\mu_{\tilde{A}}(x) = 0, -\infty < x \leq a_1$,
3. $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$,
4. $\mu_{\tilde{A}}(x) = 1, a_2 \leq x \leq a_3$,
5. $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$,
6. $\mu_{\tilde{A}}(x) = 0, a_4 \leq x < \infty$,

where a_1, a_2, a_3 and a_4 are real numbers.

Triangular Fuzzy Number

The fuzzy set $\tilde{A} = (a_1, a_2, a_3)$ where $a_1 \leq a_2 \leq a_3$ and defined on R, is called the triangular fuzzy number, if the membership function of $\tilde{A}$ is given by (Q, r) Inventory Model with Fuzzy Lead Time

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \dfrac{a_1 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{Otherwise} \end{cases}$$

Yagers' Ranking Method

If the α cut of any fuzzy number $\tilde{A}$ is $[A_L(\alpha), A_g(\alpha)]$then its ranking index $I(\tilde{A})$ is $\frac{1}{2}\int_0^1 \left[A_L(\alpha) + A_g(\alpha)\right] d\alpha$.

Total Inventory Cost

Notations :

Q	→	the order quantity
Z_0	→	standard normal safety stock future
A	→	the vendoring cost per order
D	→	the constant demand rate
h	→	the holding cost per unit time
B_2	→	shortage cost per unit
$G(Z_0)$	→	normal unit loss function
a', b'	→	Hegrerrion estimates of σ_{ELT} on T
r(ELT)	→	Cauchy sequence
σ_{ELT}	→	Standard deviation of V(ELT)

R(L) $\rightarrow$ lead time reduction

If the lead time were exponentially distributed, the total cost in the first stage including shortage and lead time reduction cost would have been

$$C(Q, Z_0) = \frac{AD}{Q} + h\left[\frac{Q}{2} + Z_0\ a'D + b'Q\ \right] + \frac{DB_2}{Q}\ a'D + b'Q\ G\ Z_0\ + \frac{D}{Q}R(L) \quad . . . (3.1.1)$$

The objective is to find the optimal order quantity which minimize the total cost

The necessary conditions for minimum $\frac{\partial C\ Q, Z_0}{\partial Q} = 0$.

Therefore the optimal order quantity is

$$Q^* = \sqrt{\frac{2\left[AD + a^1B_2\ + D^2G\ Z_0\ + DR(L)\right]}{h\ 1 + 2Z_0b'}} \quad . . . (3.1.2)$$

The EOQ Model With The Stochastic Lead Time Reduction and Fuzzy Demands

Let $\tilde{D}$ be a normal fuzzy number with parameters $\tilde{D}$ = (l, m, n, u), then the membership function of $\tilde{D}$ can be defined by a left shape function L(x) and a right shape function R(x) as :

$$\mu_{\tilde{D}}(x) = \begin{cases} L(x), & l \le x \le m \\ 1, & m \le x \le n \\ R(x), & n \le x \le u \end{cases}$$

The above equation can also be described by the terms of α-level cut of $\tilde{\lambda}$ as:

$$D(\alpha) = \left[\min .\ \mu_{\tilde{D}}^{-1}(\alpha),\ \max .\ \mu_{\tilde{D}}^{-1}(\alpha)\right]$$

$$= \left[L^{-1}(\alpha),\ R^{-1}(\alpha)\right],\ 0 \le \alpha \le 1$$

Firstly, we discussed the EOQ model with fuzzy demand, according to the extension principle, the model can be described by terms of □ as

$$\tilde{T}\ \alpha\ = \ C\left[Q \mid D = L^{-1}(\alpha)\right],\ C\left[Q \mid D = R^{-1}(\alpha)\right]\ ,\ 0 \le \alpha \le 1$$

where $C(Q) = \frac{AD}{Q} + \frac{hQ}{2}$

Since the annual cost function $\tilde{T}\ \alpha D$ is a fuzzy number, we can compare $\tilde{T}\ \alpha D$ of different Q by using some ranking methods to find the optimal solution Q* with a minimal total cost, of which, not every method is applicable to rank $\tilde{T}\ \alpha D$ of all possible Q. The method proposed by Yager (1981), does not need to know the explicit form of the membership functions, and can thus be applied here.

The Yager's ranking index ranks the fuzzy numbers by an area measurement defined as

$$I(\tilde{T}) = \frac{I_L(\tilde{T})}{2} + \frac{I_R(\tilde{T})}{2}$$

where $I_L(\tilde{T})$ represents the area bounded by the left shape function of $\tilde{T}(\alpha D)$, the x axis, they axis and the horizontal line $\mu_{\tilde{T}} = 1$ and $I_R(\tilde{T})$ represents the area bounded by the right shape function of $\tilde{T}(\alpha D)$, the x axis, the y axis and the horizontal line $\mu_{\tilde{T}} = 1$.

