Fuzzy Connectedness in Fuzzy Tri Topological Space

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Abstract

In this paper, we introduce fuzzy connectedness in fuzzy tri topological spaces and also defined separation properties in fuzzy tri topological spaces.

Keywords: Fuzzy tri topological space, fuzzy connectedness, fuzzy disconnectedness and fuzzy separation.

INTRODUCTION


Kandil A. [6,7] introduced fuzzy bitopological spaces. In this paper, we introduce fuzzy connectedness and fuzzy separated sets in fuzzy tri topological space.

PRELIMINARIES

Definition 2.1[9]: “Let \(X\) be a nonempty set and \(T_1, T_2\) and \(T_3\) are three topologies on \(X\). The set \(X\) together with three topologies is called a tri topological space and is denoted by \((X, T_1, T_2, T_3)\)”.

Definition 2.2[13]: “A subset \(A\) of a topological space \(X\) is called 123 open set if \(A \in T_1 \cup T_2 \cup T_3\) and complement of 123 open set is 123 closed set”.

Definition 2.4[1]: “A bitopological space is \((X, T_1, T_2)\) said to be connected if and only if \(X\) cannot be expressed as the union of two non empty disjoint sets \(A\) and \(B\) such that \(A\) is open and \(B\) is \(T\) open. When \(X\) can be so expressed, we write \(X = A/B\) and called call this a separation of \(X\)”.

FUZZY SEPARATED SETS IN FUZZY TRI TOPOLOGICAL SPACE

Definition 3.1: Suppose \((X, \tau_1, \tau_2, \tau_3)\) be a fuzzy tri topological space, two non-empty fuzzy subsets \(\mathcal{X}_\lambda, \mathcal{X}_\delta\) of \(X\) are called fuzzy tri separated if and only if \(\mathcal{X}_\lambda \land \text{Ftri-cl}(\mathcal{X}_\delta) = \tilde{0}_X\) and \(\text{Ftri-cl}(\mathcal{X}_\lambda) \land \mathcal{X}_\delta = \tilde{0}_X\).

Both conditions are equivalent to one condition \((\mathcal{X}_\lambda \land \text{Ftri-cl}(\mathcal{X}_\delta)) \lor (\text{Ftri-cl}(\mathcal{X}_\lambda) \land \mathcal{X}_\delta) = \tilde{0}_X\).

Example 3.2: Suppose \(X = \{1, 2, 3, 4\}\) be a non-empty fuzzy set.

Consider three fuzzy topologies on \(X\)

\[\tau_1 = \{ \tilde{1}_X, \tilde{0}_X, \mathcal{X}_{[1,3]}, \mathcal{X}_{[1,4]}, \mathcal{X}_{[1,3,4]} \}\ ,
\[\tau_2 = \{ \tilde{1}_X, \tilde{0}_X, \mathcal{X}_{[2,3]}, \mathcal{X}_{[3]}, \mathcal{X}_{[1,3,4]} \}\ ,
\[\tau_3 = \{ \tilde{1}_X, \tilde{0}_X, \mathcal{X}_{[2]} \}\.

Fuzzy open sets in fuzzy tri topological space are union of all three topologies.

Fuzzy tri open sets of \(X\)

\[X = \{ \tilde{1}_X, \tilde{0}_X, \mathcal{X}_{[1,3]}, \mathcal{X}_{[2]}, \mathcal{X}_{[3]}, \mathcal{X}_{[4]}, \mathcal{X}_{[2,4]}, \mathcal{X}_{[1,2,3]}, \mathcal{X}_{[1,3,4]} \}\ .

If we take \(\mathcal{X}_{[2]}\) and \(\mathcal{X}_{[4]}\) Then
\[\{A \land \text{Ftri-cl}(B)\} \lor \{\text{Ftri-cl}(A) \land B\} = \{\{\mathcal{X}_{[2]}\} \land \{\mathcal{X}_{[4]}\}\} \lor \{\{\mathcal{X}_{[2]}\} \land \{\mathcal{X}_{[4]}\}\} = \emptyset\ .

