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Recurrent Wavelet Network Based Control for a Class of Delayed Uncertain Nonlinear Systems Subjected to Hysteresis Nonlinearity

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Abstract
The research presents recurrent wavelet based adaptive control (RWNC) scheme for uncertain nonlinearity based delayed systems with hysteresis. For the estimation of uncertainties present in the system and hysteresis nonlinearity effect used Self recurrent wavelet neural network (SRWNN). Deployment of hysteresis phenomenon in proposed work used Prandtl-Ishlinskii (PI) model. Proposed Controller consists of wavelet based feedback system and a robust control mechanism. The simulation study carried out for stability analysis of the proposed controller using Lyapunov-Krasovskii function and verifies the effectiveness of theoretical development.

Keywords- Adaptive control; self recurrent wavelet neural networks; time delay systems; Lyapunov - Krasovskii functional; hysteresis.

INTRODUCTION

The performance of the control system decreases due to the time delay in system dynamics. Several real time systems like aircraft, Process Control System, Manufacturing System and Communication System represent an unstable behavior due to time delay. Infinite Dimensional functional differential equation based control system are more sensitive as compared to the finite dimensional differential equation, in the presence of time delay. This is one of the major challenges in controller design and opens a new branch of research in controller design.

The control strategies [3-5] are based on the application of Lyapunov-Krasovskii functional augmented with some linear matrix inequality techniques.

Hysteresis is a commonly observed nonlinearity in physical systems, especially smart material based actuators exhibits this phenomenon. Hysteresis can be defined as a multi-valued nonlinear relationship between two variables.[6]

In the system dynamic presence of hysteresis in actuators often degrades the system performance and inserts non-smooth nonlinear dynamics in the system that can lead to undesirable inaccuracies and even instability. Due to the complexity reflected by the behavior of hysteresis affected systems, the classical control tools have been proven inefficient for the control and the regulation of such systems and it demands the augmentation of the conventional tools with some adaptive dynamics to mitigate the effect of hysteresis [2, 3]. Several researchers have addressed the issue of designing of control schemes for hysteresis based systems. Most of these schemes relay on an accurate model of hysteresis nonlinearity. Preissach model, Prandtl-Ishlinskii model is commonly used mathematical models to explain the hysteresis dynamics [6].

Various Issue has been addressed [7-9] related to controller design for delay based nonlinear systems subjected to hysteresis nonlinearity.

Feedback linearization based control schemes have been proven highly effective for the control of nonlinear systems. However, a major drawback associated with these techniques is that effective implementation of these techniques needs completely known systems dynamics [10]. This requirement often confines the applicability of these techniques to a very narrow class of nonlinear systems. A common way to relax these constraints is to augment the classic controller with some adaptive tuner like neural networks, which results in some adaptive control scheme for effectively dealing withmathematical inaccuracies or model uncertainties.

A class of Wavelet neural networks (WNN), called a feed-forward neural networks in which wavelet as an activation function, having the approximation capability of neural network and localized features of wavelet functions. The performance of WNN provides better outcomes as compared to conventional neural network in terms of convergence rate, space learning capability and frequency localization functions [12-15]. Complexity of control problems is solved by WNN having the capability of convergence and recurrence [17-21].

The applicability of WNN gets restricted only to static environmental conditions owing to its feed-forward architecture. To enhance the performance of WNN under dynamic environmental conditions can be done by including recurrence characteristics in form of architectures namely output recurrent WNN (ORWNN) or self recurrent WNN (SRWNN). Application of SRWNN as a system identification tool has been cited in [19-21].

Figure 1 shows the basic diagram of the proposed system.

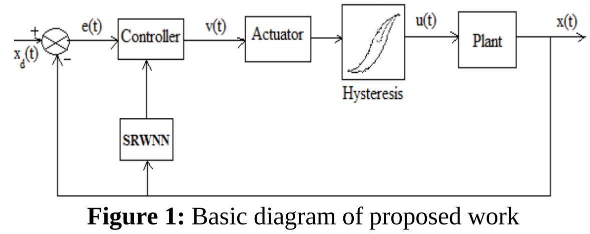


Figure 1: Basic diagram of proposed work

In this work the proposed adaptive controller (RWNC) controls the functioning of hysteresis based delayed uncertain nonlinear system (HDUNS). RWNC is based on SRWNN with the capability of system identification as well as hysteresis compensator. The stability of HDUNS computes by Lyapunov-Krasovskii function in RWNC [22, 23].

