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Constructive Methods of Forming Surfaces

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Abstract

The relevance of the study: the relevance of the researched problem is caused by the need for the implementation of the development in the field of product design and construction of complex geometric shapes, which are based on constructive ways of shaping surfaces: kinematic method; wireframe and kinematic method with directing line; method of geometric transformations; a separation method of linear-wireframe of surfaces from a plurality of multiplicity, in particular from the congruence of lines; method of non-linear projection.

Objective: the aim of the research is to study non-traditional projection systems and projecting surfaces, which were conducted by synthetic or constructive methods. The analytical interpretations of these methods are separated in forms of representation and not interfaced to software package of scientific research, design (CAD), pre-production technology.

Methods: the leading method of research of the problem is the general analytical theory of applied shaping surfaces, developed by Professor Skidanov I.A., constitutes a unified mathematical formalism of geometric modeling of surfaces, corresponding to modern requirements of the use of computer technology.

Results: The article presents the design methods of forming surfaces: a kinematic method, method of geometric transformations, a separation method of linear-wireframe of surfaces from a plurality of multiplicity; analytic interpretation of constructive ways of shaping surfaces; highlighted the basic provisions of the general technical analytic theory of morphogenesis.

Practical significance: On visualization examples projecting surfaces by computer graphics to show the applicability of analytical models in computer technology of research, design and manufacture.

Keywords: innovative projection system, congruence, plane of the parallelism, the method of transformation.

INTRODUCTION

Historically, the design methods of forming surfaces were originated at different times for meet the needs of science and technology. The first was a kinematic method, which arose with the birth of descriptive geometry by Monge [22]. At the stage of origin the method of forming of ruled surfaces has been applied with three incidents to their lines. The first milestone in the development of this method was the assignment of one of the incident lines to the infinity. So in the class of ruled surfaces was allocated subclass of ruled surfaces with plane parallelism - Catalan surfaces.

Transformation method should be recognized as the following constructive method of forming surfaces, the appearance of which is associated with the birth of a scientific specialty "Applied geometry, engineering graphics," which applies all of its provisions to solve practical problems. The first steps were a way to convert the use of affine and homologous transformations of more complex surfaces into simple solutions to simplify the tasks of descriptive geometry [14]. We associate further development of applied geometry of surfaces with the decision of purely methodological problems: the theory of surface determinant [19], the frameworks theory [40], the theory of the parameterization [37]. Finally, non-linear method of projection should be noted as a constructive. According to Egers theorem [16], all the projection methods, except the central and parallel methods, are nonlinear. Mapping the ideal line on the plane of projection in the form of a straight line is a structural feature of the linear projection.

METHODOLOGICAL FRAMEWORK

Kinematic method [11, 34]. The line is called a generatrix, it may be stable or alternating type, moves in space and on the law describes surface. Simple kinematic surface is a surface of revolution. Generatrix is some flat line, its motion is supplied axis located in the plane of the generatrix. Generatrix rotates around an axis, it describes a circle, the totality of circles is a linear frame of parallels, and meridians comprise the family of intermediate positions of generatrix. The generatrix is not necessarily flat, it can be spatial. In this case, as well as when it is flat, but axis is located outside the plane, the family of generatrixes does not coincide with the meridians family.

Law of movement of generatrix lines sometimes asks by the lines, generatrix should cross them in process of movement. These lines are called guides. As already noted, one of the guide may be located at infinity. Line at infinity is served by parallelism plane and the flat curve is served by guide cone [11]. The guide may or may not belong to surface. For example, circular surface can be submitted by three included guides, not belonging to the same plane, and by torso, which skirts the incidence plane of the circumference.

In the **wireframe and kinematic method with directing line** [29] the formation of surfaces with the directing lines is connected with the mobile Frenet trihedral. His mutually orthogonal edges - tangent, normal and binormal curve aims to create an orthogonal Cartesian system, which beginning moves on the guide. Generatrix refers to the Cartesian system.

Method of geometric transformations. There are two ways to apply geometric transformations in the shaping of surfaces: a straight line, when by the properties of the surface of the inverse image we study the properties of the surface of the image in a transformation; a return, once the prototype surface acts as a complex surface. By transformation it translates into a simple surface, through which obtain the solution of a problem. Then, the solution is returned to the surface of the pre-image. By type of transformation used affine and projective transformations [6], birational (Cremon) transformation [23], including quadratic [6], the transformation inversion [8], topological transformation [6].

