

# Saturation-type Reaching Law for Altitude Control

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## Abstract

This paper proposes a robust altitude control of a quadrotor-type VTOL (Vertical Take Off and Landing). Recently, a lot of different types of unmanned quad-rotor systems are being used for various applications. Among them, air sensing in the constant height is one example. In the case of a small size aircraft in air sensing, it can be affected by a disturbance. Thus taking these into considerations, the controller is required to keep the performance with stability irrespective of disturbances. Thus the sliding mode controller is designed with a saturation-type reaching law and the simulation demonstrates its performance.

**Keywords:** Sliding Mode Control, Reaching Law, Altitude, VTOL.

## INTRODUCTION

The automatic control of flying machines has attracted the attention of many researches in the past few years<sup>1-3</sup>. Generally, the control strategies are based on simplified models which have both a minimum number of states and a minimum number of inputs.

Its complexity is due to the versatility and maneuverability to perform many types of tasks<sup>4</sup>. The classical helicopter is conventionally equipped with a main rotor and a tail rotor. However, other types of helicopters exist, including the twin rotor or tandem helicopter and the coaxial rotor helicopter. This paper is particularly interested in controlling a mini rotorcraft having four rotors.

Quad-rotor rotorcrafts have some advantages over conventional helicopters. Given that the front and rear motors rotate counter clockwise while the other two rotate clockwise, gyroscopic effects and aerodynamic torques tend to cancel in trimmed flight<sup>5</sup>.

Sliding mode control (SMC) is desirable as a kind of approach for nonlinear and uncertain system and also the sliding mode has an advantage of the robustness against the changes always taking place in the operation environment<sup>6-14</sup>.

This paper proposes the control algorithm which can perform reliably tracking for a desired height. Control algorithm is designed based on the sliding mode with a saturation-type reaching law and the global stability of the proposed controller can be verified by sliding conditions. Simulations show that

the proposed controller has a good tracking for the desired altitude in spite of unmodeled dynamics.

## MODEL

In this section, the model of the quad-rotor rotorcraft is presented in Figure 1 and where  $p(x,y,z)$  denote the position of the center of mass of the quad-rotor rotorcraft relative to the frame  $\{G\}$  and  $(\phi, \theta, \psi)$  are the three Euler angles (yaw, pitch, and roll angles) and represent the orientation of the rotorcraft<sup>15,16</sup>.

The model for the rotorcraft dynamics is obtained as follows.

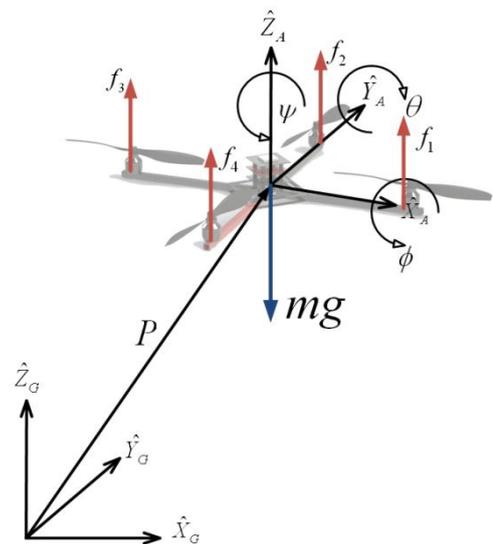


Figure 1. The quad-rotor rotorcraft.

Four-rotor quadcopters have some advantages over conventional helicopters. Given that the front and rear motors rotate clockwise while the other two rotate counter clockwise, gyroscopic effects and aerodynamic torques tend to cancel in trimmed flight. Thus the dynamics of the quad rotor rotorcraft becomes

$$m\ddot{x} = u (\sin \phi \sin \psi + \cos \phi \cos \psi \sin \theta) \quad (1)$$

$$m\ddot{y} = u (\cos \phi \sin \theta \sin \psi - \cos \psi \sin \phi) \quad (2)$$

$$m\ddot{z} = u \cos \theta \cos \phi - mg \quad (3)$$

### ALTITUDE CONTROL WITH A SATURATION-TYPE REACHING LAW

In this section, a control strategy will be developed for stabilizing the quad-rotor rotorcraft at altitude change. And the global stability of the closed-loop system will be proven. The control input  $u$  is essentially used to make the altitude reach a desired value.

