An Algorithm for Selecting Compatible Wavelet Function in Electrical Signals to Detect and Localize Disturbances

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Abstract
In power system, power quality (PQ) distortion is a topical issue. This event has spurred to develop sophisticated methods for processing and analysing the signal to detect and classify the power quality distortion. Fourier transform (FT) technique was the core of many conventional techniques However, FT being replaced by the newer method notably Wavelet Transform (WT) technique. In WT spectral leakage problem was introduced when suitable wavelet family was not available to analyse the signal. This paper presents an algorithm to select most compatible wavelet function with transient signal to detect and localized disturbances. In this paper, the transient signal is analyzed using seventeen different mother wavelets i.e. Daubechie, Coiflet, Biorbothogonal, Symlet family members etc. and then selecting most compatible mother wavelet for transient signal by calculating Percentage Root Means Square Difference (PRD) and Mean Square Error (MSE) values. Here the threshold value is used to calculate PRD and MSE values as global threshold, this work is performed using MATLAB software.

Keywords: Wavelet Transform, Mother Wavelet, Percentage Root Mean Square Difference (PRD), Mean Square Error (MSE), Power Signal, Power Quality, Transient Signal.

INTRODUCTION
Now a days, the electricity demand is growing up due to many reasons therefore, the use of power electronic equipment is gradually increasing. These power electronics devices introduce harmonics and other power quality distortions in the system [1, 2]. Basically, quality of power is introduced by the shape of power signal waveform and due to distortion; electricity waveform is deregulated from the sinusoidal wave shape. Many types of equipment used in industries are very sensitive to PQ disturbances and small amount of PQ disturbance can decrease the efficiency of the industries [3, 4]. In summary PQ is compatible issue between load and generation because variation in load and generation profile affects each other [5]. Healthy power supplied to the consumers is one of the big challenges towards the electricity utilities consequently new standards for power quality criteria [6, 7] have been fixed to establish the maximum allowed disturbance level. Several power quality mitigation devices (PQMD) are used to overcome these problems and PQMDs require knowing the type of disturbance before mitigating the problems. Many other technologies are evaluated to detect and localize the disturbances as reported by [8].

Fourier transform is one of the conventional method to detect disturbances and convert the time domain into frequency domain [9]. In FT at a time only one domain information is extracted either in frequency or time domain or in discrete fourier transform (DFT), the frequency coefficient is a function of sin and cosine. If the signal is predominantly sinusoidal, periodic, and stationary than the DFT method is useful for analysis and for the transient or non-periodic disturbances DFT is inefficient method for analysis. Fourier transform have constant time-frequency spectrum window hence, FT technique now replaced by new approach remarkably WT.

Wavelet transforms are the mathematical functions which convert time domain into time scale domain rather than frequency domain [10, 11]. The important features of wavelet are translation (shift in time) and dilation (stretches or shrinks in time). WT have flexible time-frequency window which resolve the difficulty of signal cutting by variable resolution [12, 13], consequently wavelet transform can analysis the transient signal, which is advantageous over FT. Wavelet transform uses long time interval for low frequency and short time interval for high frequency information [14].

Wavelet Packet Transform (WPT) and Discreate Wavelet Transform (DWT) are the basic types of wavelet transform techniques. WPT uses full signal decomposition tree i.e. “approximation” as well as “detailed signal” can be splits thus large number of nodes are generated that increase the calculation time, WPT also has uniform frequency window which resolve the difficulty of signal cutting by variable resolution [15, 16]. On the other hand, DWT uses only basic branches of decomposition tree and the frequency band is also flexible hence, the analyzing time is low.

In WT, signal is analyzed using mother wavelet and in doing so an appropriate mother wavelet can increase the efficiency of analysis. If compatible mother wavelet is not used for a particular signal than some information is lost, which is known as ‘spectral leakage’ and this is the main drawback in wavelet transform technique [17-19] that can be overcome by the use of suitable mother wavelet for the particular signal.

Wavelet Transforms have different kind of mother wavelets including complex wavelet function to analyze a given signal [20]. In WT technique, the signal is analyzed by mother
wavelet to detect, localize the disturbances [21, 22] and to extract the information about disturbance time and frequency. Wavelet transform is also used in many power system applications [23-27] and mostly utilized for image processing [28, 29].

