

About A Method Of Taking Bulk Viscosity Into Account In The Problems Of Computational Fluid Dynamics

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Abstract

This article offers an original method of taking bulk viscosity into account in the problems of computational fluid dynamics for liquid or gas. It allows obtaining numerical solutions of Navier-Stokes equations containing summands with bulk viscosity by means of CFD-codes. The efficiency of the offered method was checked with the aid of a test example with use of ANSYS Fluent software.

Keywords: compressible fluid, bulk viscosity, computational fluid dynamics, ANSYS

1 INTRODUCTION

It is known that hydrodynamics of a viscous fluid is generally influenced not only by shear viscosity but also by bulk viscosity which characterizes loss of momentum at time of bulk deformation. The majority of practical problems ignore bulk viscosity influence on a fluid flow due to incompressibility of the latter. Nevertheless in the problems which take into account a fluid incompressibility bulk capacity is also often excluded from motion equations by referring to the Stokes hypothesis assuming “thermodynamic” and “mechanical” pressures to be equal. However bulk viscosity should be taken into account in some cases like investigations of supersonic flows of viscous media, fluids flow in thin layers under extreme pressures and other.

Equivalently to shear viscosity bulk viscosity is characterized by a coefficient which generally depends on temperature, pressure and determines loss of momentum at time of volume flow of the medium. The Mandelstam-Leontovich relaxation theory of ultrasound wave absorption in fluid detailed description of which is given in [1] offers an analytical dependence for calculation of the bulk viscosity coefficient. How-

ever for practical use of the obtained formulas the time of medium relaxation under bulk deformation conditions which is difficult to be calculated precisely should be known. Other approach to calculation of the bulk viscosity coefficient suggests use of the correlations of Green-Kubo, Heyes and Hoover summary and comparison of which is given in [2]. However these correlations have some disadvantages such as the necessity to perform molecular dynamics simulations for calculation of the correlative function of pressure with the aid of corresponding programs and considerable computational costs.

There are also experimental methods of determining the bulk viscosity coefficient. For example a general approach to experimental bulk viscosity determination based on uniform heating (cooling) of fluid and measuring of its mechanical pressure on a membrane is described in [3]. The bulk viscosity coefficient in this case is equal to the ratio of the difference between thermodynamic and mechanical pressure to the expansion rate of fluid. A similar method was offered by [4] where the bulk viscosity coefficient was calculated depending on the pressure and the rate of change of bulk deformation of fluid at two different compression rates.

Theoretical calculations and experimental data obtained during determining of the bulk viscosity coefficient for various gases and fluids evidence that its values in many cases may exceed the value of shear viscosity coefficient by one or several digits. The references to the sources with the corresponding investigations may be found in [5], in addition this source states that, for example, in case of a lubricant film bulk viscosity should be taken into consideration if all three linear dimensions of the film are comparable. In this case even given the weak compressibility it is impossible to disregard the summands containing the bulk viscosity coefficient in the motion and energy equations.

At the present time many computational fluid dynamics problems are being solved by means of the specialized software products (CFD-codes) for example ANSYS Fluent, CFX, Star-CD, FlowVision which realize a numerical solution of model equations. However these programs use an expression for a viscous stress tensor free of a summand with bulk viscosity. In order to resolve this problem it is necessary to create such summands artificially in the form of additional source members of equations. The method of taking bulk viscosity into account offered in this article consists in forming such artificial summands and their following addition to model equations. Along with that it is necessary to establish the correlation for the bulk viscosity coefficient suitable for practical calculations within CFD-codes.

2 TAKING BULK VISCOSITY INTO CONSIDERATION IN A MOTION EQUATION

In the most general case the viscous stress tensor may be laid down as follows:

$$\mathbf{\Pi} = -p\mathbf{I} + 2\eta\mathbf{D} + \zeta(\nabla \cdot \bar{\mathbf{V}})\mathbf{I}, \quad (1)$$

where \mathbf{I} – an identity tensor, \mathbf{D} – a deformation velocity tensor, $\bar{\mathbf{V}}$ – a velocity vec-

tor, η, ζ – coefficients of shear and bulk viscosity correspondingly, p – pressure.

