

Resource Allocation In Project Scheduling Application of AHP

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Abstract

This paper presents the Analytical Hierarchy Process (AHP) as a potential method for estimating the weightages for the multiple resources. Project network is taken as an example to explain. A hierarchical structure is constructed for the resources involved in the project network, estimating the weighted sum of the resources for each activity, then ranking the activities based on the weighted sum and scheduling the activities.

Keywords: Analytical Hierarchy process; AHP; Project Network, weighted sum

Introduction

The project schedule is the tool that communicates what work needs to be performed, which resources of the organization will perform the work and the timeframes in which that work needs to be performed. The project schedule should reflect all of the work associated with delivering the project on time. Without a full and complete schedule, the project manager will be unable to communicate the complete effort, in terms of cost and resources, necessary to deliver the project.

Resource allocation is used to assign the available resources in an economic way. It is part of resource management. In project management, resource allocation is the scheduling of activities and the resources required by those activities while taking into consideration both the resource availability and the project time [18].

An **integer programming** problem is a mathematical optimization or feasibility program in which some or all of the variables are restricted to be integers. RCPSP is NP-hard optimization problem [5]. A special case, 0-1 integer linear programming, in which unknowns are binary. Mixed integer programming has many applications in industrial production, including job-shop modelling. A possible objective is to maximize the total production, without exceeding the available resources. In some

cases, this can be expressed in terms of a linear program, but variables must be constrained to be integer. The (RCPSP) consists in scheduling a set of activities linked by precedence relations in order to minimize project duration, while satisfying precedence and resource constraints [23].

The completion time under no resource constraints is less than when constraints are imposed. Optimal solution to this problem are very difficult to obtain, especially for large-scale networks. One way of approaching this problem is to use heuristic techniques. A heuristic is a set of decision rules that might lead to an optimum solution but does not guarantee optimality. In other words, the heuristic solution may result in an optimum solution for some problems and may not for other problems.

Several heuristic methods have been proposed to treat this problem in different contexts. Elsayed [9] developed a heuristic algorithm for assigning resources to activities such that project duration is minimized. The criterion for allocating a single type of resource to the activities in the network is based on ROT (resource over time), which is calculated as the maximum ROT value that an activity controls through the network on any one path.

Brook's [6] developed a heuristic algorithm ACTIM (activity time) similar to those under ROT except that the ranking of the activities in the table is dependent on the ACTIM value of an activity. The ACTIM value of an activity is calculated as the maximum time that the activity controls through the network on any one path.

Bedworth [2] developed a heuristic algorithm ACTRES (activity resources) similar to those of ACTIM with the exception that the ranking of the activities is based on their ACTRES values. The ACTRES value of an activity is calculated by multiplying each activity's time by its resource and then finding the maximum ACTRES that an activity controls through the network on any path.

Mason [17] suggested a heuristic algorithm TIMRES (time and resource combination) which is a combination of ACTIM and ACTRES. The values of TIMRES of an activity is calculated as the sum of ACTIM and ACTRES values of this activity.

Whitehouse and Brown [31] developed a heuristic algorithm GENRES by modifying the TIMRES criterion to obtain the least possible project completion time. The project scheduling can be generated using combination of ACTIM and ACTRES which are not equally weighted

Nasr [19] mentioned a criteria TMROS which is an improvement in the criteria: ACROS and ACTIM. The ACROS criterion of an activity is calculated as the maximum resource value it controls through the network on any path. The method of calculating ACROS for each activity is similar to that used in calculating ACTIM, but resource values are used instead of time values.

