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A Parallel Fast Eikonal Equation Solver Using Efficient Scanning

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Abstract

An eikonal equation is a nonlinear Partial differential equation (PDE) for finding the appropriate solution for wave equation which contains multiple unknown values. Fast sweeping method (FSM) is used to solve the eikonal equation in parallel by changing the sweeping direction and by distributing the grid points on the available processors. To improve the efficiency, progressive scanning approach and central differencing scheme are used to discretize the computational domain for updating and converging the values in all directions. Advantages of using these approaches are (1) Efficiency can be improved without increasing the complexity of the algorithm (2) Nodes in each level are divided among the available threads in a balanced manner which will provide better performance and (3) Provides second order accurate values for every node.

Index Terms— Eikonal equation, Fast sweeping method, Progressive scanning, Central differencing scheme, parallel implementation.

I. Introduction

The Eikonal equation is used to find a better solution for applications like wave propagation, image processing [7], optimal control and path planning [4,8]. The eikonal equation for n dimensions is given as:

$$|\nabla v(x)| = F(x) \quad \text{for } x \in \Omega \subset R^n,$$

$$v(x) = g(X) \text{ for } x \in \Gamma \subset \Omega \qquad (1)$$

Where v represents the unknown, f is a given inverse velocity field and g represents the values at an irregular interface. In parallel computing large problems are divided into smaller ones and computations are carried out in parallel. The parallel method for eikonal equation is developed due to the increase in need for efficiency, computational resources and computational speed.

The Fast marching method (FMM) is similar to Dijkstra's algorithm, chooses the nodes with least distance for propagating the information and the propagation takes place in the front order. It uses a sorting algorithm to update the nodes giving the algorithmic complexity of O (N log N). The Fast Iterative Method (FIM) uses heterogeneous structure for sorting and allows many nodes to get updated simultaneously, but may not show optimal efficiency for all the problems. The fast sweeping method (FSM) alternates the direction of sweeping in each iteration for propagating the information over the computational domain and uses an upwind differencing scheme for updating the nodes. In this paper, we propose a progressive scanning approach and central differencing scheme for solving the Eikonal equation that can be implemented easily and accomplishes an approximate speedup of O (N/T), where T represents the number of threads.

II. RELATED WORKS

Sethian [3] proposed a method called "A fast marching level set method" for propagating the solution where the computational domain is divided into grids. In this method, the initial value is taken as *Accepted*, the neighboring nodes of the interface are taken as *trials* from this set and the nodes with shortest path are added to the set *considered*. Then the values of the nodes in the set *considered* is updated using upwind differencing scheme and it is compared with the initial value. If the new value is better than the initial one, then the new value is added to the set *Accepted* and removed from the set *considered*. This method also uses a sorting algorithm for finding the nodes with the shortest distance which increases the complexity of finding a better solution.

The Fast sweeping method (FSM) proposed by Zhao [1] alternates the direction of sweeping in each iteration for updating the nodes in the computational domain. The proposed method takes 2^n sweeps for R^n and the sweeping directions are processed serially by a single thread.

Zhao [2] proposed an approach to parallelize the FSM implementation by assigning each sweeping direction to a separate thread. In this method all the nodes are updated simultaneously and each sub-domain is communicated through the interface at the end of every iteration. After each iteration, four solutions from each sub-domain are obtained and the solution with the minimum value is selected. For the next iteration the value is reinitialized and the process is repeated until all the nodes are propagated. This method takes high computational time and the number of iterations is also increased when compared to the serial FSM.

Jeong [9] proposed a noval method for solving an Eikonal equation in parallel architecture named as Fast Iterative Method (FIM). The FIM uses the master- slave model for solving the equation. The master divides the domain into sub-domains and allocate it to slave processes. As soon as one slave completed its work it sends back the result to the master and continue to do other work allocated by the master. If the computed node is a ghost node, then the value is communicated with the neighboring processes. Even though FIM is faster than FMM it does not follow a particular order for updating the node.