At time of inserting (1) into a general equation of continuous medium motion under stress it is necessary to take into account that in our case the coefficients of shear and bulk viscosity depend on coordinates and therefore can not be placed outside the gradient sign. Consequently the equation will take the following form:

$$\rho \left(\frac{\partial \bar{V}}{\partial t} + (\bar{V} \nabla) \bar{V} \right) = -\nabla p + \nabla \left(\eta (\nabla \bar{V} + \nabla \bar{V}^T) - \frac{2}{3} \nabla \cdot \bar{V} \mathbf{I} \right) + \nabla (\zeta \nabla \cdot \bar{V} \mathbf{I}), \quad (2)$$

where ρ – fluid density.

First of all it is necessary to determine a method of practical calculation of the bulk viscosity coefficient. Analytic dependence of the bulk viscosity coefficient on pressure is not known. The Maxwell's conclusions in regard of nature of viscosity according to which the bulk viscosity coefficient is equal to the product of a bulk compression modulus of a fluid and time of medium relaxation under the conditions of bulk deformation, i.e.

$$\zeta = K \tau_\zeta, \quad (3)$$

where $K = K_0 + n(p - p_0)$ – a bulk compression modulus of a fluid, K_0, ρ_0 – a bulk compression modulus and a fluid density at the pressure of $p_0 = 101325 \text{ Pa}$ correspondingly, n – an invariable determined on the basis of experimental data, τ_ζ – time of medium relaxation under the conditions of bulk deformation, is an alternative to the coefficient calculation based on the relaxational theory of absorption of high-frequency oscillations by a fluid or on molecular dynamics simulations.

It is difficult to obtain the precise evaluation of relaxation time τ_ζ since it depends on the potential of intermolecular interactions in a fluid and can be calculated for example by means of molecular dynamics simulations software, for instance by those contained in LAMMPS software package. However this approach requires considerable computational resources and computing time. Nevertheless a number of investigations offer data on typical times of relaxation. In [6] the authors state that for viscous media this value varies from 10^{-10} to 10^{-11} sec. The work [7] marks that relaxation time grows with the growth of medium viscosity. The author is unaware of more precise relaxation time evaluations for bulk viscosity and so much of their dependence on pressure or viscosity. In order to make calculation according to the offered model we'll take a constant value of relaxation time and given that the value K in formula (3) depends on pressure the bulk viscosity coefficient will also be variable.

The last summand in the right part of the equation (2) may be specified as a source which may be given in a form of a coordinate representation:

$$S_i = \frac{\partial}{\partial x_i} \left(\zeta \frac{\partial V_k}{\partial x_k} \right) \quad (4)$$

The bulk viscosity coefficient can not be placed outside the derivative sign since it depends on pressure and pressure is a function of coordinates. In order to calculate a source summand under the equation (4) it is necessary to calculate the product of velocity vector divergence and ζ for each computation cell and then calculate the gradient of the resulting value. Calculation of the gradient within the computational domain is based on the Gauss-Green theorem according to which for a scalar value of ϕ

$$\text{grad } \phi = \frac{1}{\Omega} \sum_f \phi_f \bar{A}_f, \quad (5)$$

where ϕ_f – a magnitude of the value ϕ at a cell edge (boundary), \bar{A}_f – a vector of normal to the edge (boundary), Ω – a cell volume (or a surface in case of a two-dimensional variant).

The magnitude of a component of the velocity vector at the edge (boundary) is calculated as the average for the adjacent cells adjoining the edge (boundary). If a cell edge belongs to the computational domain boundary then the components of the velocity vector at the edge (boundary) are taken from the previous cell adjoining this edge. Divergence is determined on the basis of the resulting velocity vector components and is multiplied by the bulk viscosity coefficient calculated for a given cell according to the formula(3).

Therefore calculation of the source summands containing bulk viscosity is made according to the formula (4) with account of the correlations (3) and (5). It is convenient to formulate the similar calculation as individual modules which then will be plugged in the principal calculation in CFD-code. Thus for example ANSYS Fluent allows adding source summands through separately compiled functions (udf-function). These functions are performed for each computation cell within the common iterative process of a numerical solution search and may be anytime activated or deactivated.

3 TEST CALCULATION OF VISCOUS FLUID FLOW WITH ACCOUNT OF BULK VISCOSITY

Let's analyze a problem of viscous compressed fluid flow in a flat duct with the width many times beneath its length as an example illustrating efficiency of the offered method for taking bulk viscosity into account. Let's assume that the flow is laminar, the duct walls roughness is not taken into account. Let's adopt in the model an assumption on the flow isothermality although in reality temperature can influence on characteristics of a compressed fluid. Let's determine analysis of the effect of bulk viscosity

on the flow characteristics under various boundary and initial conditions as the test calculation purpose.