Hence \(\mathcal{X}_{[2]}\) and \(\mathcal{X}_{[4]}\) are Fuzzy tri separated sets.

Also \(\mathcal{X}_{[1,3]}\) & \(\mathcal{X}_{[1,4]}\) etc. are Fuzzy tri separated sets.
Theorem 3.3: Suppose $\mathcal{X}_\lambda$ and $\mathcal{X}_\delta$ are two fuzzy tri separated subsets of a fuzzy tri topological space $(X, \tau_1, \tau_2, \tau_3)$ and $\mathcal{X}_\lambda \leq \mathcal{X}_\lambda$ and $\mathcal{X}_\delta \leq \mathcal{X}_\delta$, then $\mathcal{X}_\lambda$ and $\mathcal{X}_\delta$ are also fuzzy tri separated.

Proof: Since $\mathcal{X}_\lambda$ and $\mathcal{X}_\delta$ are two fuzzy tri separated sets, then $\mathcal{X}_\lambda \land \text{Ftricl}(\mathcal{X}_\delta) = \bar{0}_X$ and $\text{Ftricl}(\mathcal{X}_\lambda) \land \mathcal{X}_\delta = \bar{0}_X$.

Also $\mathcal{X}_\lambda \leq \mathcal{X}_\lambda \Rightarrow \text{Ftricl}(\mathcal{X}_\lambda) < \text{Ftricl}(\mathcal{X}_\lambda)$ and $\mathcal{X}_\delta \leq \mathcal{X}_\delta \Rightarrow \text{Ftricl}(\mathcal{X}_\delta) < \text{Ftricl}(\mathcal{X}_\delta)$.

Hence $\mathcal{X}_\lambda$ and $\mathcal{X}_\delta$ are fuzzy tri separated.

Theorem 3.4: Two fuzzy tri closed (fuzzy tri open) subsets $\mathcal{X}_\lambda$ and $\mathcal{X}_\delta$ of a fuzzy tri topological space $(X, \tau_1, \tau_2, \tau_3)$ are fuzzy separated iff they are disjoint.

Proof: Given that two fuzzy tri separated sets are disjoint, we have to prove that two disjoint fuzzy tri closed (fuzzy tri open) sets are fuzzy tri separated.

If $\mathcal{X}_\lambda$ and $\mathcal{X}_\delta$ are both disjoint fuzzy tri closed, then:

$\mathcal{X}_\lambda \land \mathcal{X}_\delta = \mathcal{X}_\lambda$, $\text{Ftricl}(\mathcal{X}_\lambda) = \mathcal{X}_\lambda$ and $\text{Ftricl}(\mathcal{X}_\delta) = \mathcal{X}_\delta$.

So that $\mathcal{X}_\lambda \land \text{Ftricl}(\mathcal{X}_\delta) = \bar{0}_X$ and $\text{Ftricl}(\mathcal{X}_\lambda) \land \mathcal{X}_\delta = \bar{0}_X$.

Therefore $\mathcal{X}_\lambda$ and $\mathcal{X}_\delta$ are fuzzy tri separated.

If $\mathcal{X}_\lambda$ and $\mathcal{X}_\delta$ are both disjoint and fuzzy tri open sets, then $\bar{1}_X - \mathcal{X}_\lambda$ and $\bar{1}_X - \mathcal{X}_\delta$ are both fuzzy tri open so that:

$\text{Ftricl}(\bar{1}_X - \mathcal{X}_\lambda) = \bar{1}_X - \mathcal{X}_\lambda$ and $\text{Ftricl}(\bar{1}_X - \mathcal{X}_\delta) = (\bar{1}_X - \mathcal{X}_\delta)$.

Also $\mathcal{X}_\lambda \land \mathcal{X}_\delta = \bar{0}_X \Rightarrow \mathcal{X}_\lambda \leq \bar{1}_X - \mathcal{X}_\delta$ and $\mathcal{X}_\delta \leq \bar{1}_X - \mathcal{X}_\lambda$.

