International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 22 (2018) pp. 15519-15529

Properties of $[\gamma, \gamma']$-Preopen Sets

Dr. S. Kousalya Devi[1] and P.Komalavalli[2]

[1]*Principal, Government Arts Colleges, Kulithalai, Tamilnadu, India.*

[2]*Assistant professor, Department of mathematics, Thanthai Hans Roever College,Perambalur, TamilNadu, India.*

Abstract:

In this paper, we investigate some more properties of bioperation-preopen sets in bioperation-topological space.

Keywords: Topological spaces, $[\gamma,\gamma']$-open set, $[\gamma,\gamma']$-preopen set.

2000 Mathematics Subject Classification. :*54A05, 54A10.*

1. INTRODUCTION

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real Analysis con-cerns the various modified forms of continuity, separation axioms etc. by utilizing generalized open sets. Kasahara [1] defined the concept of an operation on topological spaces and introduce the concept of γ- closed graphs of a function. Maki and Noiri [4] introduced the notion of $\tau_{[\gamma,\gamma']}$ which is the collection of all $[\gamma,\gamma']$-open sets in a topological space (X, τ). In this paper, we investigate some more properties of bioperation-preopen sets in bioperation-topological space.

2. PRELIMINARIES

Definition 2.1. *Let (X, τ) be a topological space. An operation $[\gamma,\gamma']$ [1] on the topology τ is function from τ on to power set* P(X) *of X such that $V \subset V^{\gamma}$ for each $V \in\tau$, where V^{γ} denotes the value of γ at V. It is denoted by $\gamma : \tau \rightarrow$* P($X$).

Definition 2.2. *Let (X, τ) be a topological space. An operation γ on τ is said to be regular* [1] *if for every open neighborhood U and V of each $x \in X$, there exists an open neighborhood W of x such that* $U^{\gamma} \cap V^{\gamma} \supset W^{\gamma}$.

Definition 2.3. *A subset A of a topological space (X, τ) is said to be $[\gamma,\gamma']$-open set [4] if for each $x\in A$ there exist open neighbourhoods Uand V of x such that $U^{\gamma} \cap V^{\gamma'} \subset A$. The complement of $[\gamma,\gamma']$-openset is called $[\gamma,\gamma']$-closed. $\tau_{[\gamma,\gamma']}$ denotes set of all $[\gamma,\gamma']$-open sets in(X, τ).*

Definition 2.4. [4] *Let A be subset of a topological space (X, τ) and γ,γ' be operations on τ. Then*

(i) *the $\tau_{[\gamma,\gamma']}$-closure of A is defined as intersection of all $[\gamma,\gamma']$- closed sets containing A. That is $\tau_{[\gamma,\gamma']}$*-Cl(A) = {F : *F is $[\gamma,\gamma']$-closed and $A\subset F$}* .

(ii) *the $\tau_{[\gamma,\gamma']}$-interior of A is defined as union of all $[\gamma,\gamma']$-open sets contained in A. That is, $\tau_{[\gamma,\gamma']}$*-Int(A) = {U : *U is $[\gamma,\gamma']$-open and $U \subset A$*}.

Definition 2.5. *A subset A of a topological space (X, τ) is said to be*

(i) $[\gamma,\gamma']$*-regular open* [3] *if* $\tau_{[\gamma,\gamma']}$-Int($\tau_{[\gamma,\gamma']}$-Cl(A)) = A.

(ii) $[\gamma,\gamma']$*-preopen* [2] *if $A\subset$* $\tau_{[\gamma,\gamma']}$-Int($\tau_{[\gamma,\gamma']}$ Cl(A)).

(iii) $[\gamma,\gamma']$*-dense* [2] *if* $\tau_{[\gamma,\gamma']}$-Cl(A)) = X.

The complement of a $[\gamma,\gamma']$-preopen set is called a $[\gamma,\gamma']$-preclosed set. The familyof all $[\gamma,\gamma']$-preopen (resp. $[\gamma,\gamma']$-preclosed) sets of (X, τ) is denoted by $[\gamma,\gamma']$-$PO(X)$ (resp. $[\gamma,\gamma']$-$PC(X)$). The family of all $[\gamma,\gamma']$-preopen sets of (X, τ) containing the point x is denoted by $[\gamma,\gamma']$- $PO(X, x)$.

