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Fuzzy Multiple Attributes Decision-Making Models Using TOPSIS Technique

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Abstract

Multi-attribute analysis is a useful tool in constructional, economical, managerial, and many other fields. The technique for order preference by similarity to ideal solution (TOPSIS) is one of the most widely used multi attribute decision making method. The fuzzy multi-criteria models in complex decision making and one multiple attributes, TOPSIS approach has been dealt with. In this paper, the triangular fuzzy numbers are converted into intervals using α cuts and calculated the normalized fuzzy numbers. Finally, implementing TOPSIS algorithm with fuzzy data, assessment of houses has been done. To improve the performance of the overall system, the set of important parameters of the decision making system is identified and the results have been tested in numerical example.

Keywords: Decision making, multiple attributes decision making, fuzzy MADM, TOPSIS method.

1. Introduction

The multiple attribute decision making (MADM) refers to making decisions in the presence of multiple, usually conflicting, attributes. Problems for multiple attributes decision making are common occurrences in every aspect of life. For example: In a personal context, the job one chooses may depend upon its prestige, location, salary, advancement opportunities, working conditions, and so on. The car one buys may be characterized in terms of price, gas mileage, style, safety, comfort, etc. The fuzzy set theory has been applied in many disciplines such as operations research, management and decision sciences, artificial intelligence, control theory, statistics, etc. It has been provided a new research direction to both concepts and methodologies to formulate and solve mathematical programming and objective decision making problems. The evaluations of alternatives with respect to some attributes are uncertain and vague, fuzzy set theory has been used.

Fuzzy MADM methods are proposed to solve problems which involve fuzzy data. Bellman and Zadeh [2] were the first to relate fuzzy set theory in decision making problems. Chen and Hwang [7] gave a comprehensive state-of-the-art in fuzzy multiple attribute decision making (FMADM). Triantaphyllou and Lin [22] evaluated five FMADM methods: fuzzy SAW model, fuzzy weighted product model, fuzzy AHP, revised fuzzy AHP and fuzzy TOPSIS. Chen [5] has used the FTOPSIS method in a group decision making problem, when the evaluations of the alternatives versus the criteria are linguistic variables. There are many works in the literature on application of FMADM methods in various fields. For example, Bender and Simonovic [3] developed a fuzzy compromise planning approach for a water management problem. Prakash [20] has used FMADM methods in an analysis of Land suitability for agricultural crops. In 1977, Baas and Kwakernaak [1] proposed a fuzzy MADM method that is widely regarded as the classic work of fuzzy MADM methods. During the past two decades several fuzzy MADM methods have been proposed. The only systematic reviews of fuzzy MADM methods have been conducted by Kickert [13] and Zimmermann [25, 26]. Zimmermann [26] among others treated the fuzzy MADM method.

The fuzzy MADM methods are complex and difficult to apply to most large size real-world problems. A good and simple method which is conceptually easy to understand and practically capable of solving real-world problems is desirable. The proposed new method to fuzzy MADM problem solving is an attempt towards that goal.

2. Fuzzy Multiple Attribute Decision Making

Fuzzy MADM methods basically consist of two phases: (i) aggregation of performance scores with respect to all attributes for each alternative, and (ii) rank ordering of alternatives according to aggregated scores. We will refer to results of the first and second phases using the terms “final rating” and “ranking order” respectively. For a crisp MADM problem, final ratings are expressed as real numbers and ranking order can be easily obtained by comparing these real numbers. The main focus of MADM problem solving is the first phase. In a fuzzy MADM problem, performance scores of an alternative with respect to all attributes may be expressed by linguistic data or fuzzy sets. As a result, the final ratings are expressed by linguistic data or fuzzy sets. Obtaining a ranking order of these fuzzy sets is not a trivial task. In this case, both phase one and phase two are important in solving a fuzzy MADM problem.

