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Single Supplier Multiple Cooperative Retailers Fuzzy Inventory Model With Quantity Discount and Permissible Delay In Payments

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Abstract

We consider in this paper an economic order quantity (EOQ) problem involving a single supplier that offers quantity discount and allow retailers to delay payments. In this model we consider a fuzzy average individual cost and trapezoidal fuzzy costs under crisp order quantity or fuzzy order quantity in order to extend the traditional inventory model to the fuzzy environment. We use Function Principle as arithmetical operations of fuzzy total average individual cost and use the Graded Mean Integration Representation Method to defuzzify the fuzzy total average individual cost. Then we use the Kuhn-Tucker Method to find the optimal order quantity of the fuzzy order inventory model.

Keywords: Economic Order Quantity, Fuzzy inventory, Function Principle, Grade mean Integration Representation, the total inventory cost, quantity discounts, optimization, Kuhn-Tucker Method, Delay in payments.

Introduction

The fuzzy set concept has been used to treat the classical inventory model recently. Park (1987) used fuzzy inventory cost in economic order quantity model. In many situations partners involved in the Supply Chain (SC) are looking forward minimizing their costs. Li and Huang (1995) discussed various cooperative mechanisms to adhere channel effectiveness: joint ownership, contacts, profit sharing and quantity discounts. Salameh Abord, El-Kassar and Ghattas (2003) studied an inventory model, assuming the delay in payment that incurs charged costs.

Suppliers generally propose discounts landing in terms of the purchased quantity. Therefore, for numerous companies are profitable. At the same time, the supplier can accept to delay the payment for companies in Lack of Cash. The supplier offers a discount quantity and allows the delay in payment to increase the sale.

Chen, Federgreen and Zheng (2001) addressed a distribution channel where a supplier delivers a single product to multiple retailers. Mendoza and Ventura (2008) incorporated the discount quantity option in the EOQ including two different models of transportation. More recently, Leopoldo and Cardenas (2009) proposed a corrected model of Sancer and Al Kindi (2006) Benton and Park (1996) presented an overview of the quantity discount. Numerous papers studied such EOQ problem with the discount option as Shin and Benton (2007), Jianli Li and Liu (2006), Oin, Tang and Guo (2007) Goossens, Maas, Spieksma and Vande Klundert (2007). Goyal (1985) was the first to propose an EOQ model under permissible delay in payment. A fully permissible delay in payments was considered by Chang, Ouyang and Tend (2003). An integrated inventory model with permissible delay in payments was developed by Chen and Kang (2007) and Huang (2005).

In this paper, we consider an all-unit discount quantity with delay payment period where an additional cost will be charged over the delay period. Jaggi Chandra, Goyal & God (2008) developed an alternative inventory model with delay in payments. We assume that the delay period is a retailer's decision variable in the time.

In section 3, the methodology is introduced. In section 3, discuss with fuzzy EOQ inventory models with different situation. A numerical example is shown in section 4 and section 5 concludes.

Methodology

Graded Mean Integration Representation Method

Chen & Hsieh [1999] introduced Graded mean Integration Representation Method based on the integral value of graded mean h-level of generalized fuzzy number for defuzzifying generalized fuzzy number. Here, we fist define generalized fuzzy number as follows :

Suppose $\tilde{A}$ is a generalized fuzzy number as shown in Fig.1. It is described as any fuzzy subset of the real line R, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions.

1. $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to [0, 1],
2. $\mu_{\tilde{A}}(x) = 0, -\infty < x \leq a_1$,
3. $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$,
4. $\mu_{\tilde{A}}(x) = W_A, a_2 \leq x \leq a_3$,
5. $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$,
6. $\mu_{\tilde{A}}(x) = 0, a_4 \leq x < \infty$,

where $0 < W_A \leq 1$ and a_1, a_2, a_3 and a_4 are real numbers.

This type of generalized fuzzy numbers are denoted as $\tilde{A} = (a_1, a_2, a_3, a_4\ ; \omega_A)_{LR}$ and $\tilde{A} = (a_1, a_2, a_3, a_4 : w_A)_{LR}$. When $\omega_A = 1$, it can be formed as $\tilde{A} = (a_1, a_2, a_3, a_4\ ; \omega_A)_{LR}$. Second, by Graded Mean Integration Representation Method, L^{-1} and R^{-1} are the inverse functions of L and R respectively and the graded mean h-level value of generalized fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4\ ; \omega_A)_{LR}$ is given by $\frac{h}{2}\left[L^{-1}(h) + R^{-1}(h)\right]$. (see fig.1).