Neumann and Zhan [20] proposed the efficient priority-rule heuristics are presented for solving the resource-constrained project-scheduling problem. Hartmann [11] proposed the branch and bound method for solving the (RCPSP). Shue and Zamani [27] proposed the Admissible heuristic Search algorithm for solving the (RCPSP). Lee et al. [15] proposed a novel market mechanism called precedence cost tatonnement (P-TATO), which solves individual resource-constrained local resource scheduling in an optimal way and searches for a precedence conflict-free scheduling

through a tatonnement type procedure. Herroelen [13] proposed a generic hierarchical project planning and control framework that serves to position the various project planning procedures and discuss important research opportunities, the exploration of which may help to close the theory-practice gap. Kolisch and Hartmann [14] proposed the forward-backward improvement technique for the (RCPSp). Banaszak and Zaremba [1] proposed a heuristic method for process planning and scheduling. The method is based on critical path approach and the branch and bound search scheme. Hasgul et al. [12] proposed an agent-based architecture for (RCPSp). Oddi et al. [21] proposed an iterative flatterring search (IFS) a met-heuristic for solving (RCPSp).

Prior research in evaluating competing system proposals have ranged from strictly quantitative, measurable considerations such as net present value, return on investment and pay-back period, to more qualitative, intangible factors, including project risk and the project's relationship with strategic needs and the organizations culture. The problem of project selection is the consideration of both qualitative and the quantitative aspects of a project. In a situation where all aspects of the system are considered simultaneously, there can be a tendency to weight the quantitative more heavily than the qualitative [28].

AHP is an easy and powerful technique, and is especially suited for problems involving both qualitative and quantitative criteria [30]. The methodology of AHP attempts to analyze the impacts of elements in the lowest levels on the overall objective or focus of a hierarchy. The strength of this approach is that it organizes tangible and intangible factors in a systematic way and provides a structured yet relatively simple solution to the problem [25].

Multiple Resource Project Scheduling

A traditional resource constrained project scheduling problem is considered to be an activity which is subjected to technological precedence constraints (i.e. An activity can start only if all its predecessor activities have been completed) and which cannot be interrupted once begun (i.e.no preemption is allowed). In this problem an activity can be performed in exactly one or more than one combination of duration and resource requirements. For any activity once initialized in a specific mode, it must be fixed without changing its mode until it is completed. Resources are available per period in a constant amount. A traditional resource constrained problem is considered to be unique in advance, in which every activity is completed successfully with probability one [16].

The Analytical Hierarchy Process (AHP)

A hierarchy of the problem is structured to encompass the basic elements. The objective is to derive priorities on the elements in the last level that best reflect their relative impact on the focus of the hierarchy. To apply the principle of comparative judgments, a matrix is set up to carry out pair wise comparisons of relative importance of the elements in the second level with respect to the overall focus of the first level. A number of criticisms have been launched at AHP over the years. Watson

and freeling [29] said that in order to elicit the weights of the criteria by means of a ratio scale by asking the decision-makers meaningless questions. Belton and Gear [3] and Dyer [7] pointed out that this method can suffer from rank reversal (an alternative chosen as the best over a set of X is not chosen when same alternative, perhaps an unimportant one, is executed from X). Belton and Gear [4] and Dyer and Wendel [8] attacked the AHP on the lack of firm theoretical basis. Harker and vergas [10] and Perez [22] discussed these major criticisms and proved that these are not valid with theoretical work and examples. Satty [24, 26] developed the following steps for AHP:

1. Define the problem and determine the objectives.
2. Structure the hierarchy from the top through the intermediate levels to the lowest levels which successfully contains the list of alternatives.
3. Construct a set of pair wise comparison matrices (size $n \times n$) for each of the lower levels. With one matrix for each element in the level immediately above by using the relative scale measurement shown in Table 1. The pair wise comparisons are done in terms of which an element dominates another. These judgments are then expressed as integers.
4. There are $n*(n-1)/2$ judgments required to develop the set of matrices in step 3. Reciprocals are automatically assigned in each pair-wise comparison.
5. Hierarchical synthesis is now used to weight the eigenvectors by the weights of the criteria and the sum is taken over all weighted eigenvector entries corresponding to those in the next lower level of the hierarchy.
6. Having made all the pair-wise comparisons and entered the data, the consistency is determined using the given value. W is solved. The consistency index, CI is calculated as follows: $CI = (\lambda_{\max} - n)/(n-1)$, where n is the matrix size. Judgment consistency can be checked by taking the consistency ratio (CR) of CI with the appropriate value in table 2. The CR is acceptable, if it does not exceed 0.10. If it is more, the judgment matrix is inconsistent. To obtain a consistent matrix, judgment should be reviewed and improved.
7. Step 3-6 are performed for all levels in the hierarchy.

