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On Distinguished GB-Compact Topological Spaces

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Abstract

The purpose of this paper is to project distinguished gb-compact topological spaces and to continue the study of their fundamental properties at a glance at one place. The notions of gb-compactness, second countable gb-space, gb-Lindelöf space, sequentially gb-compact space & countably gb-compact space are introduced. Also, the paper contains relations between (i) gb-compactness and some types of compactness, (ii) second countable gb-space & gb-Lindelöf space & (iii) a sequentially gb-compact space & countably gb-compact space. The images of these sorts of spaces under some non-continuous mappings are investigated.
The gb-convergence of a sequence due to gb-open sets in topological space has been conceptualized and the relation of gb-convergence with gb-continuity and gb-irresoluteness has been discovered here. It also deals with the relation between gb-convergent sequence and usual convergence of a sequence in a space with suitable example.

Keywords: gb-continuity, gb-convergent sequence, gb-compactness, gb-lindelöf space, second countable gb-space, sequentially gb-compact space & countably gb-compact space.

Introduction

Compactness is an important, useful and fundamental concept of not only general topology but also of other advanced branches of Mathematics, so its structural properties as emphasized in the form of gb-open sets, gb-convergent sequence, gb-lindelöf space, countably gb-compact space etc. create a new region in mathematics.

Since 1906, when frechet used for the first time the team compact, many sorts of compactness were introduced by different topologist. The author[21] described various types of compactness using α-open sets, α-convergence sequence, α-limit points etc. In this paper, we introduce and study gb-Lindelöf space, countably gb-compact space and characterization of gb-compact space. The new concept of second countable gb-space as well as sequentially gb-compact space is introduced and studied. Also the images of these sorts of spaces under gb-continuous & gb-irresolute mappings are, here, investigated & studied.

Prerequisites

We, however, know that the notions of b-open sets and regular b-closed sets have been introduced and investigated by D. Andrijevic[1] and N. Nagaveni & A. Narmadha[2] &[3], respectively. In 2007, M. Caldas & S. Jafari projected some applications of b-open sets in topological spaces[4] whereas 2009 was the year for the conceptualization of the class of generalized b-closed sets and its fundamental properties by A .Al-Omari & M.S.M. Noorami[5].

The class of generalized closed sets & regular generalized closed sets was coined & framed by N. Levine[6] and N. Palanniappan & K. Chandrasekhar Rao[7], respectively.

As usual throughout this paper, cl(A) and int(A) stand as the closure of A and the interior of A, respectively, where A is a subset of a space (X,T), with no separation axioms are assumed unless otherwise mentioned. Also, X-A = A^c represents the complements of A in X.

In this section, we recall some basic definitions of some types of near open sets defined by using the closure (cl) & interior (int) operators and represent the implications diagram among these types.

Definition (2.1)

A subset A of a space (X, T) is said to be b-open [1] if A$\subset$ int(cl(A)) $\cup$ cl(int(A)) .

Definition (2.2)

A subset A of a topological space (X, T) is called

1. regular open[8] if A = int(cl(A)).
2. an α-open[9] set if A$\subseteq$ int(cl(int(A)))
3. pre-open [10] set if A $\subseteq$ int(cl (A))
4. semi-open [11] set if A$\subseteq$ cl(int(A))
5. β-open [12] set if A$\subseteq$ cl(int(cl(A)).

The compliments of the above mentioned open sets are their respective closed sets. The smallest $\mathcal{K}$-closed set containing A is called $\mathcal{K}$cl(A) where $\mathcal{K}$ = regular, α, pre, semi, β & b. The largest $\mathcal{K}$int(A) where $\mathcal{K}$ = regular, α, pre, semi, β & b.