The Yager's ranking index of $\tilde{T}(\alpha D)$ thus can be calculated as

$$I(\tilde{T}) = \frac{1}{2}\int_0^1 C\left[Q \mid D = L_D^{-1}(\alpha_D)\right] d\alpha_D + \int_0^1 C\left[Q \mid D = R_D^{-1}(\alpha_D)\right] d\alpha_D$$

$$\text{Let } K_1(\square_0) = \frac{1}{2}\int_0^1 \left[L_D^{-1}(\alpha_D) + R_D^{-1}(\alpha_D)\right] d\alpha_D$$

Taking the partial derivative of $\tilde{T}(\alpha D)$ with respect to Q and setting to zero, the necessary condition of optimal solution of $\tilde{T}(\alpha D)$ can be found as

$$Q^* = \sqrt{\frac{2A}{h} \,.\, K_1(\alpha_D)}$$

$$\tilde{T}^*(\alpha) = \left(C\left[Q^* \mid D = L_D^{-1}(\alpha)\right],\ C\left[Q^* \mid D = R_D^{-1}(\alpha)\right]\right),\ 0 \le \alpha \le 1$$

Now if, shortage cost is permitted, the model can be described by terms of α as

$$\tilde{T}_{B_2}(\alpha) = \left(C_{B_2}\left[Q, Z_0 \mid D = L^{-1}(\alpha)\right],\ C_{B_2}\left[Q, Z_0 \mid D = R^{-1}(\alpha)\right]\right),\ 0 \le \alpha \le 1$$

The Yager's index of $\tilde{T}_{B_2}(\alpha)$ then can be derived as

$$I(\tilde{T}_{B_2}) = \frac{1}{2}\int_0^1 C_{B_2}\left[Q, Z_0 \mid D = L^{-1}(\alpha)\right] d\alpha + \frac{1}{2}\int_0^1 C_{B_2}\left[Q, Z_0 \mid D = R^{-1}(\alpha)\right] d\alpha$$

$$\text{Let } K_2(\square) = \frac{1}{2}\int_0^1 \left(\left[L^{-1}(\alpha)\right]^2 + \left[R^{-1}(\alpha)\right]^2\right) d\alpha$$

The necessary conditions for $I(\tilde{T}_{B_2})$ equal to attain the minimum are $I_Q^1(\tilde{T}_{B_2}) = 0$ which can be calculated as follows.

$$Q^2 = \frac{1}{h}\int_0^1 \left[2A \,.\, K_1(\alpha) + (h + B_2)Z_0^2 \,.\, K_2(\alpha)\right]$$

$$Q = \sqrt{\frac{1}{h}\left[2A \,.\, K_1(\alpha) + (h + B_2)Z_0^2 \,.\, K_2(\alpha)\right]} \quad \text{. . . (3.1.3)}$$

The sufficient conditions for the $I(\tilde{T}_{B_2})$ to attain the minimum are $I_Q^1(\tilde{T}_{B_2}) > 0$.

Because of $K_1(\alpha) > 0$ and $K_2(\alpha) > 0$, the sufficient conditions are clearly hold from the above equations. The optimal solutions (Q^*, t) that can be found from eqn.(3.1.3) and the optimal annual cost can be calculated as,

$$\tilde{T}^*_{B_2}(\alpha) = \left[C_{B_2}\left[Q^*, Z_0 \mid D = L_D^{-1}(\alpha)\right],\ C_{B_2}\left[Q^*, Z_0 \mid D = R_D^{-1}(\alpha)\right] \right],\ 0 \le \alpha \le 1$$

To show the characteristics of proposed models a trapezoidal fuzzy demand is employed. Let $\tilde{\lambda}$ be the trapezoidal fuzzy demands with parameters :

$\tilde{D}$ = [l, m, n, u]

It is easy to find that $K_1(\alpha) = \dfrac{l + m + n + u}{4}$

For the EOQ model with $\tilde{D}$, we have

$$Q^* = \sqrt{\frac{2A}{h}\left[\frac{1}{4}\ l + m + n + u\ \right]}$$

If $\tilde{D}$ is a symmetrical fuzzy number then

u – n = m – l

(ie) u + l = m + 1

Let $D_0 = \dfrac{m + n}{2}$, the mean of $\tilde{D}$, then

$$Q^* = \sqrt{\frac{2A}{h}\left[\frac{1}{4}\ m + u\ \right]} = \sqrt{\frac{2A}{h}D_0}$$

Q^* is the conventional EOQ with crisp demands, $\dfrac{m + n}{2}$.