Paper organized as follows: Section II explains system preliminaries. Section III describes system formulation. Section IV deal with controller designing issues. Section V analyzed Convergence aspects of RWNC. The effectiveness of RWNC is illustrated through a simulation in section VI while work concluded in section VII.

SYSTEM PRELIMINERIES

A) Hysteresis Nonlinearity

There exist several models that represent the hysteresis behavior without giving any physical insight the system like Preissach model, Prandtl-Ishlinskii model, etc. This work utilizes Prandtl-Ishlinskii model to model the hysteresis nonlinearity present in the actuators[6,8].

Prandtl-Ishlinskii (PI) model

Prandtl-Ishlinskii model is set up by composition of play and stop operators. Suppose $E_r[u]$ are basic elastic plastic elements or stop operators for all $r \in [0, R]$.For a given density function $p(r)$, the model can be expressed as[6,8]

$$w(t) = \int_0^R p(r) E_r(u)(t)\, dr \tag{1}$$

Density function $p(r)$ is supposed to be satisfied the following condition

$$p(r) \geq 0 \text{ with } \int_0^\infty r p(r) dr < \infty$$

Thus, with such density function, this operator maps $C(t_0, \infty)$ into $C(t_0, \infty)$ i.e. Lipschitz continuous inputs will yield Lipschitz continuous outputs. In terms of play operator $F_r(u)$ the PI model can be expressed as

$$w(t) = p_0 u(t) - \int_0^R p(r) F_r(u)(t)\, dr \tag{2}$$

where $p_0 = \int_0^R p(r) dr$ is a constant, which depends on the density function. Equation(2), reflects that the hysteresis nonlinearity can be considered as the combination of a linear term and the integral term which accounts for the hysteresis. In this work hysteresis model defined by equation (2) is utilized for controller design, the hysteresis part of the (2) is treated as the disturbance and is approximated by using a wavelet network.

B) Self Recurrent Wavelet Neural Network

Regression analysis of function approximation, obtained from mother wavelet function using translation and dilation, is done by wavelet network [12].

Figure2. shows Feed-Forward WNN with self feedback wavelon layer as a modified version of SRWNN. Self feedback wave-lon layer enables the wavelet network (SRWNN) to store past information, high degree approximation of dynamic nonlinearities and more convenient for adaptive control as compared with conventional.

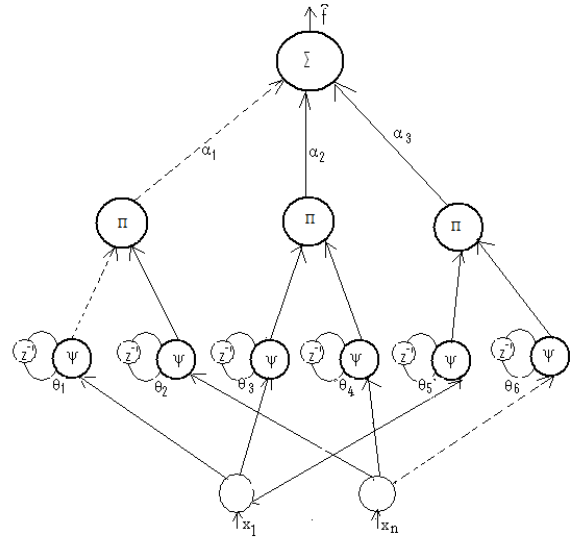


Figure 2: Self Recurrent Wavelet Neural Network

Output of an 'n' dimensional SRWNN with '*m*' wavelet nodes can be expressed as

$$f(x(t), x(t-\tau)) = \sum_{i=1}^{m} \alpha_i(t) \varphi_i \begin{pmatrix} \theta_i(t), \bar{\varphi}_i(t-\tau), x(t), \\ w_i(t), c_i(t) \end{pmatrix} \tag{3}$$

where φ_i is the i^{th} wavelet node given by

$$\varphi_i(\theta_i, \bar{\varphi}_i, x, w_i, c_i) = \prod_{j=1}^{n} \varphi_{ij}(\theta_{ij}, \bar{\varphi}_{ij}, x, w_{ij}, c_{ij}) \tag{4}$$