The control of the vertical position can be obtained by using sliding mode. For this, an error is defined

$$e = z - z_d \quad (3)$$

where  $z_d$  is a desired height and a sliding function is defined by

$$s = \dot{e} + \lambda e \quad (4)$$

In the sliding surface  $s = 0$ , the error will converge to zero for  $\lambda > 0$  in Figure 2.

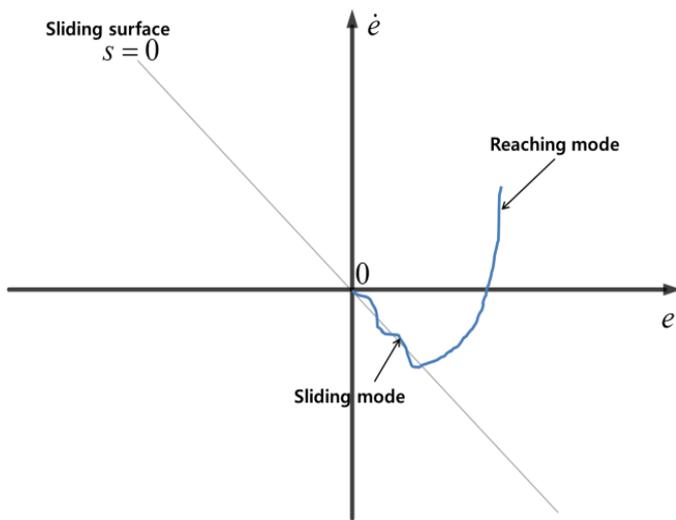


Figure 2. Phase portrait of SMC.

In the reaching mode, the dynamics will be varied along reaching law and saturation type reaching law with  $A > 0$ ,  $B > 0$  is proposed. This law can reach the sliding surface by the practical (bounded) maximum speed  $A$  from the initial condition and near the sliding surface, the reduced speed shows better in chattering.

$$\dot{s} = A \left( 1 - \frac{2}{1 + e^{-Bs}} \right) \quad (5)$$

By properly chosen  $A$  and  $B$ , the reaching dynamics results in fast response and chattering improvement in Figure 3<sup>9</sup>.

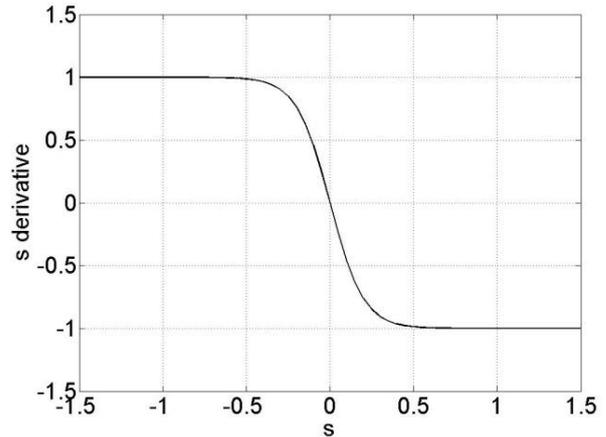


Figure 3. Reaching law ( $A=1$ ,  $B=10$ ).