Definition 2.6. [2] *Let A be subset of a topological space (X, τ) and γ,γ' be operations on τ. Then*

(i) *the $\tau_{[\gamma,\gamma']}$-preclosure of A is defined as intersection of all $[\gamma,\gamma']$- preclosed sets containing A. That is, $\tau_{[\gamma,\gamma']}$ -p* Cl(A) = {F : *F is $[\gamma,\gamma']$-preclosed and $A\subset F$}* .

(ii) *the $\tau_{[\gamma,\gamma']}$-preinterior of A is defined as union of all*

$[\gamma,\gamma']$-preopen sets contained in A. That is, $\tau_{[\gamma,\gamma']}$-p Int(A) = {U : U is $[\gamma,\gamma']$-preopen and U ⊂A}.

Theorem 2.7. [2] *Let A be subset of a topological space (X, τ) and γ,γ' be operations on τ. Then*

(i) *A is $[\gamma,\gamma']$-preopen if and only if A = $\tau_{[\gamma,\gamma']}$-p Int(A).*

(ii) *A point $x\in \tau_\gamma$ Cl(A) if and only if U ∩ A ≠ ∅ for every U∈ $[\gamma,\gamma']$-PO(X, x).*

(iii) *$\tau_{[\gamma,\gamma']}$-pCl(A) is the smallest $[\gamma,\gamma']$-preclosed subset of X containing A.*

(iv) *A is $[\gamma,\gamma']$-preclosed if and only if A = $\tau_{[\gamma,\gamma']}$-pCl(A).*

(v) $\tau_{[\gamma,\gamma']}$-pInt $(X\setminus A) = X\setminus \tau_{[\gamma,\gamma']}$-pCl(A).

(vi) $\tau_{[\gamma,\gamma']}$-pCl $(X\setminus A) = X\setminus \tau_{[\gamma,\gamma']}$-pInt (A).

Definition 2.8. [2] *Let (X, τ) be a topological space and γ,γ' be operations on τ. Then a subset A of X is said to be $[\gamma,\gamma']$-pre g.closed (written as $[\gamma,\gamma']$-pg.closed) set if $\tau_{[\gamma,\gamma']}$-pCl(A) ⊂ U whenever A⊂ U and U is $[\gamma,\gamma']$-preopen.*

Theorem 2.9. [2] *Let (X, τ) be a topological space and γ,γ' be operations on τ. Then subset A of X is $[\gamma,\gamma']$-pg.closed,then $\tau_{[\gamma,\gamma']}$-pCl(A) \ A* does not contain any nonempty $[\gamma,\gamma']$-preclosed set.

3. PROPERTIES OF $[\gamma,\gamma']$-PREOPENSETS

Throught this paper, the operators γ and γ' are defined on (X, τ) and the operators β and β' are defined on (Y, σ).

Theorem 3.1. *For any subset of a space (X, τ) the following are equivalent:*

(i) *S ∈ $[\gamma,\gamma']$-PO(X).*

(ii) *There is a $[\gamma,\gamma']$-regular open set G ⊂ X such that S ⊂ G and $\tau_{[\gamma,\gamma']}$-Cl(S) = $\tau_{[\gamma,\gamma']}$-Cl(G).*

(iii) *S is the intersection of a $[\gamma,\gamma']$-regular open set and a $[\gamma,\gamma']$- dense set.*

(iv) *S is the intersection of a $[\gamma,\gamma']$-open set and a $[\gamma,\gamma']$-dense set.*

Proof. (i) ⇒ (ii): Let S ∈ $[\gamma,\gamma']$-PO(X). Then S ⊂ $\tau_{[\gamma,\gamma']}$-Int($\tau_{[\gamma,\gamma']}$- Cl(S)). Let G = $\tau_{[\gamma,\gamma']}$-Int(S). Then G is $[\gamma,\gamma']$-regular open with S ⊂ G and S = G. (ii)⇒ (iii): Let D = S ∪ (X \ G). Then D is $[\gamma,\gamma']$-dense and S = G ∩ D. (iii)⇒ (iv): This is trivial. (iv)⇒ (i): Suppose S =G ∩ D *with* G is $[\gamma,\gamma']$-open and D$[\gamma,\gamma']$-dense. Then S = G, hence S ⊂ G ⊂ $\tau_{[\gamma,\gamma']}$-Cl(G) = S.