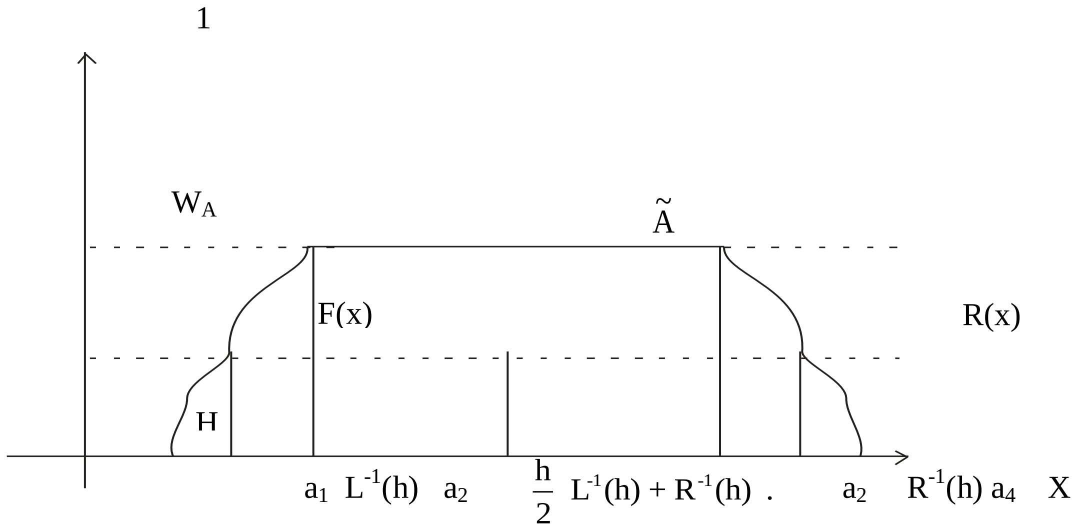

Figure 1: The graded mean h-level value of generalized fuzzy number

$\tilde{A}$ **= $(a_1, a_2, a_3, a_4 : w_A)_{LR}$.**

Then the graded Mean Integration Representation of $P(\tilde{A})$ with grade w_A, where

$$P(\tilde{A}) = \frac{\int_0^{\omega_A} \frac{h}{2}\left[L^{-1}(h) + R^{-1}(h)\right] dh}{\int_0^{\omega_A} h\, dh} \qquad \text{. . . (1)}$$

where $0 < h \leq w_A$ and $0 < w_A \leq 1$.

Throughout this paper, we only use popular trapezoidal fuzzy number as the type of all fuzzy parameters in our proposed fuzzy production inventory models. Let $\tilde{B}$ be a trapezoidal fuzzy number and be denoted as $\tilde{B} = (b_1, b_2, b_3, b_4)$. Then we can get the Graded Mean Integration Representation of $\tilde{B}$ by the formula (1) as

$$P(\tilde{B}) = \frac{\int_0^1 \frac{h}{2}\left[(b_1+b_4) + h(b_2 - b_1 - b_4 + b_3)\right] dh}{\int_0^1 h\, dh} = \frac{b_1 + 2b_2 + 2b_3 + b_4}{6} \qquad \text{. . . (2)}$$

The Fuzzy Arithmetical Operations under Function Principle

Function Principle is introduced by Chen (1985) to treat the fuzzy arithmetical operations by trapezoidal fuzzy numbers. We will use this principle as the operation of addition, multiplication, subtract, division of trapezoidal fuzzy numbers, because (1) the Function Principle is easier to calculate than the Extension Principle, (2) the Function Principle will not change the shape of trapezoidal fuzzy number after the multiplication of two trapezoidal fuzzy numbers, but the multiplication of two trapezoidal fuzzy numbers will become drum-like shape fuzzy number by using the Extension Principle, (3) if we have to multiply more than four trapezoidal fuzzy numbers then the Extension Principle cannot solve the operation, but the Function Principle can easily find the result by point wise computation. Here we describe some fuzzy arithmetical operations under the Function Principle as follows.