Chan and Hwang [6] and Chen, Hwang and Lai [8] classified fuzzy ranking methods based on two factors: (i) the comparison medium used, and (ii) the technique need to develop the comparison medium. According to Chen, Hwang and Lai [8], classification of fuzzy MADM involves the following stages.

1. The size of a MADM problem is characterized by the number of attributes and number of alternatives. Fuzzy MADM methods are suitable for solving a problem that has either less than ten alternatives and ten attributes, or any number of attributes and less than 350 attributes.
2. Data type allowed by each method can be: all fuzzy, all fuzzy singleton, all crisp, or a mixture of fuzzy and crisp. Real world MADM problems contain a mixture of fuzzy and crisp data.
3. The Basic concept of fuzzy MADM methods are derived mainly from classical MADM methods, whose basic concepts were adopted include simple additive weighing (SAW) method, technique for order performance by similarity to ideal solution (TOPSIS), analytic hierarchical process (AHP) method, conjunctive method, disjunctive method, multiple attribute utility function (MAUF) theory, out ranking method, maxi-min TOPSIS, and general classical MADM methods.
4. The techniques are required to apply each fuzzy MADM method. They include α-cut, fuzzy arithmetic operations, eigenvector methods, weight assessing method, possibility and necessity measures, human intuition, fuzzy ranking and fuzzy arithmetic, fuzzy out ranking relation, maximum and minimum operators, and semantic modeling.
5. Major approaches in any branch formed from the previous four stages are listed.

3. Preliminary

Definition 1 A fuzzy set A in X is characterized by a membership function $\mu_A(x)$ which associates with each point x a real number in the interval $[0,1]$ representing the grade of membership of x in A. Mathematically, $A = \{(x, \mu_A(x)); x \in X\}$, where $\mu_A(x): X \to [0,1]$. If $\mu_A(x) = 1$, then $x \in A$; if $\mu_A(x) = 0$, then $x \in A$. Space X is called the universe of discourse.

Definition 2 A fuzzy set A is a fuzzy number if the universe of discourse X is R and the fuzzy set A is convex, normal, the membership function of the fuzzy set $\mu_A(x)$ is piecewise continuous, and the core of the fuzzy set consists of one value only $(\mu_A(x) = 1)$.

Definition 3 A fuzzy set $\tilde{A}$ is convex iff $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \succ \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$ for all $x_1, x_2 \in R$, $\lambda \in [0,1]$, min denotes the minimum operator.

Definition 4 A fuzzy set A is normal if and only if there are one or more x values such that $\mu_A(x) = 1$.

Definition 5 The triangular fuzzy number can be denoted as $\tilde{A} = (a, m, n)$ where a is the central value, m is the left spread and n is the right spread.

Definition 6 If $\tilde{A}$ is a triangular fuzzy number and $[\tilde{A}]_\alpha^L \succ 0$ and $[\tilde{A}]_\alpha^U \leq 1$ for $\alpha \in [0,1]$ then $\tilde{A}$ is called a normalized positive triangular fuzzy number.

Definition 7 If $\tilde{A}=(a,m,n)$ and $\tilde{B}=(b,s,r)$ are two triangular fuzzy numbers then the distance between them is defined as

$$d(\tilde{A},\tilde{B})=\frac{1}{2}\{\max(|a-m-b+s|,|a+n-b-r|)+|a-b|\}$$

Remark If $\tilde{A}=\left([A]_\alpha^L,[A]_\alpha^U\right)$ then by choosing $\alpha=1$ we can identify the central value of $\tilde{A}$ and by $\alpha=0$ we can identify the left and right spreads of $\tilde{A}$.

4. Proposed Methodology

Here we introduce the technique for order performance of similarity to ideal solution (TOPSIS) with fuzzy data. TOPSIS is based upon the concept that the chosen alternative should have the shortest distance from the positive ideal solution and farthest from the negative ideal solution [10]. Assume that each alternative takes the monotonically increasing (or decreasing) utility. It is then easy to locate the ideal solution, which is a combination of all the best attribute value attainable, while the negative ideal solution is a combination of all the worst attribute values attainable. For ranking alternatives using this index, we can rank alternatives in decreasing order. The basic principle for choosing the TOPSIS technique is, the alternative should have the "shortest distance" from the positive ideal solution and the "farthest distance" from the negative ideal solution. It suggests two reference points, but it does not consider the relative importance of the distances from these points.