where φ_{ij} is the i^{th} wavelet node with j^{th} wavelon. $x(t) \in R^n$ shows input vector, with where τ is the point delay encountered in state vector. $\bar{\varphi}_i = [\bar{\varphi}_{i1}, \bar{\varphi}_{i2}, \ldots, \bar{\varphi}_{in}]$ represents the feedback argument for the i^{th} wavelet node. The Network's previous information stored in the vector $\bar{\varphi}_i = [\bar{\varphi}_{i1}, \bar{\varphi}_{i2}, \ldots, \bar{\varphi}_{in}]$. $\theta_i = [\theta_{i1}, \theta_{i2}, \ldots, \theta_{in}]$ shows weighted vector , $w_i = [w_{i1}, w_{i2}, \ldots, w_{in}]$ and $c_i = [c_{i1}, c_{i2}, \ldots, c_{in}]$ represents are dilate and translate vectors for feedback input. This input vectors applied on i^{th} wavelet node which represents as follows-

$[x_1 + \theta_{i1}\bar{\varphi}_{i1}, x_2 + \theta_{i2}\bar{\varphi}_{i2}, \ldots, x_n + \theta_{in}\bar{\varphi}_{in}]^T$.

In matrix form (3) can be rewritten as

$$f = \alpha^T \varphi(x, \theta, \bar{\varphi}, w, c) \tag{5}$$

where

$w = [w_1, w_2, ..., w_m]^T \in R^{mxn}$ and $c = [c_1, c_2, ..., c_m]^T \in R^{mxn}$ are dilation and translation parameters respectively ;

$\alpha = [\alpha_1, \alpha_2, .., \alpha_m]^T \in R^m$ and $\theta = [\theta_1, \theta_2, .., \theta_m]^T \in R^{nxm}$ are the output and feedback weights respectively. $\overline{\varphi} = [\overline{\varphi}_1, \overline{\varphi}_2 .., \overline{\varphi}_m]^T \in R^{nxm}$ shows feedback, input vector of SRWNN.

With ideal wavelet approximator, let f^* be the optimal function approximation

$$f = f^* + \Delta = \alpha^{*T}\varphi^* + \Delta \quad (6)$$

where $\varphi^* = \varphi(x, \theta^*, \overline{\varphi}, w^*, c^*)$ and the optimal parameter vectors are $\alpha^*, w^*, c^*, \theta^*$ of α, w, c, θ respectively, and the approximation error denoted by Δ with assumption $|\Delta| \le \Delta^*$,where Δ^* is a positive constant.

The optimal parameter vectors for paramount approximation are difficult to determine. The definition of estimation functions is as follows-

$$\hat{f} = \hat{\alpha}^T \hat{\varphi} \quad (7)$$

where $\hat{\varphi} = \varphi(x, \hat{w}, \hat{c}, \hat{\theta}, \overline{\varphi})$ and $\hat{\alpha}, \hat{w}, \hat{c}, \hat{\theta}$ are the estimates of $\alpha^*, w^*, c^*, \theta^*$ respectively. Defining the estimation error as

$$\tilde{f} = f - \hat{f} = f^* - \hat{f} + \Delta = \alpha^T \tilde{\varphi} + \hat{\alpha}^T \tilde{\varphi} + \tilde{\alpha}^T \hat{\varphi} + \Delta \quad (8)$$

where $\tilde{\alpha} = \alpha^* - \hat{\alpha}, \tilde{\varphi} = \varphi^* - \hat{\varphi}$

With the appropriate number of nodes, the estimation error $\tilde{f}$ can be reduced to arbitrarily small value on the compact set so that the bound $\|\tilde{f}\| \le \tilde{f}_m$ holds for all $x \in \Re$.