$\Rightarrow \text{Ftricl}(\mathcal{X}_\lambda) < \text{Ftricl}(\bar{1}_X - \mathcal{X}_\delta) = \bar{1}_X - \mathcal{X}_\delta$ and $\text{Ftricl}(\bar{1}_X - \mathcal{X}_\lambda) = \bar{1}_X - \mathcal{X}_\lambda$.

$\text{Ftricl}(\mathcal{X}_\lambda) \land \mathcal{X}_\delta = \bar{0}_X$ and $\mathcal{X}_\lambda \land \text{Ftricl}(\mathcal{X}_\delta) = \bar{0}_X$.

Hence $\mathcal{X}_\lambda$ and $\mathcal{X}_\delta$ are fuzzy tri separated in fuzzy tri topological space.

Fuzzy Connected Sets and Fuzzy Disconnected Sets in Fuzzy Tri Topological Space

Definition 4.1: Suppose $(X, \tau_1, \tau_2, \tau_3)$ fuzzy tri topological space, $\mathcal{X}_\lambda \subseteq X$ is called fuzzy tri disconnected if it is the union of two non-empty fuzzy separated sets. That is, if there exist two non-empty separated sets $\mathcal{X}_\lambda$ and $\mathcal{X}_\delta$ such that $\mathcal{X}_\lambda \land \text{Ftricl}(\mathcal{X}_\delta) = \bar{0}_X$.

$\text{Ftricl}(\mathcal{X}_\lambda) \land \mathcal{X}_\delta = \bar{0}_X$ and $\mathcal{X}_\lambda \land \mathcal{X}_\delta \land \mathcal{X}_\lambda = \mathcal{X}_\lambda$.

$\mathcal{X}_\lambda$ is called fuzzy tri connected if and only if it is not fuzzy tri disconnected.

Example 4.2: Let $X = \{1, 2, 3, 4\}$ be a non-empty fuzzy set.

Consider three fuzzy topologies on X

$\tau_1 = \{\bar{1}_X, \bar{0}_X, \mathcal{X}_1[1], \mathcal{X}_2[1], \mathcal{X}_1[2]\}$.

$\tau_2 = \{\bar{1}_X, \bar{0}_X, \mathcal{X}_1[1], \mathcal{X}_1[3], \mathcal{X}_1[3,4]\}$.

$\tau_3 = \{\bar{1}_X, \bar{0}_X, \mathcal{X}_1[1], \mathcal{X}_2[3,4]\}$.

Fuzzy open sets in fuzzy tri topological spaces are union of all three fuzzy topologies.

Fuzzy tri open sets of

$X = \{\bar{1}_X, \bar{0}_X, \mathcal{X}_1[1], \mathcal{X}_2[1], \mathcal{X}_2[3,4], \mathcal{X}_1[3,4], \mathcal{X}_1[3,4], \mathcal{X}_2[2,3,4]\}$.

If we take $A = \mathcal{X}_1[3,4], C = \mathcal{X}_1[1]$ and $D = \mathcal{X}_2[3,4]$ Then
\[A = C \lor D\] and \[\mathcal{X}_1 \land \mathcal{X}_{(3,4)} = \tilde{0}_X\]

\[C \land \text{Ftri cl}(D) = \mathcal{X}_1 \land \mathcal{X}_{(3,4)} = \tilde{0}_X,\]

\[F - \text{tri cl}(C) \land D = \mathcal{X}_1 \land \mathcal{X}_{(3,4)} = \tilde{0}_X.\]

Then \(\mathcal{X}_1\) and \(\mathcal{X}_{(3,4)}\) are fuzzy tri separated sets.

Hence the set \(\mathcal{X}_{(1,3,4)}\) is fuzzy tri disconnected.

But \(\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_{(3,4)}\) are fuzzy tri connected sets.

**Remarks 4.3:**

(i) The empty set in \(\text{Ftri } O(X)\) is trivially fuzzy tri connected.

(ii) Every singleton set in \(\text{Ftri } O(X)\) is fuzzy tri connected.