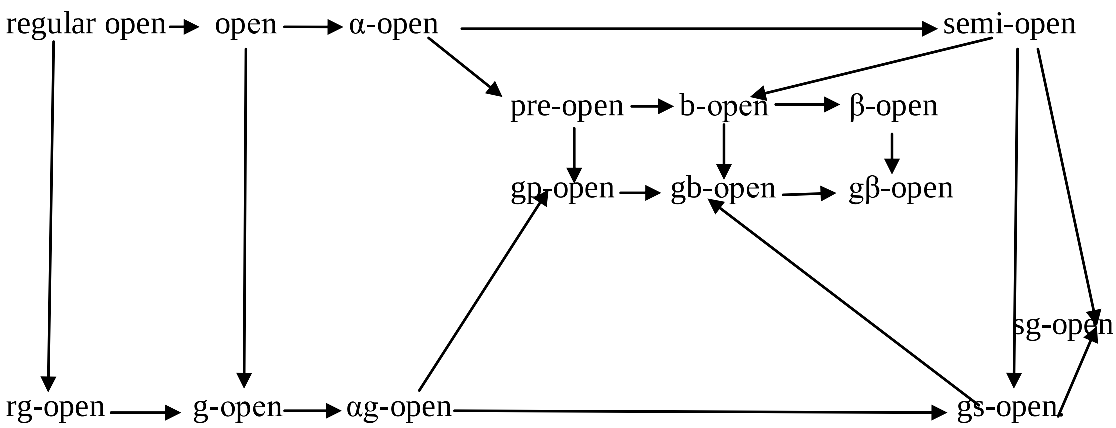
Definition (2.3): A subset A of a space (X, T) is said to be

1. generalized closed (briefly g- closed)[6]set if cl(A)$\subset$U whenever A $\subset$ U &U is open in X.
2. generalized semi-closed (briefly gs-closed) [13] set if scl(A) $\subset$U whenever A $\subset$ U & U is open in X .

3. semi- generalized closed (briefly sg-closed) [14] set if scl(A) ⊂U whenever A ⊂ U & U is semi- open in X .
4. regular generalized closed (briefly rg-closed) [7] set if cl(A) ⊂U whenever A ⊂ U & U is regular open in X.
5. generalized pre-closed briefly gp-closed) [15] set if pcl(A) ⊂U whenever A ⊂ U & U is open in X .
6. generalized b-closed (briefly gb-closed) [5] set if bcl(A) ⊂U whenever A ⊂ U & U is open in X .
7. regular b-closed (briefly rb-closed) [3] set if rcl(A) ⊂U whenever A ⊂ U & U is b- open in X.
8. α-generalized closed (briefly αg-closed) [16] set if αcl(A) ⊂U whenever A ⊂ U & U is open in X.
9. generalized β closed (briefly gβ-closed) [17] set if βcl(A) ⊂U whenever A ⊂ U & U is open in X.

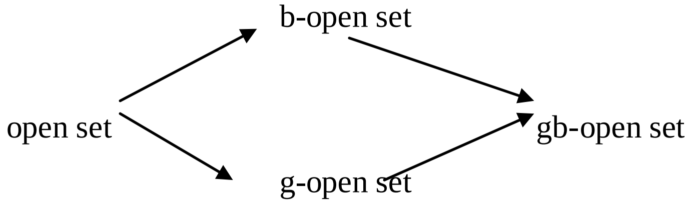
The family of all gb-open(respectively gb-closed) sets of (X, T) is denoted by GBO(X)(respectively GBC(X)).The family of gb-open sets of (X, T) containing a point x∈X is denoted by GBO(X, x).

Now, the relationships between these various types of near open sets and generalized open sets are summarized by the following implication diagram:



Implication diagram (I)

Again, for our purpose, the relations between open set, b-open set, g-open set and gb-open set in topological space are summarized by the following diagram which is a part of the above implication diagram:



Implication diagram (II)

The above implications are not reversed in general as supported by the following examples:

Example (2.4)

A. For the topological space (X, T) where X = {a, b, c, d} and T = {φ, X, {a}, {a, b, c}}, the singleton set {b} is g-open and gb-open which is neither b-open nor open.
B. For the topological space (X, T) where X = {a, b, c, d} and T = {φ, X, {a}, {b}, {a, b},{a, b, c}}, the set {a, b, d} is b-open and gb-open which is neither g-open nor open.

Next, the following lemma (2.5) & proposition (2.6) stand as a necessary and sufficient condition for a gb-open set to be b-open:

Lemma(2.5)

If A is a gb-open subset of a space (X, T), then the unique open set containing $bint(A) \cup A^C$ is X.

Proof

Let (X, T) be a topological space and $A \subset X$ such that A is a gb-open set. Let $U \in T$ in manner that

$bint(A) \cup A^C \subseteq U$………….. (1)

Again, $U^C \subseteq [bint(A)]^C \cap A \Rightarrow U^C \subseteq A$

$\Rightarrow U^C \subseteq bint(A)$ as A is gb-open.