As the wavelet network representation is nonlinear in parameter which respect to certain adjustable parameters, to facilitate the derivation of tuning laws a Taylor expansion based following partial linearization is carried out [15,21].

$$\tilde{\varphi} = A^T \tilde{w} + B^T \tilde{c} + C^T \tilde{\theta} + h \quad (9)$$

where $\tilde{w} = w^* - \hat{w}, \tilde{c} = c^* - \hat{c}, \tilde{\theta} = \theta^* - \hat{\theta}$ and h are the vectors of higher order terms and

$$A = \left[\frac{d\varphi_1}{dw}, \frac{d\varphi_2}{dw}, \ldots, \frac{d\varphi_m}{dw} \right] \Bigg|_{w=\hat{w}}$$

$$B = \left[\frac{d\varphi_1}{dc}, \frac{d\varphi_2}{dc}, \ldots, \frac{d\varphi_m}{dc} \right] \Bigg|_{c=\hat{c}}$$

$$C = \left[\frac{d\varphi_1}{d\theta}, \frac{d\varphi_2}{d\theta}, \ldots, \frac{d\varphi_m}{d\theta} \right] \Bigg|_{\theta=\hat{\theta}}$$

with

$$\frac{d\hat{\varphi}_i}{dw} = \left[0,...,0, \frac{d\hat{\varphi}_i}{dw_{1i}}, \frac{d\hat{\varphi}_i}{dw_{2i}}, ..., \frac{d\hat{\varphi}_i}{dw_{ni}}, 0...0 \right]^T$$

$$\frac{d\hat{\varphi}_i}{dc} = \left[0,...,0, \frac{d\hat{\varphi}_i}{dc_{1i}}, \frac{d\hat{\varphi}_i}{dc_{2i}}, ..., \frac{d\hat{\varphi}_i}{dc_{ni}}, 0...0 \right]^T$$

$$\frac{d\hat{\varphi}_i}{d\theta} = \left[0,...,0, \frac{d\hat{\varphi}_i}{d\theta_{1i}}, \frac{d\hat{\varphi}_i}{d\theta_{2i}}, ..., \frac{d\hat{\varphi}_i}{d\theta_{ni}}, 0...0 \right]^T$$

Substituting (9) into (8)

$$\tilde{f} = \left(\tilde{\alpha}^T \left(\hat{\varphi} - A_1^T \hat{w} - B_1^T \hat{c} - C^T \hat{\theta} \right) + \tilde{w}^T A \hat{\alpha} + \tilde{c}^T B \hat{\alpha} + \tilde{\theta}^T C \hat{\alpha} + \varepsilon \right) \quad (10)$$

the uncertain term expression are as follows-

$$\varepsilon = \left(\alpha^{*T} h + \tilde{\alpha}^T A^T w^* + \tilde{\alpha}^T B^T c^* + \tilde{\alpha}^T C^T \theta^* \right)$$

Assumption of uncertainty ε satisfies the following Lipschitz condition

$$|\varepsilon| \le q_1 \|x(t)\| + q_2 \|\overline{\varphi}_i (t - \tau)\| \quad (11)$$

where $q_i \ge 0$.

SYSTEM FORMULATION

Consider a composite delayed nonlinear system of the form [20, 24]

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_n &= f(x(t), x(t-\tau)) + w(t) \\ y &= x_1 \end{aligned} \quad (12)$$

where $x = [x_1, x_2, ..., x_n]^T$, y represents the state variable and output respectively, where as u is the control effort applied to the system through an actuator with output $w(t)$ defined by (2). $f(x(t), x(t-\tau)) : \Re^{2n} \to \Re$ is a smooth unknown, nonlinear function of present and delayed values of state variables, τ is the known time delay encountered in the state variables. In this work the function $f(x(t), x(t-\tau))$ is approximated by a self recurrent wavelet network.

The desired trajectory vector is defined as

$$\overline{y}_d = \left[y_d, \dot{y}_d, \ddot{y}_d, \ldots, \overset{n-1}{y}_d \right]^T \quad (13)$$

The design objective of an adaptive controller with wavelet network to accomplish tracking performance in the presence of uncertainties hysteresis and time delay simultaneously assuring the co-bounding of closed loop signals.

Assumption:

Desired trajectory $y_d(t)$ is assumed to be smooth, continuous C^n and available for measurement.

Considering the hysteresis model defined by(2), system model (12) can be defined as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ &\vdots \\ \dot{x}_n &= f(x(t), x(t-\tau)) + p_0 u(t) + \upsilon \\ y &= x_1 \end{aligned} \quad (14)$$

where $\upsilon = -\int_0^R p(r) F_r(u)(t) dr$ denotes the nonlinear term inserted by actuator hysteresis. In this work, it is effectively approximated and subsequently mitigated by using a SRWNN.