Suppose there exists m possible alternatives $x_1,x_2,...,x_m$ for which the decision maker has to choose on the basis on n attributes $C_1,C_2,...,C_n$ both qualitative and quantitative $A_i\ (i=1,2,...,m)$ on a attribute $C_j\ (j=1,2,...,n)$ given by the decision maker is a triangular fuzzy number $\tilde{x}_{ij}$. Then the multi-attribute decision making problem can be expressed in the matrix form as

$$\tilde{F}=\begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix}\overset{\begin{matrix} C_1 & C_2 & \cdots & C_n \end{matrix}}{\begin{pmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{pmatrix}}$$

since the physical dimensions and measurements of the n attributes are different, the fuzzy decision matrix $\tilde{F}$ must be normalized.

Algorithm of ranking order

Step 1 Identify the evaluation criteria which may be expressed in linguistic variables.

Step 2 Evaluate the alternatives in terms of the criteria.

Step 3 Identify the weight of the criteria which may also be fuzzy in nature.

Step 4 Construct the fuzzy decision matrix $\tilde{F}$. In this matrix each $\tilde{X}_{ij}$ is a triangular fuzzy number $\tilde{X}_{ij} = (x_{ij}, \alpha_{ij}, \beta_{ij})$.

Step 5 Calculate the normalized fuzzy decision matrix $\tilde{N}_{ij}$.

For each fuzzy number $\tilde{x}_{ij} = (x_{ij}, \alpha_{ij}, \beta_{ij})$, we calculate the set of α-cut as $\tilde{x}_{ij} = \left([\tilde{x}_{ij}]_\alpha^L, [\tilde{x}_{ij}]_\alpha^U \right)$, $\alpha \in [0,1]$. Each fuzzy number $\tilde{x}_{ij}$ is transformed into an interval. Now this interval is transformed into normalised interval

$$[\tilde{n}_{ij}]_\alpha^L = [\tilde{x}_{ij}]_\alpha^L \Big/ \sum_{i=1}^{m} \left[\left([\tilde{x}_{ij}]_\alpha^L \right)^2 + \left([\tilde{x}_{ij}]_\alpha^U \right)^2 \right] \qquad j = 1,2,\ldots,n$$

$$[\tilde{n}_{ij}]_\alpha^U = [\tilde{x}_{ij}]_\alpha^U \Big/ \sum_{i=1}^{m} \left[\left([\tilde{x}_{ij}]_\alpha^L \right)^2 + \left([\tilde{x}_{ij}]_\alpha^U \right)^2 \right] \qquad j = 1,2,\ldots,n$$

Now $\left([\tilde{n}_{ij}]_\alpha^L, [\tilde{n}_{ij}]_\alpha^U \right)$ is the normalized interval of $\left([\tilde{x}_{ij}]_\alpha^L, [\tilde{x}_{ij}]_\alpha^U \right)$ which is transformed into a fuzzy number $\tilde{N}_{ij} = (n_{ij}, a_{ij}, b_{ij})$. According to Remark by setting $\alpha = 1$, we have $[\tilde{n}_{ij}]_{\alpha=1}^L = [\tilde{n}_{ij}]_{\alpha=1}^U = n_{ij}$ and by setting $\alpha = 0$, we have $[\tilde{n}_{ij}]_{\alpha=1}^L = n_{ij} - a_{ij}$ and $[\tilde{n}_{ij}]_{\alpha=1}^U = n_{ij} + b_{ij}$, then $a_{ij} = n_{ij} - [\tilde{n}_{ij}]_{\alpha=0}^L$ and $b_{ij} = [\tilde{n}_{ij}]_{\alpha=0}^U - n_{ij}$. Now $\tilde{N}_{ij} = (n_{ij}, a_{ij}, b_{ij})$ is the fuzzy number of the normalized interval $\left([\tilde{n}_{ij}]_\alpha^L, [\tilde{n}_{ij}]_\alpha^U \right)$. This $\tilde{N}_{ij}$ is a normalized positive triangular fuzzy number.