$\Rightarrow [bint(A)]^C \subseteq U$……………….. (2)

Combining (1) & (2), we have

$[bint(A)]^C \cup bint(A) \cup A^C \subseteq U$

i.e. $X \cup A^C \subseteq U$ i.e. $X \subseteq U$

But $U \subseteq X$

Hence, U = X.

Hence, the theorem.

Proposition (2.6)

For a gb-open subset A of a space (X, T), A is b-open iff $bint(A) \cup A^C$ is open.

Proof

Let A be a gb-open set in a space (X, T).

Necessity

Let A be b-open, then A = bint(A). Now, $bint(A) \cup A^C = A \cup A^C = X$ which is an open set.

Sufficiency
Let bint(A)∪A^C be an open set . Then, using the lemma (2.5), we claim that the unique open set containing bint(A)∪A^C in X. So, bint(A)∪A^C = X which means that A = bint(A) and ultimately A is b-open.

Hence, the proposition.

Definition (2.7)
[18]: A topological space (X, T) is called b-compact if every b-open cover of X has a finite subcover.

Definition (2.8)
[19]: A topological space (X, T) is called g-compact if every open cover of X by g-open sets has a finite subcover.

GB-Compact Space

This section deals with the notion of gb-compactness in topological spaces. Also, the existing relations between gb-compactness, b-compactness, g-compactness are analyzed.

Definition (3.1)
In a topological space (X,T), a collection $\{G_\alpha : \alpha \in \Delta\}$ of gb-open sets in X is called a gb-open cover for X if $X = \bigcup_{\alpha \in \Delta} G_\alpha$

Definition (3.2)
[20]: A topological space (X,T) is called a gb-compact space space if every cover of X by gb-open sets has a finite subcover.

Definition (3.3)
A topological space (X, T) is called a gb-Lindelöf space if every cover of X by gb-open sets has a countable subcover.

Definition (3.4)
[20]: In a topological space (X,T), a subset A of X is said to be gb- compact relative to X if for every gb-open cover C of A, there is a finite sub collection C^* of C that covers A .

Definition (3.5)
[20]: A subspace of a topological space, which is gb-compact as a topological space in its own right, is said to be gb-compact subspace.

The following lemma (3.6) is enunciated for the above definitions to be consistent:

Lemma(3.6)

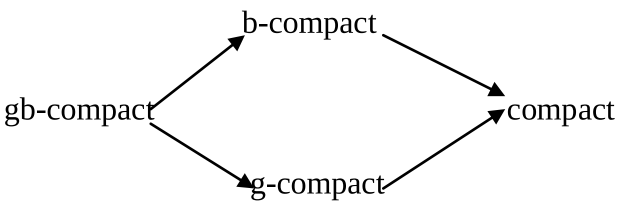
(1) Every gb-compact space is a gb-Lindelöf space.
(2) Every gb-Lindelöf space is a Lindelöf space.
(3) Every countable space is a gb-Lindelöf space.
(4) gb-compactness is not hereditary.

Proof

The statement follows from definitions (3.2), (3.3) & (3.4), (3.5).

Lemma (3.7)

[20]: If a space (X,T) is gb-compact then it is b-compact, g-compact and compact in a natural way.



The reverse implication does not hold as illustrated by the following examples:

Example (3.8)

A. The space (N,I), where I is the indiscrete topology for the set N of natural numbers, is a compact space. However, it is not b-compact (resp., not g-compact, not gb-compact) space, since $\{\{n\}: n \in N\}$ is b-open cover (resp., g-open cover, gb-open cover) of N which has no finite subcover.
B. Suppose that $X = \{0, 1, 2, 3, ..., n, ...\} = N \cup \{0\}$, and $T = \wp(N) \cup \{G \subseteq X : 0 \in G \text{ \& } G^c \text{ is finite}\}$, where $\wp(N)$ is the power set of N.

Thus,(X,T) is a g-compact topological space because any g-open cover of X must contain a g-open set A such that $0 \in A$ & A^c is finite.

But (X,T) is neither b-compact nor gb-compact, since, $(E^+ \cup \{0\}) \cup \{\{n\}: n \in O^+\}$ is b-open (also gb-open) cover of X which has no finite subcover; here $E^+ = \{2,4,6,.........2n,......\}$ & $O^+ = \{1,3,...............,2n+1,.......\}$.