**Definition 4.4:** Two points \(\mathcal{X}_1\) and \(\mathcal{X}_{(3,4)}\) of a fuzzy tri topological space \(X\) are called fuzzy tri connected if they are contained in a fuzzy tri connected subset of \(X\).

**Example 4.5:** In the last example the points \(\mathcal{X}_1\) and \(\mathcal{X}_{(3,4)}\) are fuzzy tri connected because they are contained in \(\mathcal{X}_{(1,3,4)}\) which is fuzzy tri connected subset of \(X\).

**Theorem 4.6:** A fuzzy tri topological space \((X, \tau_1, \tau_2, \tau_3)\) is fuzzy tri disconnected if there exists a non-empty fuzzy proper subset of \(X\) which is both fuzzy tri open and fuzzy tri closed.

**Proof:** Suppose \(\mathcal{X}_\lambda\) be a non-empty fuzzy proper subset of \(X\) which is both fuzzy tri open and fuzzy tri closed in \(X\). We have to show that \(X\) is fuzzy tri disconnected:

Let \(\mathcal{X}_\mu = \tilde{1}_X - \mathcal{X}_\lambda\). Then \(\mathcal{X}_\delta\) is non-empty since \(\mathcal{X}_\lambda\) is a fuzzy proper subset of \(X\). Moreover,

\[\mathcal{X}_\mu \lor \mathcal{X}_\lambda = \tilde{1}_X\]

and \(\mathcal{X}_\mu \land \mathcal{X}_\lambda = \tilde{0}_X\), since \(\mathcal{X}_\lambda\) is both fuzzy tri open and fuzzy tri closed, \(\mathcal{X}_\mu\) is also both fuzzy tri open and fuzzy tri closed. Hence

\[\text{Ftri cl}(\mathcal{X}_\lambda) = \mathcal{X}_\lambda\]

and \(\text{Ftri cl}(\mathcal{X}_\mu) = \mathcal{X}_\mu\) it follows that

\[\text{Ftri cl}(\mathcal{X}_\lambda) \land \mathcal{X}_\mu = \tilde{0}_X\]

\[\mathcal{X}_\lambda \land \text{Ftri cl}(\mathcal{X}_\mu) = \tilde{0}_X\].

Thus \(X\) has been expressed as a union of two fuzzy tri separated sets and so \(X\) is fuzzy tri disconnected.

Conversely, Let \(X\) is fuzzy tri disconnected. Then there exist non empty subsets \(\mathcal{X}_\lambda\) and \(\mathcal{X}_\mu\) of \(X\) such that

\[\text{Ftri cl}(\mathcal{X}_\lambda) \land \mathcal{X}_\mu = \tilde{0}_X\]

and

\[\mathcal{X}_\lambda \land \text{Ftri cl}(\mathcal{X}_\mu) = \tilde{0}_X\] and \(\mathcal{X}_\lambda \cup \mathcal{X}_\mu = \tilde{1}_X\).

Since \(\text{Ftri cl}(\mathcal{X}_\lambda) = \mathcal{X}_\lambda\) and

\[\text{Ftri cl}(\mathcal{X}_\mu) = \mathcal{X}_\mu\]

and

\[\mathcal{X}_\lambda \land \text{Ftri cl}(\mathcal{X}_\mu) = \tilde{0}_X\] has been non empty since \(\mathcal{X}_\mu\) is fuzzy tri separated sets.

And \(\mathcal{X}_\lambda\) is also fuzzy tri open. Now \(\mathcal{X}_\lambda \land \text{Ftri cl}(\mathcal{X}_\mu) = \tilde{0}_X\). Hence

\[\mathcal{X}_\lambda = \tilde{1}_X - \mathcal{X}_\mu\] and \(\mathcal{X}_\mu\) is non empty, \(\mathcal{X}_\lambda\) is a fuzzy proper subset of \(X\), Now \(\mathcal{X}_\lambda \lor \text{Ftri cl}(\mathcal{X}_\mu) = \tilde{1}_X\).