Table 1: Pair-wise comparison scale for AHP preferences [24, 26]

Numerical Rating	Verbal judgments of preferences
9	Extremely preferred
8	Very strongly to extremely
7	Very strongly preferred
6	Strongly to very strongly
5	Strongly preferred
4	Moderately to strongly
3	Moderately preferred
2	Equally to moderate
1	Equally preferred

Let pair wise comparison matrix be denoted by A_1 the principle vector be denoted as A_2 . Then $A_3=A_1 * A_2$ and $A_4= A_3/A_2$.

Illustrative Example

Table 2:

Activity	Duration (days)	Predecessors	Resource Requirements per day					
			R ₁	R ₂	R ₃	R ₄	R ₅	R ₆
A	6	-	5	2	2	2	7	4
B	3	-	3	5	2	3	9	6
C	4	A	2	4	4	2	3	1
D	6	-	5	4	3	5	5	4
E	7	A,B	3	5	2	3	8	0
F	5	C	4	1	4	9	2	5
G	2	D	4	1	4	3	9	8
H	2	A,B	5	5	4	0	9	1
I	2	G,H	3	2	4	3	4	2
J	6	F	1	5	4	6	7	3
K	1	C,E	3	3	2	4	5	1
L	2	E,G,H	3	2	2	8	3	4
M	4	I,K	2	2	2	2	4	8
N	2	F,L	1	4	4	3	4	1
O	3	L	5	5	4	6	2	3
P	5	J,M,N	3	2	3	4	7	8
Q	8	O	4	5	4	2	3	4
R	2	D,O	5	3	3	3	7	8
S	6	P,R	2	4	6	2	3	4
T	2	Q	1	6	2	7	5	2
Daily Resource Limit			7	10	10	16	18	13

Here Table 2 shows the example taken for illustration. Table 2 gives the information about the activities, their duration, precedence relationship of each activity and the resources required for each activity.

Solution Procedure:

Step1: developing the pair-wise comparison matrix for the resources required for the activities.

Table 3: Pair-wise comparison matrix

A1							
	R1	R2	R3	R4	R5	R6	
R1	1	0.333333	0.333333	0.2	0.142857	0.2	
R2	3	1	1	0.333333	0.2	0.333333	
R3	3	1	1	0.333333	0.2	0.333333	
R4	5	3	3	1	0.333333	1	
R5	7	5	5	3	1	1	
R6	5	3	3	1	1	1	
SUM	24	13.33333	13.33333	5.866667	2.87619	3.866667	

Step2: Developing the normalized comparison matrix for the pair-wise comparison matrix by dividing the each element in the column by the sum of that particular column.

Table 4:

	R1	R2	R3	R4	R5	R6	SUM	A2
R1	0.041667	0.025	0.025	0.034091	0.049669	0.051724	0.227151	0.037858
R2	0.125	0.075	0.075	0.056818	0.069536	0.086207	0.487562	0.08126
R3	0.125	0.075	0.075	0.056818	0.069536	0.086207	0.487562	0.08126
R4	0.208333	0.225	0.225	0.170455	0.115894	0.258621	1.203303	0.20055
R5	0.291667	0.375	0.375	0.511364	0.347682	0.258621	2.159333	0.359889
R6	0.208333	0.225	0.225	0.170455	0.347682	0.258621	1.435091	0.239182

Step3: Developing the matrix A_3 and A_4 by the formulae $A_3=A_1 \times A_2$ and $A_4=A_3/A_2$

Table 5:

A1*A2=A3							
	R1	R2	R3	R4	R5	R6	A3
R1	0.037858	0.027087	0.027087	0.04011	0.051413	0.047836	0.231391
R2	0.113575	0.08126	0.08126	0.06685	0.071978	0.079727	0.494651
R3	0.113575	0.08126	0.08126	0.06685	0.071978	0.079727	0.494651
R4	0.189292	0.243781	0.243781	0.20055	0.119963	0.239182	1.236549
R5	0.265009	0.406301	0.406301	0.601651	0.359889	0.239182	2.278333
R6	0.189292	0.243781	0.243781	0.20055	0.359889	0.239182	1.476475