For ensuring sliding condition, the following control input is proposed

$$u = \frac{m}{\cos \theta \cos \phi} \left[ A \left( 1 - \frac{2}{1 + e^{-Bs}} \right) + g - \lambda \dot{e} \right] \quad (6)$$

Angle stabilization should be handled for the altitude control<sup>5</sup>. In the rotorcraft altitude control, there are some uncertainties such as disturbances and parameter variations. For example, wind can affect the acceleration and mass change can occur by an attached camera. To find an allowable uncertain boundary, the uncertainty is considered as  $\Delta m$  and the dynamics (7) changes into

$$(m + \Delta m) \ddot{z} = u \cos \theta \cos \phi - (m + \Delta m) g \quad (7)$$

and becomes

$$m \ddot{z} = u \cos \theta \cos \phi - mg - \Delta m (\ddot{z} + g) \quad (8)$$

Assuming  $z \ll q$  from that usually the rotorcraft acceleration is below  $1 \text{ m/s}^2$ , then

$$\begin{aligned} \ddot{z} &= u \cos \theta \cos \phi - g - \frac{\Delta m}{m} (\ddot{z} + g) \\ &= u \cos \theta \cos \phi - g - \frac{\Delta m}{m} g \end{aligned} \quad (9)$$

For this uncertain system, the robust controller is proposed

$$u = \frac{m}{\cos \theta \cos \phi} \left[ A \left( 1 - \frac{2}{1 + e^{-Bs}} \right) - Q \text{sgn}(s) + g - \lambda \dot{e} \right] \quad (10)$$

and the controller still holds the sliding condition when

$$Q > \frac{\Delta m}{m} g \quad (11)$$

From (11), an admissible uncertainty is given by

$$\Delta m < \frac{m}{g} Q \quad (12)$$

and the maximum value of the uncertainty is

$$\Delta m_{\max} = \frac{m}{g} Q \quad (13)$$

### SIMULATION

Simulation environment has been developed in MATLAB / Simulink. The altitude control is simulated in four examples and the desired height is 1m. The proposed sliding mode altitude controller is (10) and the following parameters are used.

$$\lambda = 1, A = 1, B = 10, Q = 1, m = 2.08kg \quad (14)$$

The results are compared with the conventional sign function sliding mode controller (before) in four examples. The first example is for the nominal case which means no uncertainty ( $\Delta m_{\max} = 0$ ) and the result is shown in Figure 4. The state feedback control shows the fast response and the error goes to zero which means that the rotorcraft arrives the 1m height.

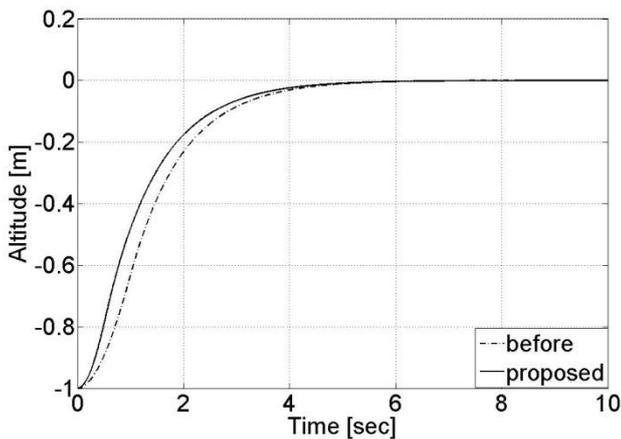


Figure 4. Error – nominal case.

The second example shows the constant uncertainty and the value is 0.1698 which is smaller (80%) than the maximum value  $\Delta m_{\max} = 0.2122$ .  $\Delta m_{\max}$ , which is obtained from (13) for  $Q = 1$ , is an admissible boundary value of the uncertainty. In Figure 5, because of the uncertainty, the error goes to zero relatively slow than the first case in the conventional SMC. But the tracking is good although the same controller in the first example is used for the proposed controller.