WAVELET CONTROLLER DESIGN

Backstepping based controller is designed as follows-

The state tracking error vector 'e' is defined as-

$$e = x - \overline{y}_d \quad (15)$$

Thus the system's error dynamics (12) becomes

$$\dot{e}_1 = \dot{x}_1 - \dot{y}_d = x_2 - \dot{y}_d = e_2$$
$$\dot{e}_2 = \dot{x}_2 - \ddot{y}_d = x_3 - \ddot{y}_d = e_3$$
$$\vdots \quad (16)$$
$$\dot{e}_n = \dot{x}_n - \overset{n}{y}_d = f(x(t), x(t-\tau)) + p_0 u(t) + \kappa - \overset{n}{y}_d$$

Defining a filter tracking error as

$$r = Ke \quad (17)$$

where $K = [k_1, k_2, \cdots, k_{n-1}, 1]$ is a suitably selected coefficient vector such that $e \to 0$ as $r \to 0$.

Defining control effort as

$$u = u_r + u_c \quad (18)$$

where u_r is the principle control term obtained by applying the concept of feedback linearization

$$u_r = \frac{1}{p_0}\left(-\hat{f}(x(t), x(t-\tau)) - \hat{\kappa} + \overset{n}{y}_d - K_e e - ar\right)$$

or $$u_r = \frac{1}{p_0}\left(-\hat{\gamma} + \overset{n}{y}_d - K_e e - ar\right) \quad (19)$$

where $\hat{\gamma}$ is the wavelet approximation of system uncertainty and the disturbance term inserted by hysteresis nonlinearity. Wavelet adjustable terms are tuned so as to effectively mitigate the undesired dynamics whereas $K_e = [0, k_1, k_2, \cdots, k_{n-1}]$ and $a > 0$.

while u_c is the robust control term inserted in order to attenuate the approximate error

$$u_c = -\frac{1}{p_0}\frac{(\rho^2+1)r}{\rho^2} \quad (20)$$

where ρ is the prescribed attenuation.

With the help of tuning laws developed in the next part of this subsection, the disturbance $\tilde{\gamma}$ (10) is reduced to a small arbitrary value; this value is further attenuated by robust control term u_c.

Adaptation laws for online tuning of wavelet parameters are as:

$$\dot{\hat{\alpha}} = -\dot{\tilde{\alpha}} = \beta_1 r(\hat{\varphi} - A^T \hat{w} - B^T \hat{c}); \dot{\hat{w}} = -\dot{\tilde{w}} = \beta_2 r A \hat{\alpha}$$
$$\dot{\hat{c}} = -\dot{\tilde{c}} = \beta_3 r B \hat{\alpha}; \dot{\hat{\theta}} = -\dot{\tilde{\theta}} = \beta_4 r C \hat{\alpha} \quad (21)$$

where $\beta_1, \beta_2, \beta_3, \beta_4$ are positive constants treated as learning rates. The convergence analysis of the closed loop systems is presented in the next section.

STABILITY ANALYSIS

To carryout convergence analysis of the closed loop systems, let us take a Lyapunov-Krasovskii functional of the form [17, 22]

$$V = r^2 + \frac{\tilde{\alpha}^T\tilde{\alpha}}{2\beta_1} + \frac{\tilde{c}^T\tilde{c}}{2\beta_2} + \frac{\tilde{w}^T\tilde{w}}{2\beta_3} + \frac{\tilde{\theta}^T\tilde{\theta}}{2\beta_4} + \int_{t-\tau}^{t} \eta(\sigma)d\sigma \quad (22)$$

Differentiating it along the trajectories of the system

$$\dot{V} = r\left(K_e e - f(x(t), x(t-\tau)) + p_0 u(t) + \kappa - \overset{n}{y}_d\right) +$$
$$\frac{\tilde{\alpha}^T\dot{\tilde{\alpha}}}{\beta_1} + \frac{\tilde{c}^T\dot{\tilde{c}}}{\beta_2} + \frac{\tilde{w}^T\dot{\tilde{w}}}{\beta_3} + \frac{\tilde{\theta}^T\dot{\tilde{\theta}}}{\beta_4} + \eta(t) - \eta(t-\tau))$$

Substitution of control law u (18,19) in the above equation yields

$$\dot{V} = r(-\tilde{\gamma} - ar + p_0 u_c) +$$
$$\frac{\tilde{\alpha}^T\dot{\tilde{\alpha}}}{\beta_1} + \frac{\tilde{c}^T\dot{\tilde{c}}}{\beta_2} + \frac{\tilde{w}^T\dot{\tilde{w}}}{\beta_3} + \frac{\tilde{\theta}^T\dot{\tilde{\theta}}}{\beta_4} + \eta(t) - \eta(t-\tau))$$