Step 6 Considering the different importance of each criterion, we can construct the weighted normalized fuzzy decision matrix as $\tilde{v}_{ij} = \tilde{N}_{ij}.\tilde{w}_j$ where $\tilde{w}_j$ is the weight of the j^{th} criterion.

Step 7 Each $\tilde{v}_{ij}$ is a normalized fuzzy number and their ranges belong to $[0,1]$. So we identify the positive ideal solution $\tilde{A}^+ = (\tilde{v}_1^+, \tilde{v}_2^+, \ldots, \tilde{v}_n^+)$ and the negative ideal solution $\tilde{A}^- = (\tilde{v}_1^-, \tilde{v}_2^-, \ldots, \tilde{v}_n^-)$ where each $\tilde{v}_j^+ = (1,1,1)$ and $\tilde{v}_j^- = (0,0,0)$, $j = 1,2,\ldots,n$ for each criteria.

Step 8 Using the distance definition we calculate the distance of each alternative from the positive ideal solution and negative ideal solution as $\tilde{d}_i^+ = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^+)$ and $\tilde{d}_j^- = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^-)$, $i = 1,2,\ldots,m$ respectively.

Step 9 A relative closeness is defined as $\tilde{C}_i = \dfrac{\tilde{d}_i^-}{(\tilde{d}_i^+ + \tilde{d}_i^-)}$, $i = 1,2,3,\ldots,m$ to determine the ranking order of all the alternatives.

An alternative is closer to $\tilde{A}_i^+$ and farther from $\tilde{A}_i^-$ as the ranking $\tilde{C}_i$ approaches 1. Therefore according to the closeness coefficient, we can determine the ranking order of all alternatives and select the best one among a set of feasible alternatives.

5. Numerical Example

Nine houses ready for sale are evaluated in such a way to find the best of nine that suits the client's needs on the basis of the factors like location, aesthetics, facilities available, expansion possibilities, cost and other benefits. Each of these is depended on a selected interest rate or discount rate to adjust cash flows at different points of time [11]. Assume that we want to choose the best house among all proposed houses. The evaluation by three decision makers are made on a nine point scale as very very low, very low, low, medium low, medium, medium high, high, very high, very very high.

Table 1: The fuzzy decision matrix and fuzzy weights of nine alternatives

	C_1	C_2	C_3	C_4	C_5	C_6
H_1	(9,1,1)	(7,1,1)	(7.7,1,1)	(9.3,1,0.7)	(6,1,1)	(7.7,1,1)
H_2	(9.7,1,0.3)	(8.3,1,1)	(8.3,1,1)	(9.7,1,0.3)	(6.7,1,1)	(6.3,1,1)
H_3	(8.3,1,1)	(9.3,1,0.7)	(5.7,1,1)	(9.3,1,0.7)	(7.7,1,1)	(6.3,1,1)
H_4	(6.3,1,1)	(7.7,1,1)	(6.7,1,1)	(7.7,1,1)	(6.3,1,1)	(6,1,1)
H_5	(5.7,1,1)	(6.3,1,1)	(8.3,1,1)	(7.7,1,1)	(5.7,1,1)	(6,1,1)
H_6	(5.3,1,1)	(5.7,1,1)	(9.7,1,0.3)	(7.3,1,1)	(7.7,1,1)	(7,1,1)
H_7	(7.7,1,1)	(7,1,1)	(6.3,1,1)	(5.7,1,1)	(5.7,1,1)	(5.7,1,1)
H_8	(8.3,1,1)	(6.3,1,1)	(5.7,1,1)	(6,1,1)	(5.7,1,1)	(9.3,1,0.7)
H_9	(6.3,1,1)	(5,1,1)	(7,1,1)	(6.3,1,1)	(5.3,1,1)	(9.7,1,0.3)
Weight	(0.97,0.1,0.03)	(0.87,0.1,0.06)	(0.73,0.1,0.1)	(0.67,0.1,0.1)	(0.6,0.1,0.1)	(0.7,0.1,0.1)