This paper introduces an algorithm to select the appropriate mother wavelet by calculating Percentage Root Means Square Difference (PRD) and Mean Square Error (MSE) values and thus the mother wavelet having minimum PRD and MSE value is the best compatible mother wavelet for the signal. Section 2 of the paper provides a synoptic introduction of WT and elaborates a WT property which explains the flexibility and procedure of wavelet decomposition. In section 3, classification of wavelet family according to their properties is presented and provides information about some extremely used wavelet family members. Section 4 provides details of the procedure to calculate PRD and MSE values and to select most appropriate mother wavelet for the signal. Power quality disturbance detection and localization by wavelet transform analysis with the use of most compatible mother wavelet is presented in section 5.

DISCRETE WAVELET TRANSFORM

WT is a mathematical tool for signal processing and it was introduced around 1980s. The wavelet transforms decompose the signal into different scales with different level of resolution by dilution of the mother wavelet.

To construct the scale analysis of a signal it is important to consider two basic functions which fulfills the properties of Lebesgue vector space \( L^2(\mathbb{R}) \). These two functions are scale function \( \varphi(t) \) and wavelet function \( \psi(t) \):

\[
\varphi(t) = \sqrt{2} \sum_k g_k \varphi(2t - k) \quad (1)
\]

\[
\psi(t) = \sqrt{2} \sum_k h_k \varphi(2t - k) \quad (2)
\]

Scaling and the Wavelet function are the two scale difference equations which are based on selecting of scaling function \( \varphi \) with properties that fulfill certain criteria, discrete filters \( h_k \) and \( g_k \) are used to solve the equation. where \( g_k = (-1)^k h_{N-1-k} \)

Let \( x(t) \) is the signal to process which is defined in \( L^2(\mathbb{R}) \) space (vector space for finite energy signal) and \( R \) is the real number.

\[
DWT_x(m,n) = 2^{-m/2} \int_{-\infty}^{\infty} x(t) \varphi\left(\frac{t - n2^m}{2^m}\right) dt \quad (3)
\]

where \( m \) and \( n \) are the scale and time dilation parameters, respectively and \( \varphi(t) \) denotes the mother wavelet. The mother wavelet used to analysis the signal must satisfy following equation:

\[
\int_{-\infty}^{\infty} \psi(t) \ dt = 0 \quad (4)
\]

As in eq. (3) the mother wavelet is dilating by a factor two for the transformation performed, thus the resulting wavelet family becomes an orthonormal basis function by which there is no data (information) lost during the compression of the signal.

By the wavelet filters \( g(n) \) (denoted by ‘H’ in figure 1) and \( h(n) \) (denoted by ‘L’ in figure 1) the sampled version of \( x(t) \) denoted by \( c(n) \) is decomposed into two signals detailed \( d(n) \) and smoothed \( c(n) \) signals respectively. Detailed value \( d(n) \) of original signal \( c(n) \) extracted by the band pass filter \( g(n) \) contains higher frequency component and by low pass filter \( h(n) \) smooth signal provides the approximation value of \( c(n) \).

\[
c_1(n) = \sum_k h(k - 2n)c_0(k) \quad (5)
\]

\[
d_1(n) = \sum_k g(k - 2n)c_0(k) \quad (6)
\]

In the wavelet transform, if \( c(n) \) have ‘N’ number of samples then coefficients \( c_1(n) \) and \( d_1(n) \) have approximate \( N/2 \) samples. Further these samples will decompose as the level increases.

In multi-level decomposition processes, signal is decomposed in two parts by low-pass filter and high pass filter. The coefficients from low pass filtering process are the “approximation coefficients” that can be processed again to decompose the data by bank of filters to generate another group of “approximation” and “detail coefficients”. This process has repeated until selected levels are reached. The two levels decomposition tree by DWT process is shown in Figure 1.

The number of filters used to analyse the signal depend on type of family members and all the family members have different kind of properties. Some type of family members present in WT are introduced in section 3.

WAVELET FAMILY

WT have several types of wavelet families to analyse the signal and all these families are further sub-divided in many family members according to their properties and application to use. Figure 2 shows the classification of different kind of wavelet families according to their properties.
Mother wavelet is the core of WT technology to analyse the signal and different type of mother wavelets are presents in WT tool i.e. Haar, Daubechies, Biorthogonal, Morlet, Symlets, Maxican-hat, Coiflets, Mayer etc. These wavelets are also classified according to the number of coefficients like -db2 & da4 (Daubechies wavelet with 2 & 4 coefficient, respectively).

In this paper work with some most potential wavelet family members is presented and the functions and properties of these mother wavelets are discussed in following section.