This result implies that no matter what the spreads of fuzzy demands, as long as the fuzzy demands are symmetric with the same mean, the Q^* will be the same and equal to the conventional EOQ with the mean of fuzzy demands. The fuzzy number of annual cost can be calculated as

$$\tilde{T}^*(\alpha) = \left[C\left[Q^*, \mid D = l + \alpha(m - l)\right],\ C\left[Q^*, \mid D = u - \alpha(u - n)\right] \right],\ 0 \le \alpha \le 1$$

The above equation shows that the annual cost will also be a trapezoidal fuzzy number and with parameter as

$$\tilde{C}\ Q^* = \left[\frac{Al}{Q^*} + \frac{hQ^*}{2},\ \frac{Am}{Q^*} + \frac{hQ^*}{2},\ \frac{An}{Q^*} + \frac{hQ^*}{2},\ \frac{Au}{Q^*} + \frac{hQ^*}{2}\right]$$

This implies that the shape of membership function of $\tilde{C}\ Q^*$ is the same as the $\tilde{D}$, but with a different scale. Accordingly, the spread of $\tilde{C}\ Q^*$ will vary according to the spread of $\tilde{D}$.

$$C_{B_2}\ Q, Z_0 \ = \frac{AD}{Q} + h\left(\frac{Q}{2} + Z_0\ \ a'D + b'Q\ \right) + \frac{DB_2}{Q}\ a'D + b'Q\ \ G\ \ Z_0\ + \frac{D}{Q}R\ \ L$$

Furthermore, besides demands, the other coefficients may also be fuzzy. Let $\tilde{h}$ and $\tilde{B}_2$ be the fuzzy number of holding cost and shortage cost of models and be defined as

$$\tilde{B}_2\ \alpha_{\tilde{B}_2} = \left[\min.\ \mu^{-1}_{\tilde{B}_2}(\alpha_{\tilde{B}_2}),\ \max.\ \mu^{-1}_{\tilde{B}_2}(\alpha_{\tilde{B}_2})\right]$$

$$= \left[L^{-1}_{\tilde{B}_2}(\alpha_{\tilde{B}_2}),\ R^{-1}_{\tilde{B}_2}(\alpha_{\tilde{B}_2})\right], 0 \le \alpha_{B_2} \le 1$$

and

$$h\ \alpha_h = \left[\min.\ \mu^{-1}_{\tilde{h}}(\alpha_h),\ \max.\ \mu^{-1}_{\tilde{h}}(\alpha_h)\right]$$

$$= \left[L^{-1}_{h}(\alpha_h),\ R^{-1}_{h}(\alpha_h)\right],\ \ 0 \le \alpha_h \le 1$$

Let $K_2\ \alpha_D\ = \frac{1}{2}\int_0^1 \left[L^{-1}_D\ \alpha_D\ \right]^2 + \left[R^{-1}_D\ \alpha_D\ \right]^2\ \ d\alpha_D$

$$K_3\ \alpha_D, \alpha_{B_2}\ = \frac{1}{4}\left\{\int_0^1 L^{-1}_{B_2}\ \alpha_{B_2}\ d\alpha_{B_2}\ .\ \int_0^1 L^{-1}_D\ \alpha_D\ d\alpha_D + \int_0^1 R^{-1}_{B_2}\ \alpha_{B_2}\ d\alpha_{B_2}\ .\ \int_0^1 R^{-1}_D\ \alpha_D\ d\alpha_D\right\}$$

$$K_4\ \alpha_L\ = \frac{1}{2}\int_0^1\left[L^{-1}_h\ \alpha_h\ + R^{-1}_h\ \alpha_h\ \right] d\alpha_h$$

$$K_5\ \alpha_h, \alpha_D\ = \frac{1}{4}\left\{\int_0^1 L^{-1}_h\ \alpha_h\ d\alpha_h\ .\ \int_0^1 L^{-1}_D\ \alpha_D\ d\alpha_D + \int_0^1 R^{-1}_h\ \alpha_h\ d\alpha_h\ .\ \int_0^1 R^{-1}_D\ \alpha_D\ d\alpha_D\right\}$$

$$K_6\ \alpha_h, \alpha_D\ = \frac{1}{4}\left\{\int_0^1 L^{-1}_h\ \alpha_h\ d\alpha_h\ .\ \int_0^1 L^{-1}_D\ \alpha_D\ d\alpha_D + \int_0^1 R^{-1}_h\ \alpha_h\ d\alpha_h\ .\ \int_0^1 R^{-1}_D\ \alpha_D\ d\alpha_D\right\}$$