Next, the following proposition provides the proper criteria for a b-compact space to be a gb-compact space:

Proposition (3.9)

If (X, T) is a b-compact space in which for every gb-open set $A \subseteq X$, bint(A) $\cup A^c$ is open , then (X, T) is gb-compact.

Proof

Let (X,T) be a b-compact space. Let $\{A_\alpha : \alpha \in \Delta\}$ be a gb-open cover of X, then $X = \bigcup_{\alpha \in \Delta} A_\alpha$. Given that for each $\alpha \in \Delta, A_\alpha$ being gb-open subset of X, bint(A) $\cup A^c$ is open. By prop.(2.6) $A_\alpha, \forall \alpha \in \Delta,$ is b-open. Thus, $\{A_\alpha : \alpha \in \Delta\}$ reduces to a b-open cover of X. Since, X is b-compact , hence, this cover has finite subcover i.e. there is finite sub collection $\{A_\alpha : \alpha \in \Delta\}$ that covers X which means that $X = \bigcup_{\alpha \in \Delta} A_\alpha$ where $\Delta_0 (\subset \Delta)$ is finite. Hence, (X, T) is a gb-compact space.

Hence, the proposition.

Proposition (3.10)
A gb-closed subset of gb-compact space (X, T) is gb compact relative to (X,T).

Proof
The proof follows as a consequence of the combination of the definitions (2.3) (vi),(3.2) & (3.4).

Characterisation of GB-compactness In Terms of GB-closed Sets
Before, we take up the characterisations of gb-compactness, we enunciate FIP as:

Finite Intersection Property (FIP)
A class $C = \{C_\alpha\}_{\alpha \in \Delta}$ of subsets of a non empty set X is said to have finite intersection property (FIP) if and only if the intersection of the members of each finite subclass $C^* = \{C_{\alpha i} : i = 1,2,3,\ldots n\}$ of C is non empty.

Remark
Obviously, a class $C = \{C_\alpha\}_{\alpha \in \Delta}$ of subsets of a non-empty set X does not have the FIP iff there exists a finite subfamily $C^* = \{C_{\alpha i} : i = 1,2,3,\ldots n\}$ of C such that $\bigcap_{i=1}^{n} C_{\alpha_i} = \phi.$

Proposition (3.11)
For a topological space (X,T) , the following statements are equivalent:

a) (X,T) is gb-compact.
b) Any family of gb-closed subsets of X satisfying 'FIP' has a non-empty intersection.
c) Any family of gb-closed subsets of X with empty intersection has a finite subfamily with empty intersection.

Proof: (a) $\Rightarrow$ (b):
Suppose that (X,T) is a gb- compact topological space. Let $\{C_\alpha\}_{\alpha \in \Delta}$ be an arbitrary collection of gb-closed subsets of X with the finite intersection property so that $\bigcap_{i=1}^{n} C_{\alpha_i} \neq \phi$ .It is to show that $\bigcap_{\alpha \in \Delta} C_\alpha \neq \phi$.

On the contrary, let $\bigcap_{\alpha\in\Delta} C_\alpha = \phi$, which implies that $\bigcup_{\alpha\in\Delta} C_\alpha^c = X$

Obviously, $\{C_\alpha^c\}_{\alpha\in\Delta}$ is a gb-open cover of X.

Since, (X,T) is a gb-ompact, there must exist finite number of sets, say, $C_{\alpha_1}^c, C_{\alpha_2}^c, \ldots\ldots\ldots C_{\alpha_n}^c$ such that $\bigcup_{i=1}^{n} C_{\alpha_i}^c = X$.

On taking complements of both sides, we have $\bigcap_{i=1}^{n} C_{\alpha_i} = \phi$, which is a contradiction of the hypothesis that $\{C_\alpha\}_{\alpha\in\Delta}$ has FIP. Hence, $\bigcap_{\alpha\in\Delta} C_\alpha \neq \phi$.

(b) ⟹ (a):
we assume that every class $\{C_\alpha\}_{\alpha\in\Delta}$ of gb-closed subsets of a topological space (X,T) with FIP has a non- empty intersection.

It is required to show that (X,T) is gb-compact.