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ & $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers. Then

1. The addition of $\tilde{A}$ and $\tilde{B}$ is
   $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
   where $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are any real numbers.
2. The multiplication of $\tilde{A}$ and $\tilde{B}$ is
   $\tilde{A} \otimes \tilde{B} = (c_1, c_2, c_3, c_4)$
   where $T = \{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$
   $T_1 = \{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$
   $c_1 = \min T_1, c_2 = \min T_1, c_3 = \max T_1, c_4 = \max T_1$
   If $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are all zero positive real numbers then
   $\tilde{A} \otimes \tilde{B} = (a_1b_1, a_2b_2, a_3b_3, a_4b_4)$
3. $-\tilde{B} = (-b_4, -b_3, -b_2, -b_1)$ then the subtraction of $\tilde{A}$ and $\tilde{B}$ is
   $\tilde{A} \ominus \tilde{B} = \{a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1\}$ where $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are any real numbers.
4. $\frac{1}{\tilde{B}} = \tilde{B}^{-1} \left(\frac{1}{b_4}, \frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1}\right)$ where b_1, b_2, b_3, b_4 are all positive real numbers.
   If $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are all nonzero positive real numbers then the division $\tilde{A}$ and $\tilde{B}$ is
   $\tilde{A} \oslash \tilde{B} = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right)$
5. Let $\alpha \in R$, then
   1. $\alpha \geq 0, \alpha \otimes \tilde{A} = (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4)$
   2. $\alpha \geq 0, \alpha \otimes \tilde{A} = (\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1)$

The Kuhn-Tucker Conditions

Taha (1997) discussed how to solve the optimum solution of nonlinear programming problem subject to inequality constraints by using the Kuhn–Tucker conditions. The development of the Kuhn–Tucker conditions is based on the Lagrangean method.

Suppose that the problem is given by

Minimize $y = f(x)$

Subject to $g_i(x) \geq 0$, $i = 1, 2, \ldots, m$.

The nonnegativity constraints $x \geq 0$, if any, are included in the m constraints.

The inequality constraints may be converted into equations by using nonnegative surplus variables. Let S_i^2 be the surplus quantity added to the i^{th} constraint $g_i(x) \geq 0$. Let $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)$, $g(x) = (g_1(x), g_2(x), \ldots g_m(x)) \ldots$ and $S_2 = S_1^2, S_2^2, \ldots, S_m^2$.

The Kuhn–Tucker conditions need x and λ to be a stationary point of the minimization problem, which can be summarized as follows :

$$\begin{cases} \lambda \leq 0, \\ \nabla f(x) - \lambda \nabla g(x) = 0, \\ \lambda_i g_i(x) = 0, \quad i = 1, 2, \ldots, m. \\ g_i(x) \geq 0, \quad i = 1, 2, \ldots, m. \end{cases}$$

The Eoq Inventory Model With Quantity Discount And Permissible Delay In Payments

In this section, we develop a sequential optimization method using Kuhn-Tucker method. We use this method to find the optimal economic order quantity (EOQ) of the fuzzy inventory model (Scoussen Krichen, Awatef Laasidi, Forad Ben Abddoziz (2011).

Notations

The following notations are used throughout to develop the EOQ inventory model.

n	$\rightarrow$	the number of retailers
N	$\rightarrow$	A set of retailers $1, \ldots, n$
i	$\rightarrow$	The retailer's index $i \in N$.
q_i	$\rightarrow$	The ordered quantity of retailer i.
d_i	$\rightarrow$	The demand for retailer i.
□	$\rightarrow$	The order cost.
h_i	$\rightarrow$	The holding cost per unit and for a unit time for retailer i.
Q_{max}	$\rightarrow$	A threshold quantity to manage the discount.
C	$\rightarrow$	The initial unit purchasing cost proposed by the supplier.
C_p	$\rightarrow$	The unit purchasing cost depending on the ordered quantity and the delay period.
C_p^{min}	$\rightarrow$	The fixed unit purchasing cost when the order quantity exceeds Q_{max}.

e	→	The discount quantity rate.
C_i	→	The ordered quantity of retailer i in a non cooperative strategy.
	→	The payment rate fixed by the supplier.

Mathematical Model

Discount

We adopt the purchasing price's formulation of Fazel Fischer and Gilbert (1998) stated in terms of the ordered quantity as follows.

$$C_p = \begin{cases} C & q_i = 0 \\ C - e \times q_i & 0 < q_i \le Q_{max} \\ C_p^{min} & q_i > Q_{max} \end{cases} \qquad \text{. . . (3.2.1)}$$

equation (3.2.1) reports three expressions where :

The initial unit purchasing cost C is the announced price of the supplier in the market (Qin. Y., Tang, H. & Guo, C. (2007))

e is a discount rate using in the landing $0 < q_i \le Q_{max}$ (Schniederjans, M.J., & Lav, O., (2006)).