Miles Detrixhe Et al [6] proposed Cuthill-Mckee ordering in fast sweeping method (FSM) that converts the sparse matrix into band matrix for propagating the information. First the axes are rotated in a particular direction and then the nodes in each level are updated in parallel. This method repeats the iteration for several times and at the end of each iteration, it initializes a new value. It continues until the information propagated is within a tolerance rate. This method suffers from performance degradation as the nodes are not equally divided among the levels.

III. ARCHITECTURE

The central differencing scheme and a progressive scanning approach are applied in FSM to solve the Eikonal equation efficiently. The overall process of the proposed system is shown in Figure 1.

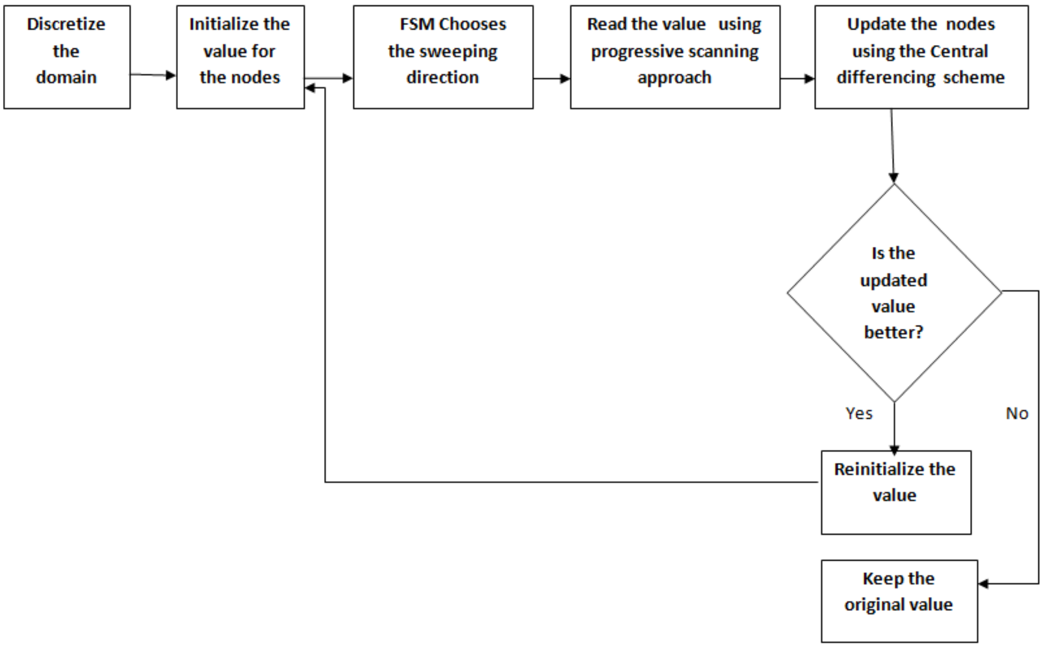

Figure 1: Work flow diagram of the FSM using central differencing scheme.

IV. PROPOSED METHOD

The proposed Fast Sweeping Method (FSM) uses progressive scanning approach for splitting the nodes into levels equally in the computational domain. Each row is considered as separate levels. To update the nodes in different directions, axes are rotated in a particular direction and then update the nodes in parallel. It chooses the sweeping direction in a manner by which the group of nodes are updated simultaneously. The default choice for sweeping is given below:

(i) $x = 1 : K, y = 1 : M$
(ii) $x = K : 1, y = 1 : M$
(iii) $x = K : 1, y = M : 1$
(iv) $x = 1 : K, y = M :1$ (2)

After choosing the sweeping direction, the nodes are divided into levels. All the threads enters into levels sequentially as given in the progressive scanning approach which is shown in Figure 2, and the nodes in each level are divided equally among the threads. The nodes can be updated in parallel using central differencing scheme [5] as shown below.

$$new\_v_{i,j} = q(c\_v_{i,j}) + r(old\_v_{i,j}) + b(c\_v_{i+1,j} +$$

$$c\_v_{i-1,j} + c\_v_{i,j+1} + c\_v_{i,y-1} - 4v_{i,j}) \quad (3)$$

Where $c\_u_{i,j}$ is the current value and the following notations are used:

$q = 2- \mu dt$, $r = -1+ \mu dt$ and $b = c^2(dt)^2 /(dx)^2$

To improve the accuracy, the process is repeated in a specific interval of time until it reaches a desired tolerance rate.