A separation method of linear-wireframe of surfaces from a plurality of multiplicity. Let a line be determined in three-dimensional space by the E - parameters, which are p parameters of position and q parameters form [7]. The space can be submited as it is filled by the E - parameters set of this type of lines. If the condition is associated as by E - 1 parameter sets, leaving one free parameter position, we obtain a one-parameters set (family, line frame) of lines that make up the surface. We bind sets parameters by the terms of lines, each of which binds a certain number of parameters [36]. It is necessary to monitor the compatibility conditions [38]. Of particular interest is the set of two-parameters - a congruence of lines, such as lines congruence. Many direct ∞3 (complex), ∞2 (congruence), ∞1 (surface) have been the subject of careful study of the synthetic method of the linear geometry, which was based on constructive geometry and constructive algebra [3].

Note that the presentation of synthetic geometry in the stage of its inception was conducted without drawings and formulas. Geometry was introduced by imaginary schemes, algebra - by relations between the algebraic-geometric characteristics of sets of lines (order, class) and by their subsets corresponding to different isolation conditions [39]. The order of congruence - the number of direct congruence passing through an arbitrary point in space is called [15]. The class of congruence - the number of direct congruence belonging to an arbitrary plane is called [15]. Congruence can have singular points and the plane in which the rule for determining the order and class is broken, namely, the number of direct lines of congruence passing through a singular point, and the number of direct congruence belonging to a singular plane, are equal to infinity [15].

The concept of the determinant surface can be extended to congruence. Congruence of lines, the order of which is equal to class is called a dual [31].

Congruence of the first order and the first-class are set [25]:

- Four straight congruence (rays);
- Collinear flat fields;
- Quadratic fields with a perspective direction;
- Focal lines, axes [28], the directrices.

Obukhova shows the transition from the first three qualifiers of the congruence of the first order and the first to the fourth grade. Starting from the first determinant [25] of the congruence of the first order and the first class they are divided into hyperbolic with two axes, one parabolic and elliptical with no axes. Let a, b, c, d - four beams of congruence, provided that each triplet has no common plane parallelism. If the line d intersects a hyperboloid of one sheet with the guides a, b and c in two points or does not have with him the common points, we have congruence of the first order and the first class are, respectively, the hyperbolic, parabolic and elliptical. One of the most common ways to release the linear frame surface from congruence is a dive a line into her. If as the line which is immersed in a congruence of the first order and the first class, we have line of 2order, we get a fourth-order surface. Research surfaces derived from the congruence of various orders and classes, dual, biplanar, bisekant mainly conducted through the study of reflections generated by the rays of the congruence. This is mainly algebraic surfaces [10] and algebraic congruence [27].

In addition to the congruence of lines in the shaping of surfaces are used congruence of circles [24], parabolas [13], a conic [20] plane curves [28], cylindrical helices [30], bevel helical lines [45].

RESULTS

Analytical interpretation of constructive ways of shaping surfaces.

The most common forms of analytic representation of surfaces are:

- Implicit coordinate

$$F(x,y,z)=0; \tag{1.1}$$

- Explicit coordinate

$$z=z(x,y); \tag{1.2}$$

- Parametric

$$x=f(u,v),\ y=\varphi(u,v),\ z=\psi(u,v). \tag{1.3}$$

Any specification of functions (1.1), (1.2) or (1.3) corresponds to its surface. There are formulas for determining the differential-geometric characteristics of surfaces represented any of the above forms. From any of the forms you can move to any other. So that on the form (1.1), go to the form (1.2), it is necessary (1.1) decide respect to z. The form (1.2) is a special case of the form (1.3), if present:

$$x=u,\ \ y=v,\ \ z=\psi(u,v) \tag{1.4}$$

To go from the form (1.3) to the form (1.2), the first two equations (1.3) must be solved for u, v, and the obtained expressions must be substituted in the third equation. To go from the form (1.1) to the form (1.3), we must assign the first two functions (1.3) arbitrarily and to determine the function of a third substitution intended functions in (1.1), and then solve the resulting expression respect to z.

The most applicable in computer modeling is a form of representation (1.3), introduced by Gauss and its special case (1.2), and proposed by Monge. It should be noted that these forms of presentation so common that do not contain any hint of geometric forming. There are a small number of surfaces, which in the above forms are known and familiar. For the rest there is no reading rules equations, that is, there are no rules of form surfaces submission with their equations

Here we are not talking about digital methods of presentation surfaces by dotted frame, using which you can create a computer model of the surface of any shape, existing in the form of an experimental sample. Along with prevalence of form independence from discrete techniques have essential shortcoming consisting in the considerable volume of input data.