Beyond Q_{max} and no matter how large the order quantity is the supplier charges a fixed minimum price equal to C_p^{min}, Q_{max} can be computed, as proposed in Fazel et al. (1998), through the following equation.

$$Q_{max} = \frac{C - C_p^{min}}{e} \qquad \text{. . . (3.2.2)}$$

Discount and Delay

Besides the discount option that generally encourages retailers to order greater quantities, delay can also be a powerful tool to increase the retailers – supplier trade. (Chen, L.H., & Kang, F.S., (2007)), Chi Kin Chan, Lee, Y.C.E., & Goyal, S.K., (2010)).

Retailers are of course motivated by the discount, however they may not dispose of enough cash and need to payoff by loan from the bank. In this case, the supplier can propose to be a source of immediate credit by delaying payments with charged interest (Salameh et al. 2003).

From the supplier's standpoint, delay in payments incurs capital loss during the delay period. This loss can be balanced by the increase of the sale volume as discussed in Sheen & Tsao (2007). We propose, an extension of the purchasing cost that takes into account both of the discount and the delay options. The additional cost of delay is C_p x a x P_i, where P_i denotes the delay period.

We develop the purchasing cost in terms of two main options, namely: the quantity discount and the delay in payments. We start by stating C_p only with discount, then add the delay option. The general purchasing cost with discount and delay is reported as follows.

$$C_p = \begin{cases} C & q_i = 0 \\ (C - e \text{ x } q_i)(1 + \alpha \text{ x } p_i) & 0 < q_i \leq Q_{max} \\ C_p^{min}(1 + \alpha \text{ x } p_i) & q_i > Q_{max} \end{cases} \quad \text{. . . (3.3.1)}$$

Goosens, D.R., Maas, A.J.T., Spieksma, F.C.R., & Vande Klundert, J.J (2007) and Teng. J., Chang, C., Goyal, S.K. (2005).

The Average Individual Cost Function for Retailer i (C_i)

If $0 < q_i \leq Q_{max}$ (Sacoussen Krichen, Awatef Laabidi, Fouad Ben Abdelaziz) (2011)

then $C_i = a \text{ x } \frac{d_i}{q_i} + h_i \text{ x } \frac{q_i}{2} + (C - e \text{ x } q_i)(1 + \alpha \text{ x } p_i) \text{ x } d_i \ \forall_i \in N$

We keep the holding and ordering costs and propose a purchasing cost that insets a fractional cost incurred by postponing the payment. Where $\frac{d_i}{q_i}$ is the number of placed orders, $\frac{q_i}{2}$ is the average size of inventory and $(C - e \text{ x } q_i)(1 + \alpha \text{ x } p_i) \text{ x } d_i$ is the annual purchasing cost. The objective is to find the optimal order quantity which minimize the average individual cost. The necessary conditions for minimum

$$\frac{\partial C_i}{\partial q_i} = 0$$

Therefore, the optimal order quantity is

$$q_i^* = \sqrt{\frac{20 \text{ x } d_i}{h_i - 2e \text{ x } (1 + \alpha \text{ x } p_i) \text{ x } d_i}} \ \forall_i \in N \quad \text{. . . (3.4.1)}$$

If $q_i = Q_{max}$

$$C_i = a \text{ x } \frac{d_i}{q_i} + h_i \text{ x } \frac{q_i}{2} + C_p^{min}(1 + \alpha \text{ x } p_i) \text{ x } d_i \ \forall_i \in N$$

The optimal quantity in this case is equal to

$$q_i^* = \sqrt{\frac{20 \text{ x } d_i}{h_i}} \ \forall_i \in N \quad \text{. . . (3.4.2)}$$