The motion equation(2) is closed by a mass balance equation for compressed medium:

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \bar{V}) = 0, \tag{6}$$

the Tait constitutive equation:

$$\left(\frac{\rho}{\rho_0}\right)^n = \frac{K}{K_0}, \tag{7}$$

and the Barus law for viscosity:

$$\eta = \eta_0 \exp[\alpha(p - p_0)], \tag{8}$$

where α – a pressure-viscosity coefficient for shear viscosity, η_0 – shear viscosity under pressure p_0 .

The materials of the work [8] evidence that the pressure-viscosity coefficient value insignificantly reduces while pressure grows therefore in a first approximation it may be taken as constant.

Let's take glycerin as an analyzed fluid, its physical properties were drawn from the data of [9] at the temperature of 20⁰C and are given in Table 1. Let's assume that the time of relaxation is equal to 10⁻¹⁰ – 10⁻⁷, provided that the pressure difference between the duct input and output is supposed to be taken as considerably high due to which the viscosity being calculated according to formula (8) may increase by 2-3 digits therefore the relaxation time will grow correspondingly. The value of pressure is set both for the flat duct input and output, where the output pressure will be equal to atmospheric. The fluid velocity on the duct walls will be taken to be equal to zero.

Table 1

Fluid	$\rho_0, \text{g/cm}^3$	$\eta_0 \cdot 10^3, \text{Pa}\cdot\text{sec}$	$\alpha \cdot 10^3, \text{MPa}^{-1}$	K_0, MPa	n
Glycerin	1.26	1480	5.8	4464	7

Numerical computation was performed with the aid of ANSYS Fluent, a coupled algorithm ensuring simultaneous solution of both motion and mass balance equations demonstrated the best rate of convergence. No significant distinctions in the

flow characteristics for the cases when bulk viscosity was and was not taken into account in the motion equations were observed as a consequence of numerical experiments with a steady-state problem statement. Such results are consistent with the theory since manifestations of bulk viscosity may be observed only during non-equilibrium processes. That's why there was applied a non-steady-state problem statement which supposes change of the input pressure according to the following law:

$$p(t) = p_0 + A \sin(\omega t). \quad (9)$$

In this case the influence of bulk viscosity on the flow characteristics may be identified by existence of a phase shift of fluctuation of pressure and of percentage change of volume which is determined by the value of the velocity vector divergence at one and the same point. Calculations of the pressure and the percentage change of volume were performed in the center of the duct medium cross-section. The following characteristics from (9) were assumed for the pressure function: $p_0 = 500$ MPa, $A = 100$ MPa, $\omega = 1000$ 1/sec. Time step made 0.0005 sec, on the whole there were calculated 30 steps.

As it can be seen from Figure 1 if bulk viscosity is not taken into account no phase shift between the fluctuations of pressure and of percentage change of volume can be observed. If on the other hand the calculations are made with account of the source summands containing bulk viscosity then the phase shift can be identified (see Figure 2).