If it not so, let (X,T) be not gb- compact, then there exists an gb-open cover $\{H_\alpha\}_{\alpha\in\Delta}$ of X which has no finite sub cover.

This means that the complement of the union of any finite number of members of the cover is non-empty.

Then, the class $\{H_\alpha^c\}_{\alpha\in\Delta}$ is a class of gb-closed sets which satisfies FIP.

Since, $\{H_\alpha\}_{\alpha\in\Delta}$ is a gb-open cover of X, hence, $\bigcup_{\alpha\in\Delta} H_\alpha = X$.

i.e. $\bigcup_{\alpha\in\Delta} H_\alpha^c = \phi$.This means that $\{H_\alpha^c\}_{\alpha\in\Delta}$, with FIP, has an empty intersection. This contradicts the hypothesis. Thus, (X,T) must be a gb -compact space.

(b) ⟹ (c):
By logic we know that the statement "**p ⟹ q**" is equivalent to the statement "**not q ⟹ not p**" (contrapositive). Hence, obvious.

Hence, the proposition.

Second Countable GB-Space

In analogy to well known "second Axiom of countability" in general topology, we coin "second Axiom of gb-countability" in the following manner:

Definition (4.1)
A topological space (X, T) posseses second axiom of gb-countability if there exists a countable gb-open base for the topology T.

By a gb-open base for the space (X, T) , we mean a subcollection B⊆ GBO(X) such that every member of T is a union of member of B.

Definition (4.2)
A topological space (X, T) is said to be a second countable gb- space or a second axiom gb-space if it carries second axiom of gb-countability.

In other words, a topological space (X,T) is called a second countable gb-space iff there exists a countable gb-open base for the topology T.

Proposition (4.3): (Theorem (2.1)[22])
Every second countable gb-space is a gb- Lindelöf space.

GB-Convergent Sequences & GB-Continuity/ Irresoluteness At A Point:

In this section, we project the notions of gb-accumulation point of a set , gb-limit of a sequence, gb-convergence of a sequence in a space (X,T) alongwith the gb-continuity /irresoluteness at a point of a mapping f(X,T)⟶(Y,σ) as prepared by the author's mathematical paper [22] & accepted for publication in Acta Ciencia Indica.

Definition (5.1): GB-limit point of a set:(Definition (3.1)[22])
Let (X,T) be a topological space and A⊆X.

A point p∈X is called a gb-limit point(or a gb-cluster point or a gb-accumulation point) of A iff every gb-open set containing p contains a point of A other than p.

i.e. symbolically [p∈(X,T)∧A⊆X]⇒[p= a gb-limit point for A]

⇔ [∀N∈GBO(X)∧p∈N⇒ [N-{p}]∩A≠φ]

Definition (5.2): GB-limit point of a sequence:(Definition (3.3)[22])
A point x_0 in X is said to be gb-limit point of a sequence $\{x_n\}$ in a topological space(X,T) iff every gb-open set L containing x_0 there exists a +ve integer n for each+ve integer m such that $n\geq m \Longrightarrow x_n \in L$.

This means that a sequence $\{x_n\}$ in a topological space(X,T) is said to have $x_0 \in X$ as a gb-limit point iff for every gb-open set containing x_0 contains x_n for finitely many n.

Definition (5.3): GB-convergent sequences:[Definition (3.2)[22])
A sequence $\{x_n\}$ in a topological space(X,T) is said to be gb-convergent to a point x_0 or to converge to a point x_0 ∈Xwith respect to gb-open sets, written as $x_n \xrightarrow{gb-cgt} x_0$, if for every gb-open set L containing x_0 ,there exists a positive integer m, s.t. $n\geq m \Rightarrow x_n \in L$.

This concept is symbolically presented as:

$$x_n \xrightarrow{gb-cgt} x_0 \Leftrightarrow gb-\lim_{n\to\infty} x_n = x_0$$

Obviously, a sequence$\{x_n\}$ in a topological space (X,T) is said to be gb-convergent to a point x_0 in X iff it is eventually in every gb- open set containing x_0.

Definition(5.4): GB-continuity/GB-irresolute at a point:(Definition (3.6) & (3.6) (a) [22])

A mapping $f:(X,T)\rightarrow(Y,\sigma)$ from one topological space (X,T) to another topological space(Y,σ) is said to be gb-continuous/ gb-irresolute at a point x_0∈X if for every σ-open set V(resp. σ-gb open set V) containing f(x_0) there exists a gb-open set L in (X,T) containing x_0 such that f(L)⊆V.