Substituting $\tilde{\gamma}$ (6) and adaptation laws (21) in above equation,

$$\le -ar^2 + (p_0 u_c r + r\varepsilon) + \eta(t) - \eta(t-\tau))$$
$$\le -ar^2 + p_0 u_c r + |r||\varepsilon| + \eta(t) - \eta(t-\tau))$$

Selecting $\eta(t)$ and substituting in above equation

$$\le \begin{pmatrix} -ar^2 + p_0 u_c r + q_1\|x(t)\||r| + \\ q_2\|\overline{\varphi}_i(t-\tau)\||r| + \eta(t) - \eta(t-\tau)) \end{pmatrix}$$

Assuming $\eta(t) = q_2|r|\|\overline{\varphi}_i(t)\|$

$$\le -ar^2 + p_0 u_c r + q_1\|x(t)\||r| + \eta(t)$$
$$\le -ar^2 + p_0 u_c r + q_1\|x(t)\||r| + q_2|r|\|\overline{\varphi}_i(t)\| \quad (23)$$

Substituting robust control term (20) in the above equation,

$$\le -p_1 r^2 + p_2\|x(t)\|^2 + p_3\|\overline{\varphi}_i(t)\|^2 \quad (24)$$

where

$$p_1 = (a + \frac{(q_1+q_2)}{2}); p_2 = \frac{q_1\rho^2}{2}; p_3 = \frac{q_2\rho^2}{2}$$

Stability is assured as long as

$$p_1 r^2 \le p_2\|x(t)\|^2 + p_3\|\overline{\varphi}_i(t)\|^2 \quad (25)$$

It indicates that $\dot{V}$ is negative outside a compact set which indicates the ultimate upper boundedness of all the closed loop signals.

VII. SIMULATION RESULTS

Simulation is performed to illustrate the efficiency of proposed control strategy. Considering a system of the form

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = x_4 \quad (26)$$
$$\dot{x}_4 = 0.1x_1(t-\tau) + 0.1x_2^2 x_3 + w(t)$$
$$y = x_1$$

System (26) represents uncertain nonlinear systems with composite delay defined by (12) with $n = 4$.The time delay τ is taken as 1 sec. To solve the tracking problems of this system by using control strategies discussed in equations (18,19,20,21).

For simulation, the desired trajectory is taken as $y_d = 0.1\sin\frac{\pi}{3}t + 0.1\cos\frac{\pi}{4}t$. Initial conditions are taken as $x(0) = [0.3, 0.1, 0, 0]^T$.Attenuation levels for robust controller are taken as 0.01. Controller gain vectors are taken as $k = [4.34, 2.4, 1.96, 1]; a = 2.32; \rho = 0.01$. The control input(18,19,20) applied to the system is generated using the hysteresis model $w(t) = p_0 u(t) - \int_0^R p(r) F_r(u)(t) dr$ with

$p_0 = 2$ and $p(r) = \zeta e^{-\kappa(s-1)^2}$ where, $\zeta = 0.5, \kappa = 0.0014$ Wavelet network is constructed by using a Mexican hat as the mother wavelet. Initialization of wavelet parameter is set to zero and the adjusted value of parameters is tuned online uses adaptation laws (21) with unity learning rates. Figure 3 and Figure 4 shows simulation results and indicate the system response. The adaptive control term developed in this work drives the state variables towards the desired trajectory after a short duration initial transient.

Conclusion

An adaptive control scheme is presented for a class of uncertain delayed nonlinear systems with delay and subjected to hysteresis nonlinearity. Proposed adaptive scheme integrates the classical state feedback controller with SRWNN in order to insure effective control of the system under consideration. SRWNN, which is highly suitable for the approximation of uncertain dynamics is used for approximating the uncertain system dynamics as well as to compensate the undesired dynamics due to hysteresis. Tuning laws are invented for online adjustment of the wavelet system parameters. A Theoretical system is validated by the simulation results.