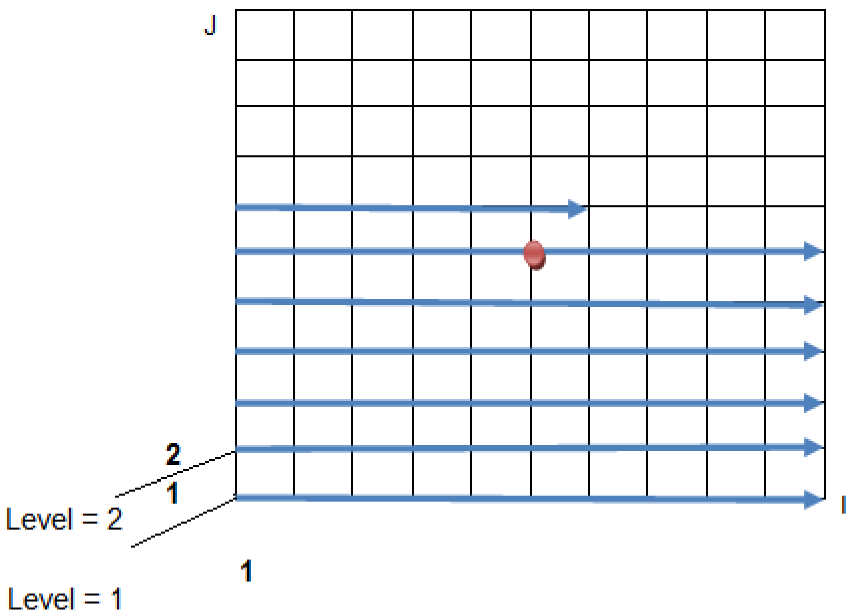

Figure 2: Example of progressive scanning approach for a 2D problem.

Algorithm 1: Proposed fast sweeping method

Input: Number of grid points in the computational domain.
Output: Updated nodes.
1. initialize the node value (v_{ij})
2. for (t = 1: maximum) do
3. for order = 1: 2^2
4. for i = 1: K do
5. Level = i
6. Parallel for j = 1: M do
7. Update the node (v_{ij})

The initialization given in step (1) is the same as to original Fast Sweeping Method [3]. This step is executed in parallel by all the threads. Next, sweeping direction is chosen for node updation. In FSM R^n has 2^n sweeps. The number of levels is equivalent to the total number of rows in the domain. The notation "parallel for" specifies that nodes in each level are divided among the available threads. The nodes are updated by using equation (3). This process is repeated for a specific number of times.

Advantages
The Fast Sweeping algorithm uses progressive scanning approach which offers the following advantages: (1) Each level has an equal number of nodes that are updated simultaneously by the available threads which will reduce the performance degradation and computational time. (2) The central differencing scheme provides better accuracy compared to the upwind scheme (3) This approach is suitable for any number of threads and no need for domain decomposition.

V. EXPERIMENTAL RESULT
In this section the efficiency of the proposed work is compared with the detrixhe et al cuthill-Mckee approach by using the following example. Figure 3 shows the initialization of a circle at center. Here a 200 x 200 velocity grids are used with dx = 0.1km, resulting in a 20 x 20 km model. In this example the nodes are updated using a central differencing scheme which gives better results.