**Daubechies Wavelet Family**

Ingrid Daubechies was firstly introduced daubechies mother wavelet and piloted much research in the wavelet transform domain [30-34]. The daubechies family members represented as ‘dbN’, where surname of daubechies family is ‘db’ and the order of family is represented by ‘N’ (N=1, 2, 4, 6, 8, 10). Db1 is the basic daubechies wavelet, which is also known as ‘Haar wavelet’. Figure 3 presents the scaling and wavelet function of haar mother wavelet.

Almost all the Daubechies members are unsymmetrical therefore; this wavelet is mostly used to the analysis of non-periodic signal. Figure 4 shows the wave-shape of daubechies wavelet family members db2, db4, db6, db8 with their scaling function.

**Coiflet Wavelet family**

Ingrid daubechies invented coiflet mother wavelet, on the request of R. Coifman. ‘Coif’ represents the surname of coiflet family member and five members (coif1 to coif5) are present in this family. This wavelet function and scaling function has 2N and 2N-1 moments equal to zero respectively. Orthogonal, compactly supported but nearly symmetry are the general properties of the coiflet family members. Figure 5 shows waveform of some basically used wavelet family members along with their scaling function.
Symlet Wavelet family

Symlet mother wavelet is modified form of daubechies family. Due to unsymmetrical property of ‘db’ mother wavelet, symlet mother wavelet was invented which have modified symmetry property of ‘db’ family wavelets. It is also known as Daubechies least asymmetric wavelet and represented as ‘symN’ and there are seven members (sym2 to sym8) are belonging to this family. “Orthogonal” and “compactly supported in time” are the most important properties of symlet family members. The scaling and wavelet function waveforms for some mostly used symlet family members are shown in Figure 6.

![Figure 6. Scaling and Wavelet Function of Symlet Family Members sym4, sym6, sym8](image)

Biorthogonal Wavelet family

These wavelet family members have biorthogonal property. The surname of these family members is represented by ‘bior’ and their order is denoted by Nd and Nr. The general characteristics of this family are compactly supported but do not have orthogonal property. Figure 7 present the scaling and wavelet function waveforms of bior3.3 and bior3.5 mother wavelet.

![Figure 7. Scaling and Wavelet Function of bior3.5 and bior3.3 wavelet](image)

All These ‘wavelet 1 D’ family members are used to analyze the signal and to calculate their PRD values. The processor to calculate PRD and MSE values is described in next section.

PERFORMANCE EVALUATION CRITERIA

The main intention of wavelet transform analysis is extract the important information from the signal in the form of energy and remove the redundancy, unwanted and irrelevant information. Error criterion is the one of the most difficult problems in signal compression and reconstruction applications. Consequently, to measure the ability of the reconstructed signal to preserve the relevant information, error criterion has to be defined and the distortion is defined as the difference between the original and the reconstructed signal. The PRD and MSE are the most prominently used distortion measurement techniques and defined as below:

\[
PRD = \left[ \frac{\sum_n \left[ x_n(n) - x_n^r(n) \right]^2}{\sum_n [x_n(n)]^2} \right]^{1/2} \times 100 \tag{7}
\]

\[
MSE = \left[ \frac{1}{N} \sum_n \left( x_n(n) - x_n^r(n) \right)^2 \right] \tag{8}
\]

Here original signal is denoted by ‘x0’ and reconstruction signal is denoted by ‘xr’, ‘N’ introduces the number of samples in both the signals.

In WT analysis, most appropriate mother wavelet for the signal is selected by PRD and MSE values in three steps. Firstly, the wavelet transformed to be applied on the signal is analyzed. Secondly, by the wavelet coefficients which are extracted in step1, the threshold values are calculated. Finally, inverse wavelet transform is applied to decomposed signal and then by eq. (7) PRD values and by eq. (8) MSE values are calculated for the signal. All the process covered in these steps is known as decomposition and reconstruction sequence. The appropriate level for decomposition and reconstruction process is selected by considering the dominant frequency components of the signal. In wavelet transform analysis, the signal is decomposed into P+1 sub-bands with detailed and approximation coefficients by selecting the sampling frequency. Here ‘P’ denotes the level of the wavelet transform. In this paper, level 5 is chosen for wavelet decomposition & reconstruction and thus Global threshold value is used.
Figure 8 shows the algorithm to find the most suitable mother wavelet among various mother wavelets for analyzing the electric signal by calculating PRD and MSE value.

![Flow chart of performance evaluation criteria](image)

By this algorithm the family member which has lowest value of PRD and MSE is considered the most compatible mother wavelet for the signal analysis with high efficiency.