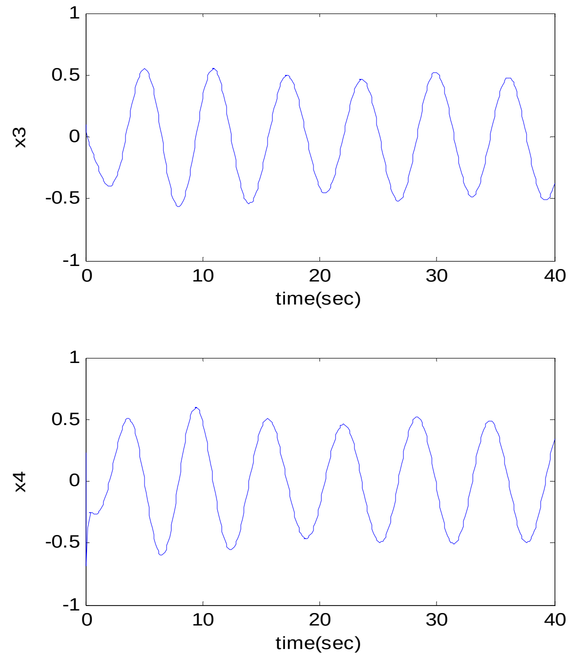
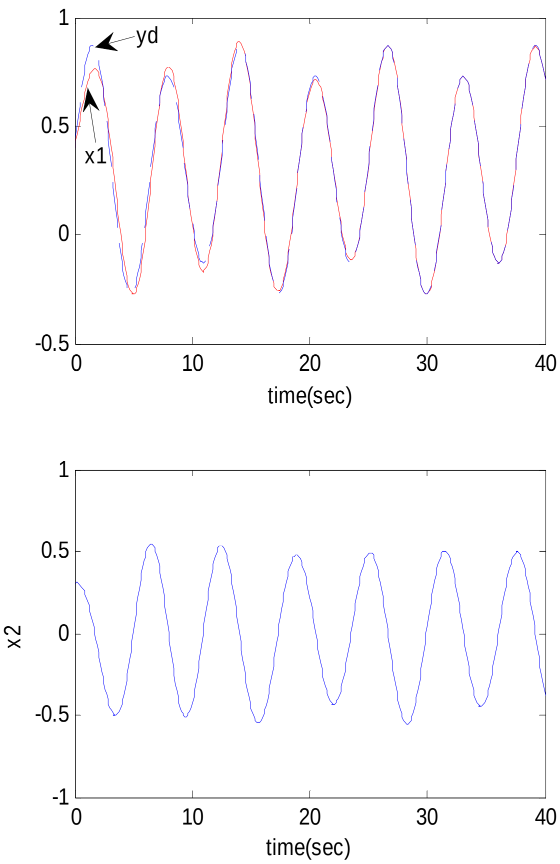


Figure 3: State variables and desired trajectory.



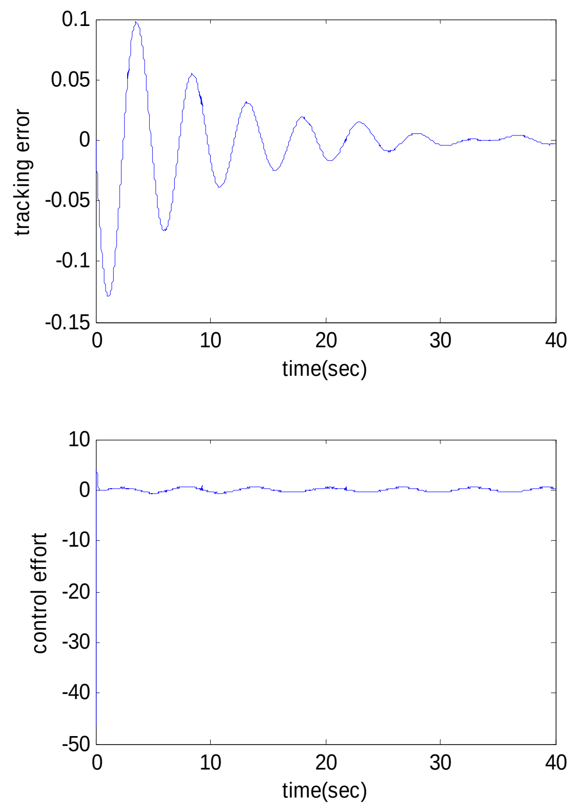


Figure 4: Tracking error and control signal

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