[Since \(\mathcal{X}_\lambda \lor \mathcal{X}_\mu = \tilde{1}_X\) and

\[\mathcal{X}_\lambda \land \text{Ftri cl}(\mathcal{X}_\mu) = \tilde{0}_X\]

but \(\mathcal{X}_\lambda \lor \text{Ftri cl}(\mathcal{X}_\mu) < \tilde{1}_X\) always] Also

\[\mathcal{X}_\lambda \land \text{Ftri cl}(\mathcal{X}_\mu) = \tilde{0}_X\]

and similarly \(\mathcal{X}_\mu = \tilde{1}_X - (\text{Ftri cl}(\mathcal{X}_\lambda))\).

Since \(\text{Ftri cl}(\mathcal{X}_\lambda)\) and \(\text{Ftri cl}(\mathcal{X}_\mu)\) are fuzzy tri closed sets, it follows that \(\mathcal{X}_\lambda\) and \(\mathcal{X}_\mu\) are fuzzy tri open sets, and since \(\mathcal{X}_\lambda = \tilde{1}_X - \mathcal{X}_\mu\), \(\mathcal{X}_\lambda\) is also fuzzy tri closed. Thus \(\mathcal{X}_\lambda\) is non empty fuzzy proper subset of \(X\) which is both fuzzy tri open and fuzzy tri closed.

In the same way we can show that \(\mathcal{X}_\mu\) is also non empty fuzzy proper subset of \(X\) which is both fuzzy tri open and fuzzy tri closed.

**Theorem 4.7:** Let \((X, \tau_1, \tau_2, \tau_3)\) be a fuzzy tri topological space and \(\mathcal{X}_\lambda < \tilde{1}_X\). If \(\mathcal{X}_\lambda\) is fuzzy tri connected, then so is \(\text{Ftri cl}(\mathcal{X}_\lambda)\).

**Proof:** Let \(\mathcal{X}_\lambda\) be the fuzzy tri connected subset of a fuzzy tri topological space \((X, \tau_1, \tau_2, \tau_3)\). To prove that...
Ftri cl(χ₁) is fuzzy tri connected. Suppose contrary. Then Ftri cl(χ₁) is fuzzy tri disconnected. Then there exist non empty tri fuzzy sets χ₁, χ₂ such that
\[(\tilde{1}_X - χ₁) \cap χ₂ = \tilde{0}_X, χ₁ \cap (\tilde{1}_X - χ₂) = \tilde{0}_X.\]

Ftri cl χ₁ = χ₁ ∨ χ₂
χ₁ ∨ χ₂ = Ftri cl χ₁ > χ₁,
⇒ χ₁ < χ₁ ∨ χ₂, χ₁ is fuzzy tri connected.

χ₁ < χ₁ ⇒ Ftri cl χ₁ < Ftri cl χ₁
⇒ χ₁ < χ₁ ∨ χ₂ = Ftri cl χ₁ ∧ χ₂ = \tilde{0}_X ......(i)

Ftri cl χ₁ = χ₁ ∨ χ₂
⇒ χ₂ < Ftri cl χ₁
⇒ Ftri cl χ₁ ∧ χ₂ = χ₂
⇒ χ₂ = \tilde{0}_X

For Ftri cl χ₁ ∧ χ₂ = \tilde{0}_X (from (i))
χ₁ in X such that Fbcl(χ₁) = χ₁ \lor χ₁.

Since χ₁ = (χ₁ \lor χ₁) \lor (χ₁ \lor χ₁) and
Fcl(χ₁ \lor χ₁) \lor Fcl(χ₁ \lor χ₁) and
Fcl(χ₁ \lor χ₁) \lor Fcl(χ₁ \lor χ₁)
χ₁ \lor χ₁ = \tilde{0}_X then
Fcl(χ₁ \lor χ₁) χ₁ \lor χ₁ = \tilde{0}_X. Hence
Fcl(χ₁ \lor χ₁) \lor (χ₁ \lor χ₁) = \tilde{0}_X. Similarly
(Fcl(χ₁ \lor χ₁) \lor (χ₁ \lor χ₁) = \tilde{0}_X. Therefore
χ₁ is fuzzy connected a contradiction for χ₁ ≠ \tilde{0}_X

Similarly, χ₁ ∧ χ₂ ⇒ χ₁ = \tilde{0}_X. Again we get a contradiction. Hence, if χ₁ is fuzzy tri connected, then so is
Ftri cl(χ₁).