Theorem 3.2. *If every subset of X is either $[\gamma,\gamma']$-open or $[\gamma,\gamma']$- closed, then every $[\gamma,\gamma']$-preopen set in X is $[\gamma,\gamma']$-open.*

Proof.Let A be a $[\gamma,\gamma']$-preopen in X. If A is not $[\gamma,\gamma']$-open, then A is $[\gamma,\gamma']$-closed by hypothesis. Hence A = $\tau_{[\gamma,\gamma']}$-Cl(A), and $\tau_{[\gamma,\gamma']}$- Int($\tau_{[\gamma,\gamma']}$-Cl(A)) = $\tau_{[\gamma,\gamma']}$-Int(A) is a proper subset of A. Thus, $A \not\subseteq \tau_{[\gamma,\gamma']} - Int(\tau_{[\gamma,\gamma']} - Cl(A))$,so that Ais not $[\gamma,\gamma']$-preopen, contradiction.

Theorem 3.3. *Let (X, τ) be a topological space in which every $[\gamma,\gamma']$- preopen set in X is $[\gamma,\gamma']$-open. Then each singleton in X is either $[\gamma,\gamma']$-open or $[\gamma,\gamma']$-closed.*

Proof.Let x ∈ X, and suppose that {x} is not $[\gamma,\gamma']$-open. Then {x} is not $[\gamma,\gamma']$-preopen. Hence {x} ⊈ $\tau_{[\gamma,\gamma']}$-Int($\tau_{[\gamma,\gamma']}$-Cl({x}), so that $\tau_{[\gamma,\gamma']}$-Int($\tau_{[\gamma,\gamma']}$-Cl({x}) = ∅. We have that $\tau_{[\gamma,\gamma']}$-Int($\tau_{[\gamma,\gamma']}$-Cl(X \ {x}) ⊃ $\tau_{[\gamma,\gamma']}$-Int($\tau_{[\gamma,\gamma']}$-Cl(X \ ($\tau_{[\gamma,\gamma']}$-Cl({x}))) = $\tau_{[\gamma,\gamma']}$-Int(X \ ($\tau_{[\gamma,\gamma']}$-Int($\tau_{[\gamma,\gamma']}$-Cl({x})))) = X ⊃ X \ {x}. Thus, X \ {x} is $[\gamma,\gamma']$-preopen and hence$[\gamma,\gamma']$-open. Therefore, {x} is $[\gamma,\gamma']$-closed.

Theorem 3.4. *For a topological space (X, τ) and γ,γ' be regular op- erations on τ, the following are equivalent:*

(i) *Every $[\gamma,\gamma']$-preopen set is $[\gamma,\gamma']$-open.*

(ii) *Every $[\gamma,\gamma']$-dense set is $[\gamma,\gamma']$-open.*

Proof.(i)⇒ (ii): Let A be a $[\gamma,\gamma']$-dense subset of X. Then $\tau_{[\gamma,\gamma']}$- Int($\tau_{[\gamma,\gamma']}$-Cl(A) = X, so that $A \subset \tau_{[\gamma,\gamma']}$-Int($\tau_{[\gamma,\gamma']}$-Cl(A)) and A is $[\gamma,\gamma']$-preopen. Hence A is $[\gamma,\gamma']$-open. (ii)⇒ (i): Let B be a $[\gamma,\gamma']$- preopen subset of X, so that $B \subset \tau_{[\gamma,\gamma']}$-Int($\tau_{[\gamma,\gamma']}$-Cl(B)) = G , say.hen $\tau_{[\gamma,\gamma']}$-Cl(B) = $\tau_{[\gamma,\gamma']}$-Cl(G), so that $\tau_{[\gamma,\gamma']}$-Cl($X \setminus G$) ∪ B) = $\tau_{[\gamma,\gamma']}$-Cl(X \ G) ∪ $\tau_{[\gamma,\gamma']}$-Cl(B) = (X \ G) ∪ $\tau_{[\gamma,\gamma']}$-Cl(G) = X, and thus ($X \setminus G$) ∪ B is $[\gamma,\gamma']$ -dense in X. Thus, ($X \setminus G$) ∪ B is $[\gamma,\gamma']$-open. Now, B = (($X \setminus G$) ∪ B) ∩ G, the intersection of two $[\gamma,\gamma']$-open sets is $[\gamma,\gamma']$ -open ([4], Proposition 2.7), so that B *is* $[\gamma,\gamma']$-open.