We, here, produce the following two propositions concerned with gb-convergence & convergence of a sequence and its image sequence under gb-continuity and gb-irresoluteness.

Proposition (5.5):(Theorem (3.2) [22])

In a topological space if a sequence {x_n} is gb-convergent to a point x_0∈X, then it is simply convergent to that point .

But the converse may not be true.

Proposition (5.6):(Theorem (3.3) [22])

If f(X,T)⟶(Y,σ) be a gb-continuous mapping from a topological space (X,T) into (Yσ) and {x_n} be gb-convergent to x_0∈X, then {f(x_n)} is convergent to f(x_0)∈ Y.

Corollary (5.7):

If f:(X,T) ⟶(Y,σ) be a gb-irresolute mapping and {x_n} be gb-convergent to x_0∈X, then

$$x_n \xrightarrow{gb-cgt} x_0 \Rightarrow f(x_n) \xrightarrow{gb-cgt} f(x_0), \forall \text{ gb-irresolute maps f.}$$

Proof

The proof is straightforward &natural & so omitted.

Sequentially GB-Compact & Countably GB-Compact Spaces

This section introduces the notions of sequentially & countably gb-compactness in topological spaces and their interrelation.

Definition (6.1): Sequentially GB-compact spaces:(Definition (3.4)[22])

A topological space (X, T) is said to be sequentially gb-compact iff every sequence in X contains a sub-sequence which is gb- convergent to a point of X.

Definition (6.2): Countably GB-compact spaces: (Definition (3.5)[22])

A topological space(X,T) is said to be countably gb-compact(or to have gb-Bolzano Weierstrass Property) iff every infinite subset of X has at least one gb- limit point in X.

Or

A topological space(X, T) is known as countably gb-compact iff every countable T-gb-open cover of X has a finite sub- cover.

Remark (6.3):[22]

1. Every finite subspace of a topological space is sequentially gb-compact.
2. Every gb-compact space is a countably gb-compact space.
3. Every cofinite topological space is a countably gb-compact space.

Theorem (6.4): (Theorem (3.1)[22])

Every sequentially gb-compact topological space(X,T) is countably gb-compact .

Remark (6.5)

A countably gb-compact space is not necessarily sequentially gb-compact as illustrated by following example:

Example (6.6)

Let N = {n: n is a natural number}.

Let T be topology on N generated by the family H = {{2n-1,2n}: n∈ N} of subsets of N.

Let E be a non-empty subset of N.

Let m_0∈ E. If m_0 is even m_0-1 is a gb-accumulation point of E and if m_0 is odd m_0+1 is a gb-accumulation point of E. Hence, every non-empty subset of N has a gb-accumulation point, so that (N,T) is countably gb-compact.

Also,(N,T) is not sequentially gb-compact because the sequence {2n-1: n∈N} has no gb-convergent sub-sequence.

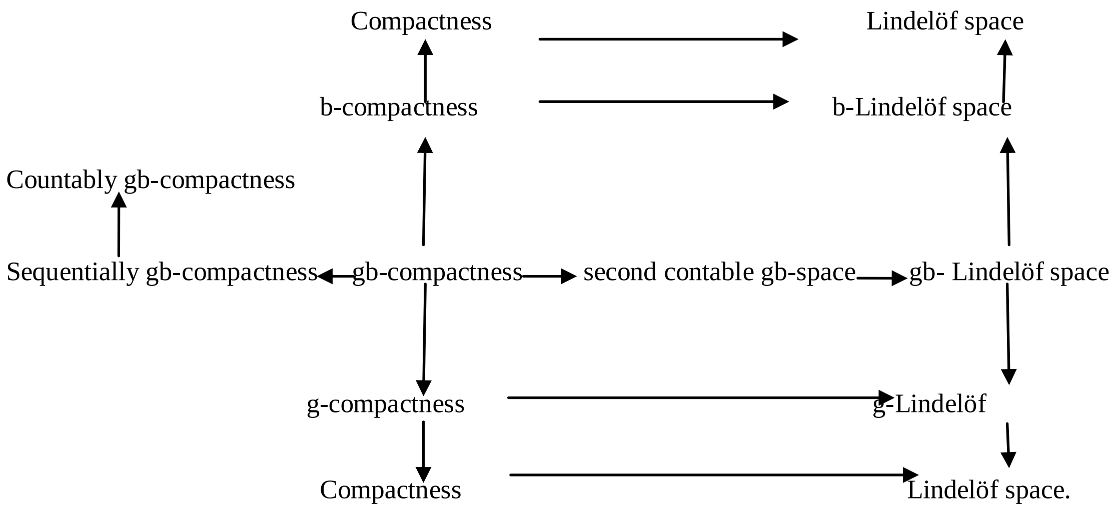
Therefore,

Countably gb-compactness ⇏ gb-sequentially compactness.