Table 2: The normalized fuzzy decision matrix is

	C_1	C_2	C_3	C_4	C_5	C_6
H_1	(0.2807, 0.0312, 0.0312)	(0.2319, 0.0331, 0.0332)	(0.2456, 0.0319, 0.0319)	(0.2822, 0.0303, 0.0213)	(0.2195, 0.0366, 0.0365)	(0.2507, 0.0325, 0.0326)
H_2	(0.3025, 0.0311, 0.0094)	(0.2750, 0.0331, 0.0331)	(0.2647, 0.0319, 0.0319)	(0.2944, 0.0304, 0.0091)	(0.2451, 0.0366, 0.0365)	(0.2051, 0.0325, 0.0326)
H_3	(0.2589, 0.0312, 0.0312)	(0.3081, 0.0331, 0.0232)	(0.1818, 0.0319, 0.0319)	(0.2822, 0.0303, 0.0213)	(0.2816, 0.0365, 0.0366)	(0.2051, 0.0325, 0.0326)
H_4	(0.1965, 0.0312, 0.0312)	(0.2551, 0.0331, 0.0332)	(0.2137, 0.0319, 0.0319)	(0.2337, 0.0304, 0.0303)	(0.2304, 0.0366, 0.0366)	(0.1954, 0.0326, 0.0325)
H_5	(0.1778, 0.0312, 0.0312)	(0.2087, 0.0331, 0.0332)	(0.2647, 0.0319, 0.0319)	(0.2337, 0.0304, 0.0303)	(0.2085, 0.0366, 0.0366)	(0.1954, 0.0326, 0.0325)
H_6	(0.1653, 0.0312, 0.0312)	(0.1889, 0.0332, 0.0331)	(0.3094, 0.0319, 0.0095)	(0.2215, 0.0303, 0.0304)	(0.2816, 0.0365, 0.0366)	(0.2279, 0.0325, 0.0326)
H_7	(0.2402, 0.0936, 0.0312)	(0.2319, 0.0331, 0.0332)	(0.2009, 0.0319, 0.0319)	(0.1730, 0.0304, 0.0303)	(0.2085, 0.0366, 0.0366)	(0.1856, 0.0326, 0.0326)

H_8	(0.2589, 0.0312, 0.0312)	(0.2087, 0.0331, 0.0332)	(0.1818, 0.0319, 0.0319)	(0.1821, 0.0304, 0.0303)	(0.2085, 0.0366, 0.0366)	(0.3028, 0.0325, 0.0228)
H_9	(0.1965, 0.0312, 0.0312)	(0.1657, 0.0332, 0.0331)	(0.2233, 0.0319, 0.0319)	(0.1912, 0.0304, 0.0303)	(0.1938, 0.0365, 0.0366)	(0.3158, 0.0325, 0.0098)