Presentation of the final surface (1.1), (1.2), (1.3) is characterized by the absence of form controls. It is the lack of controls led to the development of processes for surface modeling that would suggest variations of form. For example, the variation of form in the kinematic method of the formation achieved an arbitrary choice of generatrix and directing. For a separation method of linear-wireframe of surfaces from a plurality of multiplicity, in particular from the congruence of lines - by the arbitrary choice of a plurality of elements and its determinant, and by a conditions whuch are randomly assigned, with allocating a linear frame of the plurality. In the formation process of geometric transformations - by the arbitrary choice of the surface of the pre-image, the type of transformation and its parameters.

Analytical method for forming the kinematic interpretation we can see in Bubennikovs work [2]. This work points to the common ways of obtaining the final kinematic equations of surfaces without any analysis of the impact on the ultimate objective of a problem of eliminating the problem parameters. As specific we can recognize the following works, in which the kinematic method is used for a certain class of surfaces: linear [45], including deployable surfaces [33,34]; cyclic, including Dupin cyclide [46]; screw [21] and kvazi screw [43].

Regarding frame and kinematic method [29], it should be seen as analytical, since the provisions of the Frenet trihedral on a spatial guideline determined analytically by the curvature and torsion of the guide. Methods for transformation of space in the analytic representation are not often used to describe the surface.

Analytic interpretation of a separation method of linear-wireframe of surfaces from a plurality of multiplicity, depends on solving systems of equations linking parameters, ie, as the "full-length," there is a problem removing the parameters. Here, great attention is paid to ruled surfaces, rectilinear frames which are separated from the congruence. Since the very same analytical framework in this matter is algebraic geometry [12], surface equation is obtained in the form of (1.1). The most significant results are due to A. Cayley [3], J. Darboux [4] and V.S. Obukhovoy [26]. Individual results for specific congruence of direct and specific conditions for the allocation of these ruled surfaces found in the work of Akhonin [1] - for bisekant congruence in the work of Kochetkova [17] - for the congruence of the first order and first class. Analytical interpretations of the separation surfaces of the congruence of circles and parabolas are known. Equations of surfaces of these cogruences are derived in the form of (1.1).

Parametric representation of congruence of helical cylindrical and tapered lines, spiral lines, as well as the corresponding representation of the surface shape of the congruence (1.3) is in the work of Tevlins [45]. In this paper we consider the congruence transformation, which produce the helical surfaces of the frame.

Formation of surfaces rays by nonlinear projection methods

As a rule, a plurality of lines of a projection system is a two-parameter, ie, is congruence. The congruence of lines has focal figures: lines that cross all rays of the congruence, or surfaces that envelop all the rays. Namely a focal figure often used to represent both congruence and as a determinant of the projection system. Determinant of congruence can be a determinant of the correspondence point’s two flat fields. The beams pass through the congruence of the corresponding points. This determinant is often convenient for the study of the congruence of surfaces. For the formation of projection systems are best suited congruence of the first order.

The study of surfaces of congruence is to identify the critical points and lines belonging to the surface. First of all, we should include the focal line as the special line. The focal surfaces limit the scope of the existence of congruence, and all its surfaces. On the appearance of specific lines or points of congruence with respect to the congruences surface, location of the line has an impact to the focal lines of figures, which is immersed in congruence and highlights from her surface.

Among the special points and lines the n - fold line is called by us as the line n - fold self-intersection of surfaces, forming torsovyh - rays of congruence on the surface at a constant tangent plane to the surface at all points of the beam. Torsovyh forming of a ruled surface cross double lines of this surface in the limit (cuspidal) points. Plane passing through the point kuspidalni intersect the surface of the lines for which kuspidalnye points are cusps.

From all the systems of the projection rays congruence of the first order and first-class we highlight biaxial and uniaxial. Their name is derived from the number of focal lines (headmistress) congruence. Rays congruence of the first order and the first-class in general is crossed by two skew lines, which are the directrices (focal lines) congruence. A one-parameter family of planes passing through the headmistress, a number of projective points located on the second headmistress. Thus, all of the surfaces of such congruences are called hyperbolic, have double lines, which are the axes of the plane beams (directrices, focal lines) congruence. Surfaces are called two-axis.

Let us combine axis and establish a correspondence between the plane of the beam passing through the common axis, and a number of points on the same axis. We obtain the so-called parabolic congruence with one axis, the surface of which is called the single-axis. The hyperbolic congruence can have one infinity focal line (axis headmistress). In this case all the parallel rays are fixed to its plane.

An essential attribute of the projection system is the presence of one or more projection planes. The projection plane is selected in such a position with respect to the elements of the determinant projecting congruence, which would be most convenient for the solution of a class of problems. So, the two planes of the projections on which the set point line, you can apply congruence and you can study the properties of the congruence of the projections surfaces. As the rays projecting systems are used not only directly, but also other lines: a circle [35], parabola, ellipse, conic, flat curve, involute, similar curves, spiral curves, cylindrical and bevel helical lines.