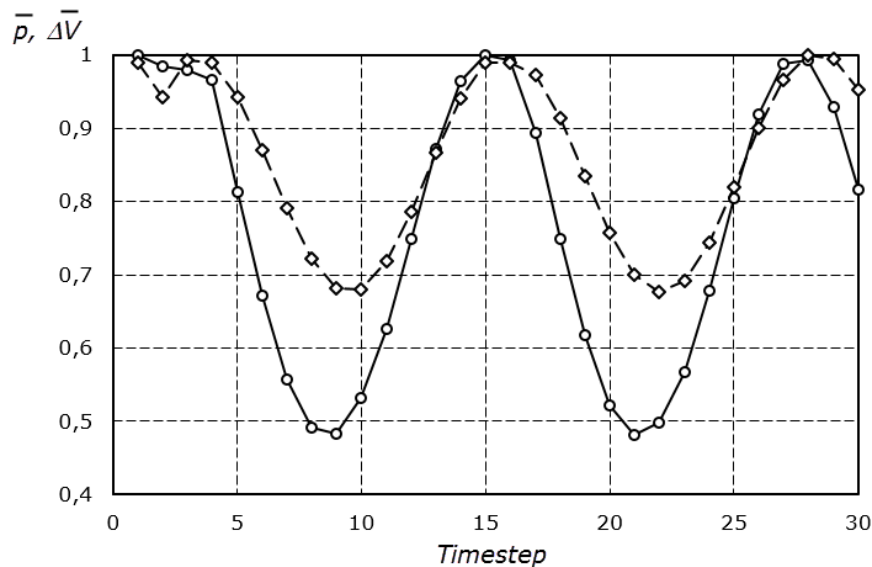


Fig. (1). Fluctuations of pressure and of percentage change of volume in a set point with no account of bulk viscosity: \circ - relative volume $\overline{\Delta V} = \Delta V / \Delta V_{\max}$, \diamond - pressure $\overline{p} = p / p_{\max}$

By comparing diagrams on Figures 1 and 2 a conclusion may be drawn that the pressure fluctuations in the both cases are nearly similar however the fluctuations of the percentage change of volume undergo a phase shift. The relaxation time influence on the nature of the periodic processes under consideration in practice appeared to be very insignificant although in theory a phase shift should increase with the relaxation time growth. Apparently it is connected with selection of the range for τ_ζ and the values of initial and amplitude pressures for the boundary condition (9). It's worth noting that considerable relaxation time increase results in the loss of stability and divergence of a numerical solution algorithm.

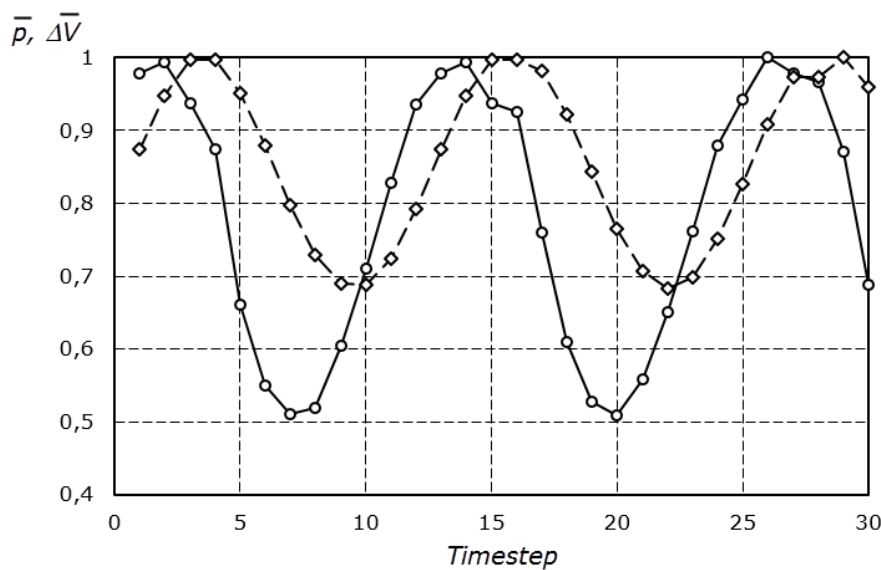


Fig. (2). Fluctuations of pressure and of percentage change of volume in a set point with account of bulk viscosity: \circ - relative volume $\bar{\Delta V} = \Delta V / \Delta V_{\max}$, \diamond - pressure $\bar{p} = p / p_{\max}$

4 FINDINGS

The offered method allows taking into account the influence of bulk viscosity on liquid or gas flow characteristics by means of adding source summands to the motion equations. The described algorithm of development of the necessary source summands for a motion equation may be realized both via ANSYS Fluent and via other popular CFD-codes. The results of test calculations of viscous fluid flow in a flat duct showed that the influence of bulk viscosity may be found only in a non-steady-state problem statement and manifests itself as a phase shift of the fluctuations of pressure and of percentage change of volume in a set point.

5 CONCLUSION

Therefore the offered method may be used for taking into account bulk viscosity in the problems of computational fluid dynamics. Since there is some kind of uncertainty while choosing the value of relaxation time of volume deformation τ_ζ the results of calculation made by this method will always contain a complementary error. Calculation accuracy may be improved for example through waving the bulk viscosity coefficient calculation according to the formula (3) and setting it as a function approximating the values of bulk viscosity (obtained by an experimental approach or by direct calculation with the help of software packages for molecular dynamics simulations) depending on pressure.

CONFLICT OF INTERESTS

The author confirms that the presented data do not contain any conflict of interests.

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