⇏ gb-compactness.

Remark (6.7)

The following implication diagram depicts the relation between the mentioned sorts of compactness.



Implication diagram (III)

Images of Sorts of GB-Compact Spaces

The images of the sorts of gb-compact spaces under some non-continuous mappings find place in this section so the following definitions of such mappings are, here, recalled:

Definition (7.1)[5]

A function f:(X,T) ⟶ (Y,σ), from one topological space to another topological space (Y,σ), is said to be gb-continuous(resp.gb-irresolute) if $f^{-1}(V)$ is gb-closed in X for every closed(resp. gb-closed) set V of Y.

Definition (7.2) [5]

A function f:(X,T) ⟶ (Y,σ), from one topological space to another topological space (Y,σ), is said to be gb-open if f(V) is gb-open in Y whenever V is gb- open subset of X.

Proposition (7.3):

A function f:(X,T) ⟶ (Y,σ) is gb –continuous(resp.gb-irresolute)iff $f^{-1}(V)$ is gb-open in X for every open(res. gb-open) set V in Y.

Proof

Straightforward followed from definitions (7.1), so omitted.

Proposition (7.4)

A continuous and open mapping f:(X,T) ⟶ (Y,σ) relating topological spaces (X,T) & (Y,σ) is always gb-irresolute mapping.

Proof

Let f:(X,T) ⟶ (Y,σ) be continuous and open mapping from one topological space (X,T) to other **(**Y,σ).

Let V be a gb-open subset of Y. Let F be a closed set in X such that $F \subseteq f^{-1}(V)$.

Since, f is an open mapping hence, f(F) is a closed set in Y.

As, $F \subseteq f^{-1}(V)$ so $f(F) \subseteq f(f^{-1}(V)) = V$. Since, V is a gb-open set in Y, hence, by definition $f(F) \subseteq V \Rightarrow f(F) \subseteq \sigma\text{-bint}(V)$ and f(F) is a closed set in Y.

Again, $f(F) \subseteq \sigma\text{-bint}(V) \Rightarrow f^{-1}(f(F)) \subseteq f^{-1}(\sigma\text{-bint}(V)) \Rightarrow F \subseteq T\text{-bint}(f^{-1}(V))$.

Thus, we have, $F \subseteq f^{-1}(V) \Rightarrow F \subseteq T\text{-bint}(f^{-1}(V))$ F is a closed subset of X. Equivalently, $f^{-1}(V)$ is a gb-open set in X.

Consequently, for a gb-open set V in Y, $f^{-1}(V)$ is a gb-open set in X. i.e. f is a gb-irresolute mapping.

Proposition (7.5)

The homeoporphic image of a gb-compact space is gb-compact.

Proof

Let f:(X,T) ⟶ (Y,σ)be a homeomorphism between topological spaces (X,T)& (Y,σ).

Then, the mapping f is a surjective, continuous & open mapping. Obviously f is a surjective irresolute mapping by using prop. (7.4).

Given that (X,T) is a gb-compact space. Let $\{A_\alpha : \alpha \in \Delta\}$ be a gb-open cover of Y. Then, $\{f^{-1}(A_\alpha) : \alpha \in \Delta\}$ is a gb-open cover of X. Since,(X,T) is gb-compact, it has a finite subcover, say, $\{f^{-1}(A_{\alpha i}) : i = 1,2,3,......n\}$. since, f is onto, $\{(A_{\alpha i}) : i = 1,2,....n\}$ is a finite gb-open cover of Y. therefore, (Y,σ) is gb-compact.

Corollary (7.6)

A gb-irresolute image of a gb-compact space is gb-compact.

Proposition (7.6)

A surjective gb-continuous image of a gb-compact space is compact.