By analytical interpretation method of forming a projection surfaces nonlinear line, the majority of the surfaces formed by this method is described implicit coordinate form (1.1) when projecting the beam is an algebraic curve. Exceptions are ruled surfaces of 4th order, which investigated the synthetic method as the ray family hyperbolic congruence, which projects a curve of order 2, and their analytical description obtained in parametric form [9], which helped to get their by computer-graphics .

Description projection systems of transcendental curves [45] and the surfaces formed by the rays of these systems carried out in parametric form (1.3), but without computer visualization.

Application of nonlinear and projection surfaces formed by these methods cover different areas: the tasks of descriptive geometry, profiling of cutting tools, construction executive organs of tillers, the modeling of the space plane, modeling fields, construction of shells.

DISCUSSIONS

Main provisions of the general analytical theory of a technical shaping of surfaces

With the advent of the computer on the arena of the technical development , with the rapid spread of computer technology applications in research, design and development, the role of analytical modeling methods for forming surfaces and control their shape has changed dramatically: the role of analytical model became priority before computer model. By the new role of the analytical model and derive new requirements for her. We formulate these requirements and explain their nature.

Hierarchy of structure of analytical model

The effectiveness of the software depends on the amount of objects and scope of tasks with objects that are distributed software system. Efficiency is achieved by the simultaneous observance of the principles of synthesis and analysis: a generalized representation of the model at the highest levels and decay on the individual cases on the bottom [32].

Pairing input parameters of the analytical model with inputs of computer visualization and computer production management.

Imaging surface is very important in automation systems research (ASR) and computer-aided design (CAD). As a rule, visualization of curved surfaces in the scientific purposes is realized by the axonometric image of the grid applied on a surface.

It is desirable to incorporate into the net one or two families of lines characteristic of the surface. So, for a ruled surface is characterized by a family of straight generators for cyclic - family circles, for screw - screw family lines and axial sections. Not always this desire can be realized in the case of an implicit surface (1.1) or explicit (1.2) coordinate form. Presentation of the same in parametric form (1.3) allows you to do this always. In the case of representation of the surface parametric equations (1.3) Net consists of two families of coordinate lines u = const and v = const. The same surface can present many options functions (1.3), i.e., curvilinear coordinate transformation on the surface can change one or both of the grid of the family, which facilitates the display surface of the frame linear characteristic. By setting limits and changing steps curvilinear coordinates’ u, v limit surface compartment and prescribed mesh density.

Ability to assign on the surface of an arbitrary grid plays an important role not only in computer-graphic visualization of surfaces, but also contributes to the selection of products toolpaths curved movements, taking into account the workpiece and the machining tool.

Considering ways of shaping surfaces it can be concluded that most of them differ from each other only by interpretation. For example, a ruled surface with three guide lines can be obtained by kinematic method, if a direct counts forming. On the other hand, any two of the three guides can be of local lines of congruence. Same ruled surface formed by the release of its linear frame congruence with the immersion in the last third of the guide. Finally, the surface of the carcass line can be regarded as a family of rays of congruences provided by two of the three guides, which project the third rail.

It is natural to wonder if there is a basis for combining analytical interpretations of the main ways of forming surfaces in general analytical model, which would correspond to the formulated requirements of this section? A positive answer to this question is in the works of Skidan [41]. The Skidan's dissertation [42] provides an overview of forming the kinematic surface, according to which the presentation of a generator associated with the representation of the surface (plane) to which it belongs. Generatrix together with the surface of the carrier move in space according to this law. This describes the surface. In the particular case of forming the carrier may be a characteristic surface shaping movement.

On the top level of the hierarchy forming process is modeled by three functions of three variables:

$$x = f(t,u,v), \quad y = \varphi(t,u,v), \quad z = \psi(t,u,v). \qquad (1.5)$$

The method of kinematic surfaces forming the connection between the function expressed by rectangular Cartesian coordinates x, y, z and special parameterization space using parameters u, v, coordinates that are curved on the surface, and the surface position parameter t in its movement.