**RESULTS AND ANALYSIS**

In this paper, the transient signal is analyzed by most compatible wavelet function to detect and localize the distortion. For calculating most compatible mother wavelet, transient signal with 1500 data samples is analyzed using wavelet transform and decomposed signal up to 5 levels. The global threshold value is calculated using the detailed coefficients and then the signal is reconstructed. The original signal data and reconstructed signal data are used to calculate the PRD and MSE values for various mother wavelets. This process is repeated till all the mother wavelets are used. The minimum PRD and MSE values give the better way to select compatible mother wavelet function for signal i.e. the mother wavelet with minimum PRD and MSE value is best wavelet for the signal.

Figure 9 and Figure 10 shows the transient signal with approximate and detailed signals up to 5 levels, analyzed by Bior3.5 mother wavelet.

![5 Level Wavelet Decomposition by Bior3.5 Mother Wavelet detailed signal](image)

Figure 11. Original signal, reconstructed signal and error signal using Bior3.5 mother wavelet

where ‘S’ represents original signal, ‘a5’ is 5th level approximate coefficient and d1 to d5 are detail coefficient of
corresponding level. Considering level one detailed coefficient (D1) signal, it can be concluded that when the distortion occurs in the power signal then spikes are introduced in detailed signal which represent the distortion energy by which the disturbance is detected and localized in time format.

Figure 11 shows the original signal which is to be analyzed by the wavelet transform and reconstructed signal after the decomposition up to five levels by bior3.5 mother wavelet, with error signal is the difference between these two signals.

From the Table 1 and Figure 12 to Figure 15 it can be observed that bior3.5 mother wavelet has lowest value of PRD and MSE among all the 17 mother wavelets therefore, bior3.5 mother wavelet is best compatible mother wavelet for this transient signal.

Table 1. PRD and MSE Values for Various Mother Wavelets

<table>
<thead>
<tr>
<th>Wavelet Family</th>
<th>PRD Values</th>
<th>MSE Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Haar</td>
<td>0.7338</td>
<td>2.8604</td>
</tr>
<tr>
<td>Db2</td>
<td>0.4367</td>
<td>1.0130</td>
</tr>
<tr>
<td>Db4</td>
<td>0.2628</td>
<td>0.3669</td>
</tr>
<tr>
<td>Db6</td>
<td>0.2628</td>
<td>0.4167</td>
</tr>
<tr>
<td>Db8</td>
<td>0.2888</td>
<td>0.4429</td>
</tr>
<tr>
<td>Db10</td>
<td>0.3149</td>
<td>0.5269</td>
</tr>
<tr>
<td>Sym4</td>
<td>0.2601</td>
<td>0.3594</td>
</tr>
<tr>
<td>Sym6</td>
<td>0.3144</td>
<td>0.5249</td>
</tr>
<tr>
<td>Sym8</td>
<td>0.2967</td>
<td>0.4676</td>
</tr>
<tr>
<td>Coif1</td>
<td>0.4620</td>
<td>1.1339</td>
</tr>
<tr>
<td>Coif2</td>
<td>0.2558</td>
<td>0.3475</td>
</tr>
<tr>
<td>Coif4</td>
<td>0.2467</td>
<td>0.3232</td>
</tr>
<tr>
<td>Bior 1.3</td>
<td>0.7263</td>
<td>2.8022</td>
</tr>
<tr>
<td>Bior 2.2</td>
<td>0.2841</td>
<td>0.4287</td>
</tr>
<tr>
<td>Bior 3.3</td>
<td>0.2073</td>
<td>0.2282</td>
</tr>
<tr>
<td><strong>Bior 3.5</strong></td>
<td><strong>0.1959</strong></td>
<td><strong>0.2038</strong></td>
</tr>
<tr>
<td>Bior 3.9</td>
<td>0.2413</td>
<td>0.3092</td>
</tr>
</tbody>
</table>

The information lost from original signal to reconstructed signal by bior3.5 is low in comparison to other 16 mother wavelets. In other words, haar wavelet leaks more informations in comparison to other mother wavelet due to highest PRD and MSE value.

CONCLUSION

This paper presents a method to select the most compatible mother wavelet for the transient signal to minimize the leakage problem and to detect the distortion in signal. The main advantage of wavelet transform technique is step by step analysis of the signal and possibly provides good quality of resultant signal with all the information about the disturbances. In term of PRD and MSE choice of appropriate mother wavelet for particular signal at any level is found optimal.

In summary, for the transient signal biorthogonal family member “bior3.5” has minimum PRD and MSE values and it means information lost between original and reconstructed.
signal is minimum. Therefore, among 17 mother wavelets “bior3.5” appears to be the most compatible function for transient signals.

REFERENCES