Throughout this paper, we use the following variables in order to simplify the treatment of the fuzzy inventory models. $\tilde{a}, \tilde{h}_i, \tilde{c}, \tilde{e}, \tilde{\alpha}, \tilde{p}_i$ are fuzzy parameters. The fuzzy average individual cost function for retailer i is

$$\tilde{T}C_i(q_i) = \begin{cases} a_1 \text{ x } \frac{d_i}{q_i} + h_{i_1} \text{ x } \frac{q_i}{2} + \ C_1 - e_4 \text{ x } q_i \ \ 1 + \alpha_1 \text{ x } P_{i_1} \ \ \text{x } d_i, \end{cases}$$

$$a_2 \times \frac{d_i}{q_i} + h_{i_2} \times \frac{q_i}{2} + \quad C_2 - e_3 \times q_i \quad 1 + \alpha_2 \times P_{i_2} \quad \times d_i,$$

$$a_3 \times \frac{d_i}{q_i} + h_{i_3} \times \frac{q_i}{2} + \quad C_3 - e_2 \times q_i \quad 1 + \alpha_3 \times P_{i_3} \quad \times d_i,$$

$$a_4 \times \frac{d_i}{q_i} + h_{i_4} \times \frac{q_i}{2} + \quad C_4 - e_1 \times q_i \quad 1 + \alpha_4 \times P_{i_1} \quad \times d_i \Big\}$$

$$\tilde{T}C_i(q_i) = a \otimes (d_i \oslash q_i) \oplus h_i \otimes \frac{q_i}{2} \oplus (c \ominus e \otimes q_i) \otimes (1 \oplus \alpha \otimes \text{equal}p_i) \otimes d_i$$

where $\oslash$, $\otimes$, $\ominus$, $\oplus$ are the fuzzy arithmetical operations under function principle.

Suppose, $\tilde{a} = (a_1, a_2, a_3, a_4)$ $\tilde{h}_i = (h_{i1}, h_{i2}, h_{i3}, h_{i4})$

$\tilde{C} = (C_1, C_2, C_3, C_4)$ $\tilde{e} = (e_1, e_2, e_3, e_4)$

$\tilde{\alpha} = (\square_1, \square_2, \square_3, \square_4)$ $\tilde{P}_i = (P_{i1}, P_{i2}, P_{i3}, P_{i4})$

are non-negative trapezoidal fuzzy numbers. Then we solve the optimal order quantity as the following steps. Second we defuzzify the fuzzy total inventory cost, using the Graded Mean Integration Representation Method. The result is

$$P\ \tilde{T}C_i(q_i) = \frac{1}{6}\Big\{ a_1 \times \frac{d_i}{q_i} + h_{i_1} \times \frac{q_i}{2} + \quad C_1 - e_4 \times q_i \quad 1 + \alpha_1 \times P_{i_1} \quad \times d_i$$

$$+ 2\left[a_2 \times \frac{d_i}{q_i} + h_{i_2} \times \frac{q_i}{2} + \quad C_2 - e_3 \times q_i \quad 1 + \alpha_2 \times P_{i_2} \quad \times d_i \right]$$

$$+ 2\left[a_3 \times \frac{d_i}{q_i} + h_{i_3} \times \frac{q_i}{2} + \quad C_3 - e_2 \times q_i \quad 1 + \alpha_3 \times P_{i_3} \quad \times d_i \right]$$

$$+ \ a_4 \times \frac{d_i}{q_i} + h_{i_4} \times \frac{q_i}{2} + \quad C_4 - e_1 \times q_i \quad 1 + \alpha_4 \times P_{i_1} \quad \times d_i \Big\}$$

Third, we can get the optimal order quantity q_i^*, when $P\ \tilde{T}C_i(q_i)$ is minimization. In order to find the minimization of $P\ \tilde{T}C_i(q_i)$ the derivative of $P\ \tilde{T}C_i(q_i)$ with q_i is

$$\frac{\partial P\ \tilde{T}C_i(q_i)}{\partial q_i} = 0$$

Hence we find the optimal order quantity q_i^*

$$q_i^* = \sqrt{\frac{2\left[\ a_1 \text{ x } d_i \ + 2\ a_2 \text{ x } d_i \ + 2\ a_3 \text{ x } d_i \ + \ a_4 \text{ x } d_i\ \right]}{\begin{array}{l} h_{i_1} + 2h_{i_2} + 2h_{i_3} + h_{i_4} \ - 2\Big[\left[e_4 \text{ x } (1+\alpha_1 \text{ x } P_{i_1}) \text{ x } d_i\right] \\ + 2\left[e_3 \text{ x } (1+\alpha_2 \text{ x } P_{i_2}) \text{ x } d_i\right] + 2\left[e_2 \text{ x } (1+\alpha_3 \text{ x } P_{i_3}) \text{ x } d_i\right] \\ \quad + \left[e_1 \text{ x } (1+\alpha_4 \text{ x } P_{i_4}) \text{ x } d_i\right]\Big] \end{array}}}$$