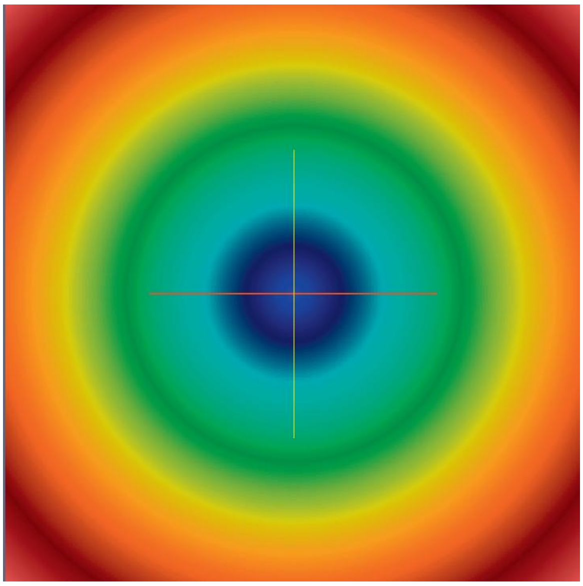

Figure 3: Center test

The computational time of the proposed method using progressive scanning approach for this example is lesser than that of detrixhe et al method which is shown in Figure 4.

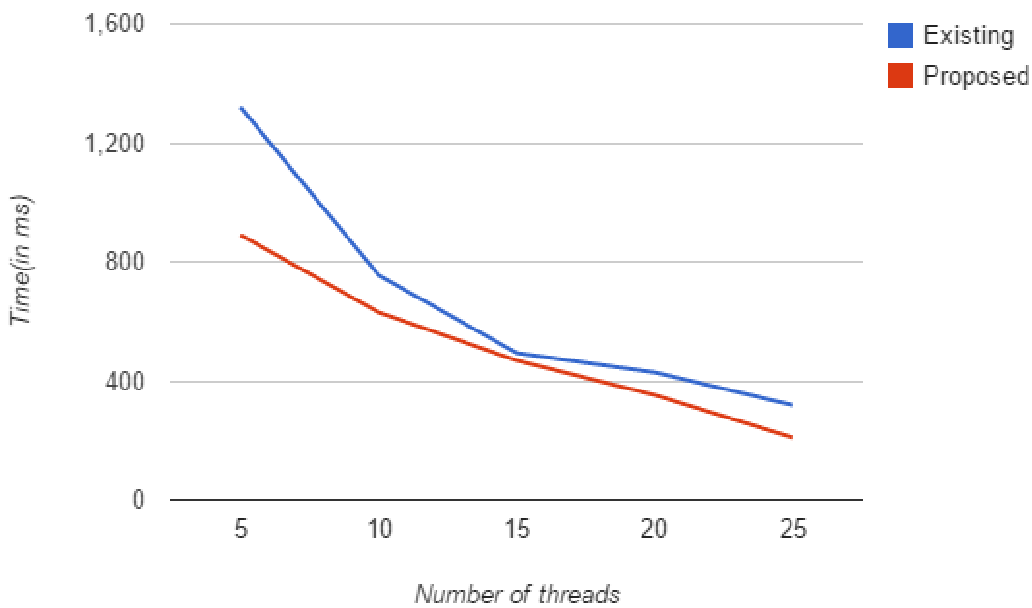

Figure 4: In this fig the total number of nodes is $N=200^2$.Comparison of Computational time of the proposed Progressive Scanning Approach and the existing Cuthill Ordering.

VI. CONCLUSION

In this paper, a progressive scanning approach is proposed which uses the Fast Sweeping Method (FSM) to read the values. The number of levels is equal to N and the nodes in this level are updated by using a central differencing scheme which provides better accuracy than the upwind scheme. This approach is effective for any number of threads by balancing the loads among the threads. Thus the proposed work is efficient compared to the Miles detrixhe et al parallel fast sweeping method using cuthill-McKee ordering.In future, it is planned to concentrate on the speculation of this methodology and the investigation of its proficiency for problems with multiple dimensions.

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