Table 6: $A_4 = A_3/A_2$

A3	A2	A4
0.231391	0.037858	6.112009
0.494651	0.08126	6.087244
0.494651	0.08126	6.087244
1.236549	0.20055	6.165775
2.278333	0.359889	6.330659
1.476475	0.239182	6.173023

Step 4: Estimating the eigen-vector λ_{\max} by the formula $\lambda_{\max} = \text{Sum of } A_4 / \text{no. of resources}$, estimating the value of CI and CR.

$$\lambda_{\max} = 36.9525/6 = 6.1588$$

$$CI = (\lambda_{\max} - n) / (n - 1)$$

$$= (6.1588 - 6) / (6 - 1)$$

$$= 0.03176$$

$$CR = CI / \text{Random Index}$$

$$= 0.03176 / 1.24$$

$$= 0.0256$$

$$CR > 0.1 \text{ Accepted}$$

As the value of CR is less than 0.1, the judgments are acceptable. Similarly the pair-wise comparison matrices and priority vector for the remaining criteria can be found is shown in Tables 3-6.

Activity	Duration (days)	Predecessors	Resource Requirements per day					
			R ₁	R ₂	R ₃	R ₄	R ₅	R ₆
			W ₁	W ₂	W ₃	W ₄	W ₅	W ₆
			0.0376	0.0811	0.0811	0.2005	0.2005	0.2394
A	6	-	5	2	2	2	7	4
B	3	-	3	5	2	3	9	6
C	4	A	2	4	4	2	3	1
D	6	-	5	4	3	5	5	4
E	7	A,B	3	5	2	3	8	0
F	5	C	4	1	4	9	2	5
G	2	D	4	1	4	3	9	8
H	2	A,B	5	5	4	0	9	1
I	2	G,H	3	2	4	3	4	2
J	6	F	1	5	4	6	7	3
K	1	C,E	3	3	2	4	5	1
L	2	E,G,H	3	2	2	8	3	4
M	4	I,K	2	2	2	2	4	8
N	2	F,L	1	4	4	3	4	1

O	3	L	5	5	4	6	2	3
P	5	J,M,N	3	2	3	4	7	8
Q	8	O	4	5	4	2	3	4
R	2	D,O	5	3	3	3	7	8
S	6	P,R	2	4	6	2	3	4
T	2	Q	1	6	2	7	5	2
Daily Resource Limit			7	10	10	16	18	13

Step 5: Estimating the weighted sum for each activity using the weightages of the resources for each activity using the formula and ranking the activities according to the weighted sum of the activities.

$$Ws = \sum_{j=1}^k w_j \left(\frac{ij - \min_j}{\max_j - \min_j} \right)$$

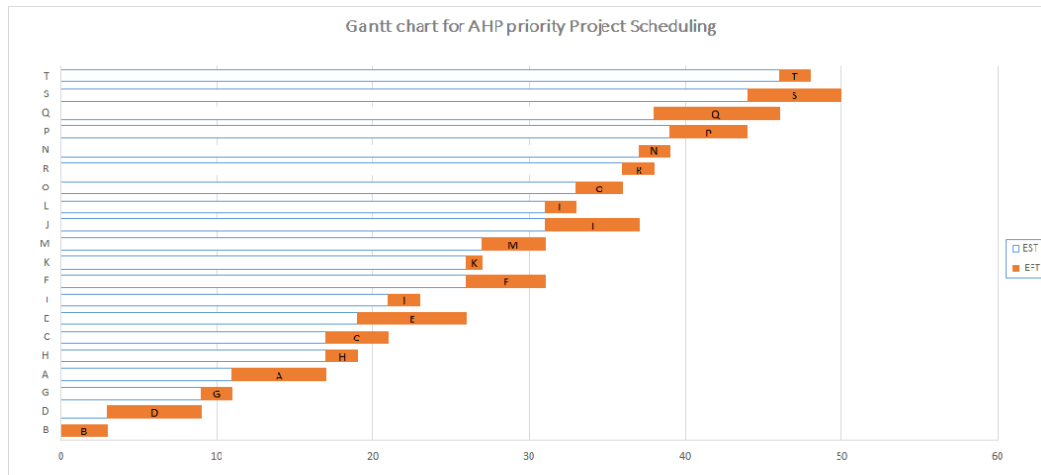
Where i_j is the value of the resource required for the activity j ; \max_j and \min_j are the maximum and minimum values of criterion j among all activities.