Fuzzy Inventory EOQ Model with Fuzzy Order Quantity

In this section, we introduce the fuzzy inventory EOQ models by changing the crisp order quantity $\tilde{q}_i$ be a trapezoidal fuzzy number $\tilde{q}_i = q_{i_1}, q_{i_2}, q_{i_3}, q_{i_4}$ with $0 < q_{i1} \le q_{i2} \le q_{i3} \le q_{i4}$. Then we get the fuzzy average individual cost function for retailer (i) as

$$P\ \tilde{T}C_i(q_i) = \left\{ a_1 \text{ x } \frac{d_{i_1}}{q_{i_4}} + h_{i_1} \text{ x } \frac{q_{i_1}}{2} + \ C_1 - e_4 \text{ x } q_{i_4} \ \ 1 + \alpha_1 \text{ x } P_{i_1} \ \ \text{x } d_{i_1}, \right.$$

$$a_2 \text{ x } \frac{d_{i_2}}{q_{i_3}} + h_{i_2} \text{ x } \frac{q_{i_2}}{2} + \ C_3 - e_2 \text{ x } q_{i_3} \ \ 1 + \alpha_2 \text{ x } P_{i_2} \ \ \text{x } d_{i_2},$$

$$a_3 \text{ x } \frac{d_{i_3}}{q_{i_2}} + h_{i_3} \text{ x } \frac{q_{i_3}}{2} + \ C_3 - e_2 \text{ x } q_{i_2} \ \ 1 + \alpha_3 \text{ x } P_{i_3} \ \ \text{x } d_{i_3},$$

$$\left. a_4 \text{ x } \frac{d_{i_4}}{q_{i_1}} + h_{i_4} \text{ x } \frac{q_{i_4}}{2} + \ C_4 - e_1 \text{ x } q_{i_1} \ \ 1 + \alpha_4 \text{ x } P_{i_4} \ \ \text{x } d_{i_4} \right\}$$

Secondly we defuzzify the fuzzy average individual cost function for retailer (i) using the Graded Mean Integration Representation Method. The result is

$$P\ \tilde{T}C_i(q_i) = \frac{1}{6}\left\{ a_1 \text{ x } \frac{d_{i_1}}{q_{i_4}} + h_{i_1} \text{ x } \frac{q_{i_1}}{2} + \ C_1 - e_4 \text{ x } q_{i_4} \ \ 1 + \alpha_1 \text{ x } P_{i_1} \ \ \text{x } d_{i_1} \right.$$

$$+ 2\left[a_2 \text{ x } \frac{d_{i_2}}{q_{i_3}} + h_{i_2} \text{ x } \frac{q_{i_2}}{2} + \ C_3 - e_2 \text{ x } q_{i_3} \ \ 1 + \alpha_2 \text{ x } P_{i_2} \ \ \text{x } d_{i_2} \right]$$

$$+ 2\left[a_3 \text{ x } \frac{d_{i_3}}{q_{i_2}} + h_{i_3} \text{ x } \frac{q_{i_3}}{2} + \ C_3 - e_2 \text{ x } q_{i_2} \ \ 1 + \alpha_3 \text{ x } P_{i_3} \ \ \text{x } d_{i_3} \right]$$

$$\left. + \ a_4 \text{ x } \frac{d_{i_4}}{q_{i_1}} + h_{i_4} \text{ x } \frac{q_{i_4}}{2} + \ C_4 - e_1 \text{ x } q_{i_1} \ \ 1 + \alpha_4 \text{ x } P_{i_4} \ \ \text{x } d_{i_4} \right\}$$

with $0 < q_{i1} \le q_{i2} \le q_{i3} \le q_{i4}$.

It will not change the meaning of formula, if we replace inequality conditions $0 < q_{i1} \le q_{i2} \le q_{i3} \le q_{i4}$ into the following inequality constraints $q_{i2} - q_{i1} \ge 0$, $q_{i3} - q_{i2} \ge 0$, $q_{i4} - q_{i3} \ge 0$ and $q_{i1} > 0$.