**Corollary 4.8:** A fuzzy tri topological space
(X, τ₁, τ₂, τ₃) is fuzzy tri connected if and only if the only non-empty fuzzy subset of X which is both fuzzy tri open and fuzzy tri closed in X is X itself.

**Corollary 4.9:** A fuzzy subset Y of a fuzzy tri topological space X is fuzzy disconnected if and only if Y is the union of two non-empty disjoint fuzzy sets both fuzzy tri open (fuzzy tri closed) in Y.

**Theorem 4.10:** (X, τ₁, τ₂, τ₃) be a fuzzy tri topological space. If χ₁ is a fuzzy connected set of X and
χ₁, χ₂ are fuzzy tri separated sets of X with
χ₁ < χ₁ ∧ χ₂, then either χ₁ < χ₁ or χ₁ < χ₂.

**Proof:** Let χ₁ < χ₁ ∧ χ₂. Since
χ₁ = (χ₁ ∧ χ₁) ∨ (χ₁ ∧ χ₂) then
(χ₁ ∧ χ₁) ∧ Ftri cl(χ₁ ∧ χ₂) < χ₁ ∧ cl(χ₁ ∧ χ₂) = \tilde{0}_X.

By similar reasoning, we have
(χ₁ ∧ χ₁) ∧ Ftri cl(χ₁ ∧ χ₂) < χ₁ ∧ Ftri cl(χ₁ ∧ χ₂) = \tilde{0}_X

Suppose that (χ₁ ∧ χ₁) and χ₁ ∧ χ₂ are nonempty. Then χ₁ is not fuzzy tri connected. This is a contradiction. Thus either χ₁ ∧ χ₁ = \tilde{0}_X or χ₁ ∧ χ₂ = \tilde{0}_X. This implies that χ₁ < χ₁ or χ₁ < χ₂.

**Theorem 4.11:** Let (X, τ₁, τ₂, τ₃) and (Y, τ'_1, τ'_2, τ'_₃) are two fuzzy tri topological spaces. Let f : I^X → I^Y be a fuzzy tri continuous function. If χ₁ is fuzzy tri connected in X, then f(χ₁) is fuzzy tri connected in Y.
Proof: Suppose that $f(\mathcal{X}_1)$ is fuzzy tri disconnected in $Y$. There exist two fuzzy tri separated sets $\mathcal{X}_{\lambda_1}$ and $\mathcal{X}_{\lambda_2}$ of $Y$ such that $f(\mathcal{X}_{\lambda_1}) = \mathcal{X}_{\lambda_1} \cup \mathcal{X}_{\lambda_2}$. Set

$\mathcal{X}_{\delta_1} = \mathcal{X}_{\lambda_1} \land f^{-1}(\mathcal{X}_{\lambda_1})$ and

$\mathcal{X}_{\delta_2} = \mathcal{X}_{\lambda_2} \land f^{-1}(\mathcal{X}_{\lambda_2})$.

Since $f(\mathcal{X}_{\lambda_1}) \land \mathcal{X}_{\lambda_1} \neq \tilde{0}_X$ then $\mathcal{X}_{\lambda_1} \land f^{-1}(\mathcal{X}_{\lambda_1}) \neq \tilde{0}_X$ and so $\mathcal{X}_{\delta_1} \neq \tilde{0}_X$. Similarly $\mathcal{X}_{\delta_2} \neq \tilde{0}_X$. Since $\mathcal{X}_{\delta_1} \land \mathcal{X}_{\delta_2} = \tilde{0}_X$, $\mathcal{X}_{\delta_1} \land \mathcal{X}_{\delta_2} = \tilde{0}_X$. Since $f$ is continuous then by Lemma 4.7, $Ftricl(f^{-1}(\mathcal{X}_{\lambda_2})) < f^{-1}(Ftricl(\mathcal{X}_{\lambda_2}))$ and $\mathcal{X}_{\delta_2} < f^{-1}(\mathcal{X}_{\lambda_2})$.