Activity	Duration (days)	Predecessors	Resource Requirements per day						Weighted sum W_s	Rank
			R ₁	R ₂	R ₃	R ₄	R ₅	R ₆		
			W ₁	W ₂	W ₃	W ₄	W ₅	W ₆		
			0.0376	0.0811	0.0811	0.2005	0.2005	0.2394		
A	6	-	5	2	2	2	7	4	0.4754	8
B	3	-	3	5	2	3	9	6	0.6903	2
C	4	A	2	4	4	2	3	1	0.2245	20
D	6	-	5	4	3	5	5	4	0.492	7
E	7	A,B	3	5	2	3	8	0	0.4754	9
F	5	C	4	1	4	9	2	5	0.4188	11
G	2	D	4	1	4	3	9	8	0.7351	1
H	2	A,B	5	5	4	0	9	1	0.5331	6
I	2	G,H	3	2	4	3	4	2	0.3051	18
J	6	F	1	5	4	6	7	3	0.5862	5
K	1	C,E	3	3	2	4	5	1	0.3246	17
L	2	E,G,H	3	2	2	8	3	4	0.3844	13
M	4	I,K	2	2	2	2	4	8	0.4124	12
N	2	F,L	1	4	4	3	4	1	0.2888	19
O	3	L	5	5	4	6	2	3	0.3706	14
P	5	J,M,N	3	2	3	4	7	8	0.6411	4
Q	8	O	4	5	4	2	3	4	0.3493	16
R	2	D,O	5	3	3	3	7	8	0.6538	3

S	6	P,R	2	4	6	2	3	4	0.3548	15
T	2	Q	1	6	2	7	5	2	0.4512	10
Daily Resource Limit			7	10	10	16	18	13		

Step 6: Arranging the activities as per their rank in the ascending order and scheduling the activities as per the precedence relationship by considering the weighted sum ranking.

Rank	Activity	Predecessor	Duration (days)	R ₁	R ₂	R ₃	R ₄	R ₅	R ₆
1	G	D	2	4	1	4	3	9	8
2	B	-	3	3	5	2	3	9	6
3	R	D,O	2	5	3	3	3	7	8
4	P	J,M,N	5	3	2	3	4	7	8
5	J	F	6	1	5	4	6	7	3
6	H	A,B	2	5	5	4	0	9	1
7	D	-	6	5	4	3	5	5	4
8	A	-	6	5	2	2	2	7	4
9	E	A,B	7	3	5	2	3	8	0
10	T	Q	2	1	6	2	7	5	2
11	F	C	5	4	1	4	9	2	5
12	M	I,K	4	2	2	2	2	4	8
13	L	E,G,H	2	3	2	2	8	3	4
14	O	L	3	5	5	4	6	2	3
15	S	P,R	6	2	4	6	2	3	4
16	Q	O	8	4	5	4	2	3	4
17	K	C,E	1	3	3	2	4	5	1
18	I	G,H	2	3	2	4	3	4	2
19	N	F,L	2	1	4	4	3	4	1
20	C	A	4	2	4	4	2	3	1



The Critical path is B-D-G-A-H-E-K-M-I-O-R-P-S and the project duration is 50 days.

Conclusions

The discerning ability and methods to make sound decisions are involved in the complex decision making situation in the project management. The project scheduling with the help of Analytical Hierarchy Process in developing the weightages for the activities are done in this paper and the corresponding project schedule is shown in this paper. This paper provides the basis for applications of AHP weightages in the project scheduling. The final project scheduling and the project duration are obtained from the Gantt chart.

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