Table 3: The weighted normalized fuzzy decision matrix

	C_1	C_2	C_3	C_4	C_5	C_6
H_1	(0.2723, 0.0552, 0.0396)	(0.2017, 0.0486, 0.0448)	(0.1793, 0.0447, 0.0510)	(0.1891, 0.0455, 0.0446)	(0.1317, 0.0403, 0.0475)	(0.1504, 0.0413, 0.0479)
H_2	(0.2934, 0.0573, 0.0185)	(0.2392, 0.0529, 0.0473)	(0.1932, 0.0465, 0.0530)	(0.1972, 0.0467, 0.0365)	(0.1471, 0.0429, 0.0500)	(0.1231, 0.0368, 0.0433)
H_3	(0.2511, 0.0530, 0.0390)	(0.2680, 0.0563, 0.0401)	(0.1327, 0.0383, 0.0447)	(0.1891, 0.0455, 0.0446)	(0.1290, 0.0065, 0.0937)	(0.1231, 0.0368, 0.0433)
H_4	(0.1906, 0.0468, 0.0371)	(0.2219, 0.0510, 0.0462)	(0.1560, 0.0415, 0.0478)	(0.1566, 0.0407, 0.0467)	(0.1382, 0.0413, 0.0487)	(0.1172, 0.0358, 0.0423)
H_5	(0.1725, 0.0450, 0.0365)	(0.1816, 0.0464, 0.0434)	(0.1932, 0.0465, 0.0530)	(0.1566, 0.0407, .0467)	(0.1251, 0.0391, 0.0465)	(0.1172, 0.0358, 0.0423)
H_6	(0.1603, 0.0436, 0.0362)	(0.1643, 0.0444, 0.0422)	(0.2259, 0.0511, 0.0388)	(0.1484, 0.0394, 0.0456)	(0.1690, 0.0465, 0.0537)	(0.1367, 0.0390, 0.0456)
H_7	(0.233, 0.0512, 0.0384)	(0.2017, 0.0486, 0.0448)	(0.1466, 0.0401, 0.0466)	(0.1159, 0.0346, 0.0406)	(0.1251, 0.0391, 0.0465)	(0.1114, 0.0349, 0.0413)
H_8	(0.2511, 0.053, 0.039)	(0.1816, 0.0464, 0.0434)	(0.1327, 0.0383, 0.0447)	(0.1220, 0.0355, 0.0415)	(0.1251, 0.0391, 0.0465)	(0.1817, 0.0466, 0.0462)
H_9	(0.1906, 0.0468, 0.0371)	(0.1442, 0.0422, 0.0407)	(0.1630, 0.0424, 0.0488)	(0.1281, 0.0365, 0.0424)	(0.1163, 0.0377, 0.0450)	(0.1895, 0.0479, 0.0384)

Table 4: Closeness coefficient

	$\tilde{d}_j^+$	$\tilde{d}_j^-$
H_1	5.0133	1.2269
H_2	4.9483	1.3175
H_3	5.0252	1.2457
H_4	5.1481	1.1149
H_5	5.1806	1.0804
H_6	5.1274	1.1356
H_7	5.1906	1.0628
H_8	5.1352	1.1249
H_9	5.1957	1.0579

Table 5: Ranking order

	R_i	Rank
H_1	0.19661	3
H_2	0.21027	1
H_3	0.19865	2
H_4	0.17802	6
H_5	0.17256	7
H_6	0.18133	4
H_7	0.16996	8
H_8	0.17969	5
H_9	0.16917	9

These data and the vectors of corresponding weight of each criteria, the normalized fuzzy decision matrix and weighted normalized fuzzy decision matrix are given in Table 1, Table 2, and Table 3, respectively. The closeness coefficients, which are defined to determine the ranking order of all alternatives by calculating the distance of both the "fuzzy positive-ideal solution" and the "fuzzy negative-ideal solution" are given in Table 4. According to the closeness coefficient, ranking the preference order of these alternatives is in Table 5. The proposed approach presented in this paper can be applied in many areas of management and decision making problems.

6. Conclusion

In this paper, we will provide a thorough, systematic review of the existing fuzzy MADM methods. Theoretical background as well as the algorithms is presented for each method. The normalized fuzzy decision matrix is calculated by using the concept of α cuts. Here, we are considering the distance of an positive ideal solution and negative ideal solution. i.e. the less distance from the fuzzy ideal solution and the more distance from the fuzzy negative ideal solution.

Future Research

The existing fuzzy MADM methods are complex and difficult to apply in most large size real-world problems. A good and simple method which is conceptually easy to understand and practically capable of solving real-world problems is desirable. The proposed new method to fuzzy MADM problem solving is an attempt towards that goal.

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