$Ftricl(\mathcal{X}_{\lambda_1}) \prec Ftricl(\mathcal{X}_{\lambda_1})$ and $\mathcal{X}_{\lambda_1} \land Ftricl(\mathcal{X}_{\lambda_1}) = \tilde{0}_X$. This implies that $\mathcal{X}_{\delta_1} \prec \mathcal{X}_{\lambda_1} \land Ftricl(\mathcal{X}_{\lambda_1})$ and $\mathcal{X}_{\delta_1} = Ftricl(\mathcal{X}_{\lambda_1}) \land \mathcal{X}_{\delta_1} = \tilde{0}_X$. Thus $\mathcal{X}_{\delta_1}$ is an fuzzy tri empty set for if $\mathcal{X}_{\delta_1}$ is fuzzy tri nonempty, this is a contradiction. Suppose that $\mathcal{X}_{\delta_1} \prec \mathcal{X}_{\delta_1}$. By similar way, it follows that $\mathcal{X}_{\delta_1}$ is empty. This is a contradiction. Hence, $\mathcal{X}_{\lambda_2}$ is fuzzy tri connected.

Theorem 4.13: If $\mathcal{X}_{\lambda_1}$ and $\mathcal{X}_{\lambda_2}$ are fuzzy tri connected sets which are fuzzy tri separated, then $\mathcal{X}_{\lambda_1} \lor \mathcal{X}_{\lambda_2}$ is fuzzy tri connected.

Proof: Suppose $\mathcal{X}_{\lambda_1} \lor \mathcal{X}_{\lambda_2}$ is tri disconnected and suppose $\mathcal{X}_{\delta_1} \lor \mathcal{X}_{\delta_2}$ is fuzzy tri disconnected sets of $\mathcal{X}_{\lambda_1} \lor \mathcal{X}_{\lambda_2}$. Since $\mathcal{X}_{\delta_1}$ is a fuzzy tri connected subsets of $\mathcal{X}_{\lambda_1} \lor \mathcal{X}_{\lambda_2}$, then $\mathcal{X}_{\lambda_1} \lor \mathcal{X}_{\lambda_2}$ is fuzzy tri connected. Similarly either $\mathcal{X}_{\lambda_2} \prec \mathcal{X}_{\delta_1}$ or $\mathcal{X}_{\lambda_2} \prec \mathcal{X}_{\delta_2}$.

If $\mathcal{X}_{\lambda_1} \prec \mathcal{X}_{\delta_1}$ and $\mathcal{X}_{\lambda_2} \prec \mathcal{X}_{\delta_2}$ then $(\mathcal{X}_{\lambda_1} \lor \mathcal{X}_{\lambda_2}) \land \mathcal{X}_{\lambda_2} = \mathcal{X}_{\lambda_2}$ and $(\mathcal{X}_{\lambda_1} \lor \mathcal{X}_{\lambda_2}) \land \mathcal{X}_{\lambda_1} = \mathcal{X}_{\lambda_1}$ are fuzzy tri separated but this contradicts the hypothesis. Hence either

$\mathcal{X}_{\lambda_1} \lor \mathcal{X}_{\lambda_2} \prec \mathcal{X}_{\delta_1}$ or $\mathcal{X}_{\lambda_1} \lor \mathcal{X}_{\lambda_2} \prec \mathcal{X}_{\lambda_2}$ and

$\mathcal{X}_{\delta_1} \lor \mathcal{X}_{\delta_2}$ is not a fuzzy tri disconnected subset of $\mathcal{X}_{\lambda_1} \lor \mathcal{X}_{\lambda_2}$. In other words $\mathcal{X}_{\lambda_1} \lor \mathcal{X}_{\lambda_2}$ is fuzzy tri connected.

CONCLUSION:

In this paper the idea of tri connectedness and tri separation were introduced and studied in fuzzy tri topological spaces.
REFERENCES