Specific terms of the space coordination functions (1.5) is determined from the condition:

$$\frac{D(x,y,z)}{D(t,u,v)} = \begin{vmatrix} \frac{\partial f}{\partial t} & \frac{\partial \varphi}{\partial t} & \frac{\partial \psi}{\partial t} \\ \frac{\partial f}{\partial u} & \frac{\partial \varphi}{\partial u} & \frac{\partial \psi}{\partial u} \\ \frac{\partial f}{\partial v} & \frac{\partial \varphi}{\partial v} & \frac{\partial \psi}{\partial v} \end{vmatrix} = 0. \qquad (1.6)$$

The points at which the equality (1.6) is not satisfied, the coordination of the functions of the space (1.5) is holds. At these points a possible solution of equations (1.5) with respect to t, u, v:

$$t = f_1(x,y,z), \quad u = \varphi_1(x,y,z), \quad v = \psi_1(x,y,z). \quad (1.7)$$

Functions (1.7) express the inverse relationship of special coordinate's t, u and v from Cartesian. If the surface of the carrier properly represented and defined the law of its motion, the kinematic surface supplied functions (1.5) up to feed generator on the surface of the carrier its domestic equation, for example, function

$$v = v(t,u). \qquad (1.8)$$

The Skidan's research [42] shows the formula for determining the coefficients of the quadratic form of the surface, provided the internal equation (1.8): a) for arbitrary functions (1.5) and (1.8), b) for an arbitrary function (1.8) and certain functions (1.5). We studied two classes: C - surfaces and the G - flat surfaces forming. Kinematic scheme and give rise to generalize hyperbolic cylindrical coordinate system. Determine the internal equation (1.8), from which the direct substitution in (1.5) can go to the parametric equations of line deployed, cyclic, spiral C - kvazivintovih surfaces and the G - surfaces with canal - surfaces Monge, minimum screw C - surfaces. Further application of the formulas (1.5) - (1.8) for analytical modeling of other ways the formation of surfaces and obtain the equations was accomplished by disciples of Professor I.A. Skidan.

The frame-kinematic method of forming a carrier plane is a normal plane of the guide, the parameter position of the point at which a movement parameter. The position of the movable Frenet trihedral also depends on the setting of motion, so that the generator can be given for not only flat, but also the spatial image, relating it to the Frenet trihedral. In the paper of Frolov [5] frame and kinematic manner guide the production line parametric equation canal surfaces and surfaces Monge. The study also conducted using formulas (1.5) - (1.8).

It is noted that the function (1.5) can be treated as such, which serves a special coordinate space if x, y, z and t, u, v define the same point. If they determine various points, the function (1.5) represents the point of space transformation. So, according to formulas (1.5) - (1.8) is carried out simulations of the formation of a way to convert surfaces. The Jeger's paper [16] summarized the functions given expression (1.5) for the group of Möbius transformations.

To obtain a separate isometric, affine, projective, inversion transformation, it is necessary to relate the coefficients of the general expression corresponding relations.

It is known [44], that the congruence of lines also defines the functions (1.5). Moreover, since the same functions introduced a special coordination of space, they define three coordinate lines of the congruence: congruence t - intersections of lines is formed of two families of coordinate surfaces u = const, v = const; congruence u - lines - Intersection of two families of coordinate surfaces t = const, v = const; congruence v - lines - Intersection of two families of coordinate surfaces t = const, u = const.

The paper of Kolomoets [18] first considered the coordination of space by means of a normal congruence supplied to the supporting surface. Function parameters (1.5) acquire content: u, v - curvilinear coordinates on the support surface, t - setting the position along the normal to the support surface.

Coordination of space normal congruence has important applications in science in the study of surfaces, equidistant to the support. Equidistant surface is a surface corresponding to the coordinate of the coordinate system. In addition, the curvature of the base line and equidistant surfaces has a general curvilinear coordinates in the corresponding system, which facilitates the solution of the problem referring to the lines of curvature of the equidistant surfaces.

In practice, the normal coordination of space useful for building spatial diagrams normal stress and normal displacement. The paper of Zvereva [47] in parametric form developed a method of forming the release surface of the frame with a linear congruence: straight, circular, cylindrical and conical screw lines, cycloidal curves. For the first time in the construction of an analytical model of the determinant of the congruence of straight lines and curves cycloidal included focal surface.

CONCLUSION

Thus, it is proved that the basic design methods of forming surfaces require joint analytical interpretation expressed by formulas (1.5) - (1.8), which, thanks to the parametric representation of the input data is combined with the automated system technology preparation of production

RECOMMENDATIONS

The contents of this article can be useful for graduate students, faculty, students design a model of non-traditional projection systems.

In the course of the study there are new questions and problems to its decision. It is necessary to continue research on the development of the general algorithm of drawing up the parametric equations of the set (congruence) rays unconventional projection systems and projecting surfaces and apply it for projection systems rays' hyperbolic and parabolic congruence of straight lines, linear and continuous congruence circles.

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