Thirdly, the Kuhn-Tucker condition is used to find the solution of $q_{i1}, q_{i2}, q_{i3}, q_{i4}$ to minimize $P\ \tilde{T}C_i(q_i)$, subject to $q_{i2} - q_{i1} \ge 0$, $q_{i3} - q_{i2} \ge 0$, $q_{i4} - q_{i3} \ge 0$ and $q_{i1} > 0$. The Kuhn-Tucker conditions are, □ ≤ 0.

$$\nabla f\ \tilde{P}TC_i\ q_i\ - \lambda \nabla g(q_i) = 0$$

$$\lambda_i g_i(q_i) = 0$$

$$g_i(q_i) \ge 0.$$

These conditions simplify to the following □₁□..□..□≨

and $\nabla f\ \tilde{P}TC_i\ q_i\ - \lambda \nabla g(q_i) = 0$

$$\Rightarrow \frac{1}{6}\left\{ a_1\ x\ \frac{d_{i_1}}{q_{i_4}} + h_{i_1}\ x\ \frac{q_{i_1}}{2} + \ C_1 - e_4\ x\ q_{i_4}\ \ 1 + \alpha_1\ x\ P_{i_1}\ \ x\ d_{i_1} \right.$$

$$+ 2\left[a_2\ x\ \frac{d_{i_2}}{q_{i_3}} + h_{i_2}\ x\ \frac{q_{i_2}}{2} + \ C_3 - e_2\ x\ q_{i_3}\ \ 1 + \alpha_2\ x\ P_{i_2}\ \ x\ d_{i_2} \right]$$

$$+ 2\left[a_3\ x\ \frac{d_{i_3}}{q_{i_2}} + h_{i_3}\ x\ \frac{q_{i_3}}{2} + \ C_3 - e_2\ x\ q_{i_2}\ \ 1 + \alpha_3\ x\ P_{i_3}\ \ x\ d_{i_3} \right]$$

$$\left. + \ a_4\ x\ \frac{d_{i_4}}{q_{i_1}} + h_{i_4}\ x\ \frac{q_{i_4}}{2} + \ C_4 - e_1\ x\ q_{i_1}\ \ 1 + \alpha_4\ x\ P_{i_4}\ \ x\ d_{i_4} \right\}$$

$$-\lambda_1\ q_{i_2} - q_{i_1}\ \ - \lambda_2\ q_{i_3} - q_{i_2}\ \ - \lambda_3\ q_{i_4} - q_{i_3}\ \ - \lambda_4 q_{i_1} = 0$$

which implies

$$\frac{1}{6}\left\{ \frac{h_{i_1}}{2} - \frac{a_4\ x\ d_{i_4}}{q_{i_1}^2} - e_1\ x\ \ 1 + \alpha_4\ x\ P_{i_4}\ \ x\ d_{i_4} \right\} + \lambda_1 - \lambda_4 = 0$$

$$\frac{2}{6}\left\{ \frac{h_{i_2}}{2} - \frac{a_3\ x\ d_{i_3}}{q_{i_2}^2} - e_2\ x\ \ 1 + \alpha_3\ x\ P_{i_3}\ \ x\ d_{i_3} \right\} - \lambda_1 + \lambda_2 = 0$$

$$\frac{2}{6}\left\{ \frac{h_{i_3}}{2} - \frac{a_2\ x\ d_{i_2}}{q_{i_3}^2} - e_3\ x\ \ 1 + \alpha_2\ x\ P_{i_2}\ \ x\ d_{i_2} \right\} - \lambda_2 + \lambda_3 = 0$$

$$\frac{1}{6}\left\{\frac{h_{i_4}}{2} - \frac{a_1 \text{ x } d_{i_1}}{q_{i_4}^2} - e_1 \text{ x } \left(1+\alpha_1 \text{ x } P_{i_1}\right) \text{ x } d_{i_1}\right\} - \lambda_3 = 0$$

$$\lambda_1 \left(q_{i_2} - q_{i_1}\right) = 0$$

$$\lambda_1 \left(q_{i_3} - q_{i_2}\right) = 0$$

$$\lambda_3 \left(q_{i_4} - q_{i_3}\right) = 0$$

$$\lambda_4 q_{i_1} = 0$$

$$q_{i_2} - q_{i_1} \geq 0$$

$$q_{i_3} - q_{i_2} \geq 0$$

$$q_{i_4} - q_{i_3} \geq 0$$

$$q_{i_1} > 0$$

Because $q_{i_1} > 0$ and $\lambda_4 q_{i_1} = 0$ then $\lambda_4 = 0$.