Then the Yager's ranking index can be derived as

$$I(\tilde{T})\ = \frac{AD}{Q}K_1\ \alpha_D\ + \frac{Q}{2}K_4\ \alpha_h\ + Z_0\ .\ K_5\ \alpha_h, \alpha_D$$

$$+\ \frac{D^2B_2}{Q}G\ \ Z_0\ \ K_6\ \alpha_h, \alpha_D\ + DB_2G\ \ Z_0\ \ K_3\ \alpha_D, \alpha_{B_2}$$

and the optimal solution can form found as :

$$Q^* = \sqrt{\frac{1}{h}\left[\frac{2AK_1(\alpha_D)}{K_4\ \alpha_h} + \frac{(h + B_2)K_5^2\ \alpha_h, \alpha_D}{K_4\ \alpha_h\ \left[K_6\ \alpha_h, \alpha_D\ + K_3\ \alpha_D, \alpha_{B_2}\ \right]}\right]}$$

Numerical Example

$D = 600$, $h = 20$, $A = 200$, $B = 1000$, $b' = 0.396$, $a' = 0.360$, $R(L) = 5.6$, $Z_0 = 2.3$, $G(2.3) = 0.0036$, $L = 6$, $\sigma = 5.999$

$Q^* = 144.5$

$C(Q, Z_0) = 18951.06$

$\tilde{D} = (560, 580, 620, 640)$

$\tilde{C}(Q^*, Z_0) = (17758.57, 18351.23, 19553.07, 20172.25)$

Conclusion

The purpose of this paper is to study the EOQ model with impact of stochastic lead time reduction on inventory cost under order crossover and fuzzy demands. Because demands are fuzzy, the quantities of all usually treated as a decision variable will be also fuzzy. The results of this study indicate that, for the EOQ model no matter what the spreads of fuzzy demands, as long as the fuzzy demands are symmetric with the same mean, the optimal ordering quantities will be the same and equal to the conventional EOQ with the mean of fuzzy demands. Fuzzier demands only yield fuzzier annual costs. In this paper, we are using the exponential distribution to characterize lead time. In reducing the lead time we take advantage of order crossover. With order crossover the lead times are transferred to effective lead times whose mean is the same as that of the original variance.

References

[1] Yager, R.R., “A procedure for ordering fuzzy subsets of the unit interval”, Information Sciences, 24, 143-161 (1981).

[2] Jack C. Hayya, Terry P. Harrison, X.James He, “The impact of stochastic lead time reduction on inventory cost under cross over”, European Journal of Operational Research, 211(2011) 274-281.

[3] Ben-Daya, M., Raouf, A., 1994. Inventory models involving lead time as a decision variable. Journal of the Opeational Research Society, 45 (5), 579-582.

[4] Hayya, J.C., Bagchi, U., Kim, J.G., Sun, D., 2008. On Static Stochastic Order Crossover, International Journal of Production Economics. 114(11), 404-413.

[5] Lan, S.P., Chu, P., Chung, K.J., Wan, W.J., Lo, R., 1999. A simple method to locate the optimal solution of the inventory model with variable lead time. Computers and Operations Research, 26, 599-605.

[6] Silver, E.A., Pyke, D.F., Peterson, R., 1998. Inventory Management and Production Planning and Scheduling, Third ed. John Wiley & Sons, New York.

[7] Park, K.S., “Fuzzy set theoretic interpretation of economic order quantity”, IEEE Trans. System, Man, Cybernetics, SMC. M, 1082-1084 (1987).

[8] Chang, H.C., "An application of fuzzy theory to the EOQ model imperfect quality items", Computers and Operations Research 31, 2079-2092 (2004).

[9] Chang, S.C. and J.S.Yao., Economic reorder point for fuzzy backorder quantity", European Journal of Operational Research, 109, 183-202 (1998).

[10] IShii, H and T. Konno, "A Stochastic inventory problem with shortage cost", European Journal of Operational Research, 106, 90-94 (1998).

[11] Kao, C. and W.K. Hsu, "A single-period inventory model with fuzzy demand", Computer and Mathematic with Application, 43, 841 – 898 (2002).

[12] Lee, H.M and J.S. Yao, "Economic Production Quantity for fuzzy demand and fuzzy production quantity", European Journal of Operational Research, 109, 203-211 (1998).

[13] Petrovic D., R. Petrovic and M. Vujosevic, "Fuzzy model for the newsboy problem", International Journal of Production Economics, 45, 435-441 (1996).

[14] Roy, T.K., and M. Maiti, "A fuzzy EOQ model with demand dependent unit cost under limited storage capacity", European Journal of Operational Research, 99, 425-432 (1997).

[15] Yao, J.S. and J. Chiang, "Inventory without backorder with fuzzy total cost and fuzzy storing cost defuzzified by centroid and sign distance", European Journal of Operational Research, 148, 401-409 (2003).