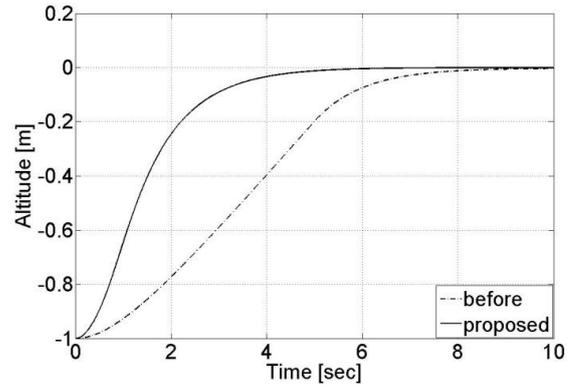


Figure 5. Error – constant uncertainty.

The third example is simulated for random uncertainty ( $\Delta m_{\max} = 0.2122$ ) and the result is good in Figure 6. Although that random is not admissible in practical cases, the stability of the proposed controller is ensured which is the same with the first example.

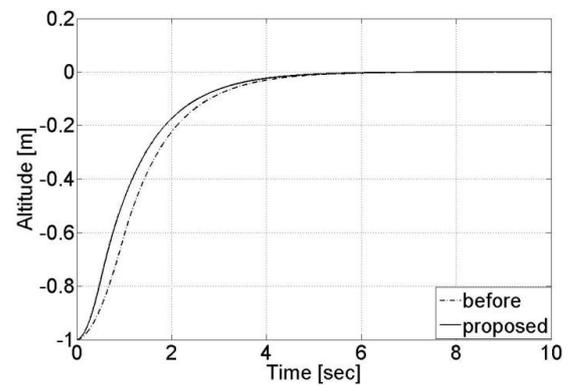


Figure 6. Error – random uncertainty.

The fourth example is for sinusoidal case which can occur by wind. Wind direction can change slowly this way and that way.  $\Delta m_{\max} = 0.2122$  is used for the amplitude of the sinusoidal uncertainty and the trajectory is slightly changed in the conventional SMC but tracking is still good for the proposed controller in Figure 7.

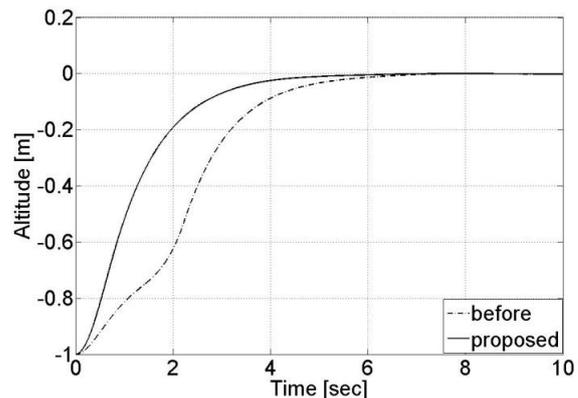


Figure 7. Error – sinusoidal uncertainty.

Four examples show the good tracking performance although the same controller is used and it also shows how the proposed controller works good and robust for the quad-rotor rotorcraft altitude control with uncertainty. And the result table of the transient responses such as rising time  $t_r$  and settling time  $t_s$  also shows the good performance of the proposed controller in Table 1.

**Table 1.** Transient response.

Uncertainty	Responses	Conventional	Proposed
1st (nominal)	$t_r$	2.36	2.23
	$t_s$	4.45	4.17
2nd (constant)	$t_r$	4.47	2.41
	$t_s$	7.22	4.49
3rd (random)	$t_r$	2.35	2.23
	$t_s$	4.4	4.15
4th (sinusoidal)	$t_r$	3.26	2.28
	$t_s$	5.48	4.15

## CONCLUSION

This paper proposes a sliding mode control with a saturation-type reaching law for altitude control. The proposed controller has a good convergence characteristic and robustness from sliding mode control. Control algorithm is designed based on the sliding mode with the proposed reaching mode and the global stability of the proposed controller is shown by using the Lyapunov function. Also the boundary of the uncertainty is derived which ensures the stability of the proposed controller. MATLAB/ SIMULINK simulations show that the proposed controller has a good tracking for the desired altitude in spite of disturbances.

## ACKNOWLEDGMENTS

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