If $\lambda_1 = \lambda_2 = \lambda_3 = 0$ then $q_{i_4} < q_{i_3} < q_{i_2} < q_{i_1}$, it does not satisfy the constraints $0 < q_{i_1} \leq q_{i_2} \leq q_{i_3} \leq q_{i_4}$.

Therefore $q_{i_2} = q_{i_1}$, $q_{i_3} = q_{i_2}$ and $q_{i_4} = q_{i_3}$ (ie) $q_{i_1} = q_{i_2} = q_{i_3} = q_{i_4} = q_i^*$. Hence from we find the optimal order quantity q_i^* as

$$q_i^* = \sqrt{\frac{2\left[a_1 \text{ x } d_{i_1} + 2\, a_2 \text{ x } d_{i_2} + 2\, a_3 \text{ x } d_{i_3} + a_4 \text{ x } d_{i_4}\right]}{\begin{gathered} h_{i_1} + 2h_{i_2} + 2h_{i_3} + h_{i_4} - 2\Big[\left[e_4 \text{ x } (1+\alpha_1 \text{ x } P_{i_1}) \text{ x } d_{i_1}\right] \\ + 2\left[e_3 \text{ x } (1+\alpha_2 \text{ x } P_{i_2}) \text{ x } d_{i_2}\right] + 2\left[e_2 \text{ x } (1+\alpha_3 \text{ x } P_{i_3}) \text{ x } d_{i_3}\right] \\ + \left[e_1 \text{ x } (1+\alpha_4 \text{ x } P_{i_4}) \text{ x } d_{i_4}\right]\Big]\end{gathered}}}$$

Numerical Example

Consider an inventory system with the following characteristics. Retailer's data and costs for n = 1, a = 60, c = 0.05, □ = 0.01, d_1 = 500, h_1 = 15 ; P_1 = 3 ; m_1 = 7.

q_i^* = 78.047

C_i = 26554.384

Suppose fuzzy initial unit purchasing cost "more or less than 50"

$\tilde{C} = (C_1, C_2, C_3, C_4) = (40, 45, 55, 60)$;

Fuzzy annual demand for retailer (i = 1) is "more or less than 500"

$\tilde{d}_1 = (d_{11}, d_{12}, d_{13}, d_{14}) = (460, 480, 520, 540)$;

Fuzzy holding cost per unit and for a unit time for retailer (i = 1) is "more or less than 15"

$\tilde{h}_1 = (h_{11}, h_{12}, h_{13}, h_{14}) = (13, 14, 16, 17)$;

Fuzzy delay period for retailer (i = 1) is "more or less than 3"

$\tilde{P}_1 = (P_{11}, P_{12}, P_{13}, P_{14}) = (1, 2, 4, 5)$;

Fuzzy number of orders per period for retailer (i = 1) is "more or less than 7"

$\tilde{m}_1 = (m_{11}, m_{12}, m_{13}, m_{14}) = (5, 6, 8, 9)$;

Fuzzy order cost is "more or less than 60"

$\tilde{a} = (a_1, a_2, a_3, a_4) = (40, 50, 70, 80)$;

Fuzzy discount quantity rate is "more or less than 0.005"

$\tilde{e} = (e_1, e_2, e_3, e_4) = (0.003, 0.004, 0.006, 0.007)$;

Fuzzy payment rate fixed by the supplier is "more or less than 0.01"

$\tilde{\alpha} = (\Box_1, \Box_2, \Box_3, \Box_4) = (0.008, 0.009, 0.011, 0.012)$;

Fuzzy order quantity

$q_i^* = (77.01, 77.01, 77.01, 77.01)$

Optimal average individual cost function for retailer i(i = 1)

(19036.31, 22614.02, 31091.67, 35427.91)

Conclusion

We prepare in this paper a fuzzy model for the EOQ problem where retailers are cooperatively to benefit from the supplier's options namely : the quantity discount and the delay in payments. In this mode, initial unit purchasing cost (c), annual demand for retailer (d_i), holding cost (hi), delay period for retailer (Pi), number of orders discount quantity rate (e), payment rate fixed by the supplier (a) and the average individual cost function for retailer i (i) are represented by fuzzy numbers. For each fuzzy model, a method of defuzzification, namely the Graded Mean Integration Representation is employed to find the estimate of average individual cost function for retailer i(ci) in the fuzzy sense and then the corresponding optimal order lot size is derived from Kuhn-Tucker Method. Numerical examples are carried out to investigate the behavior of our proposed model.

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