Proof

Let f:(X,T) ⟶ (Y,σ) be gb-continuous mapping from a gb-compact space (X,T) onto a space (Y,σ). Let $\{A_\alpha : \alpha \in \Delta\}$ be an open cover of Y. Then, $\{f^{-1}(A_\alpha) : \alpha \in \Delta\}$ is a gb-open cover of X. Since,(X,T) is gb-compact, it has a finite subcover, say, $\{f^{-1}(A_{\alpha i}) : i = 1,2,3,......n\}$. Since, f is onto, $\{(A_{\alpha i}) : i = 1,2,....n\}$ is a finite gb-open cover of Y. therefore, (Y,σ) is gb-compact.

Proposition (7.7)

If f:(X,T) ⟶ (Y,σ) is bijective & gb-open and (Y,σ) is gb-compact, then (X,T) is also gb-compact.

Proof

Let f:X⟶Y be bijective and gb-open mapping from a topological space (X,T) to a gb-compact space (Y,σ).

Let $\{V_\alpha\}_{\alpha \in \Delta}$ be an gb-open cover for X. Then X = $\bigcup_{\alpha \in \Delta} V_\alpha$

This means that f (X) = $f\left(\bigcup_{\alpha \in \Delta} V_\alpha\right)$

$$\Rightarrow Y = \bigcup_{\alpha \in \Delta} (f(V_\alpha))$$

Since, f is gb-open mapping, we have, $\{f(V_\alpha)\}_{\alpha \in \Delta}$ is a gb-open cover of Y. Again as(Y,σ) is gb-compact , so this gb-open cover contains finite sub cover, namely $\{f(V_{\alpha_1}), f(V_{\alpha_2}),......, f(V_{\alpha_n})\}$.

This means that Y = $\bigcup_{r=1}^{n} f(V_{\alpha_r})$

$$\Rightarrow f^{-1}(Y) = f^{-1}\left(\bigcup_{r=1}^{n} f(V_{\alpha_r})\right)$$

$$X = \bigcup_{r=1}^{n} f^{-1} f(V_{\alpha_r})$$

$$\Rightarrow X = \bigcup_{r=1}^{n} (V_{\alpha_r})$$

$\Rightarrow \{V_{\alpha_r} : r = 1,2,......n\}$ is a finite sub cover of gb-open cover $\{V_\alpha\}_{\alpha\in\Delta}$ for X .

$\Rightarrow$ (X,T) is a gb- compact.

Proposition (7.8)
A gb-continuous image of a sequentially gb-compact set is sequentially compact.

Proof
Let f(X,T)⟶(Y,σ).Suppose, f is a gb-continuous mapping. Let A be a sequentially gb-compact set in topological space (X,T) and we have to show that f(A) is sequentially compact subset of (Y,σ)

Let $\{y_n\}$ be an arbitrary sequence of points in f(A), then for each n∈N there exists x_n∈A such that $f(x_n) = y_n$ and thus we obtain a sequence$\{x_n\}$ of points of A.

But A is sequentially gb–compact w.r.t. T so that there is a subsequence $\{x_{nk}\}$ of $\{x_n\}$ which is gb-convergent to a point say,x of A.

Therefore, $x_{nk} \xrightarrow{gb\text{–}cgt} x \Rightarrow f(x_{nk}) \xrightarrow{cgt} f(x) \in f(A)$ as f is gb-continuous.[prop.(5.7)]

Hence, $f(x_{nk})$ is a subsequence of the sequence$\{y_n\}$ of f(A), converging to a point f(x) in f(A). Consequently, f(A) is sequentially compact.

Corollary (7.9)
1. The gb-irresolute image of a sequentially gb-compact set is sequentially gb-compact.
2. The gb-continuous(resp. gb-irresolute) image of a countably gb-compact set is countably compact (countably gb-compact).
3. The gb-compactness, sequentially gb-compactness as well as countably gb-compactness is a topological property under gb-irresoluteness of mappings.
4. The second countable gb-space as well as gb- Lindelöf space is a topological property under homeomorphism.

Conclusion
The structures projected & discussed in the paper have wide applications and it surely pleases the mathematician if one of his abstract structures finds an application,. The future scope of the study is to obtain observations & results in respective

paracompactness & connectedness and also in pairwise respective compactness in Bitopological spaces.

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