Theorem 3.5. *(X, τ) is a topological space in which every subset is $[\gamma,\gamma']$-preopen if and only if every $[\gamma,\gamma']$-open set in (X, τ) is $[\gamma,\gamma']$- closed.*

Proof. Let G *be* $[\gamma,\gamma']$-open. Then $X \setminus G$ = $\tau_{[\gamma,\gamma']}$-Cl($X \setminus G$) which is $[\gamma,\gamma']$-preopen, so that $\tau_{[\gamma,\gamma']}$-Cl($X \setminus G$) ⊂ $\tau_{[\gamma,\gamma']}$-Int($\tau_{[\gamma,\gamma']}$-Cl($\tau_{[\gamma,\gamma']}$-Cl($X \setminus G$))) = $\tau_{[\gamma,\gamma']}$-Int($\tau_{[\gamma,\gamma']}$-Cl($X \setminus G$)) = $\tau_{[\gamma,\gamma']}$-Int($X \setminus G$). Thus, $X \setminus G$ = $\tau_{[\gamma,\gamma']}$-Int($X \setminus G$), so that $X \setminus G$ is $[\gamma,\gamma']$-open, and G is $[\gamma,\gamma']$-closed. Conversely, let A be any subset of X. Then X \ $\tau_{[\gamma,\gamma']}$-Cl(A) is $[\gamma,\gamma']$- open, and hence $[\gamma,\gamma']$-closed. Thus, $X \setminus \tau_{[\gamma,\gamma']}$-Cl($A$) = $\tau_{[\gamma,\gamma']}$-Cl($X \setminus \tau_{[\gamma,\gamma']}$-Cl($A$)) = $X \setminus \tau_{[\gamma,\gamma']}$-Int($\tau_{[\gamma,\gamma']}$-Cl(A)), so that $A \subset \tau_{[\gamma,\gamma']}$-Cl($A$) = $\tau_{[\gamma,\gamma']}$- Int ($\tau_{[\gamma,\gamma']}$-Cl(A)), and hence A is $[\gamma,\gamma']$-preopen.

Theorem 3.6. *Let (X, τ) be a topological space, G be a $[\gamma,\gamma']$-open subset of X and b be a point of* $\tau_{[\gamma,\gamma']}$-Cl(G) \ G. *Then {b} is not $[\gamma,\gamma']$- preopen in (X, τ).*

Proof. Suppose {b} is $[\gamma,\gamma']$-preopen, so that {b} ⊂ $\tau_{[\gamma,\gamma']}$-Int($\tau_{[\gamma,\gamma']}$-Cl({b})). Thus, G ∩ $\tau_{[\gamma,\gamma']}$-Int($\tau_{[\gamma,\gamma']}$-Cl({b})) ≠ ∅. Let $c \in G$ ∩ $\tau_{[\gamma,\gamma']}$-Int($\tau_{[\gamma,\gamma']}$-Cl({b})), so $c \in \tau_{[\gamma,\gamma']}$-Cl({$b$}) and hence {$b$} ∩ ($G$ ∩ $\tau_{[\gamma,\gamma']}$-Int($\tau_{[\gamma,\gamma']}$-Cl({b})) ≠ ∅. This contradicts the fact that {b} ∩ G = ∅. Hence {b} is not $[\gamma,\gamma']$-preopen.

Theorem 3.7. *Let (X, τ) be a topological space, G be a $[\gamma,\gamma']$-regular open subset of X and b be a point of* $\tau_{[\gamma,\gamma']}$-Cl(G) \ G. *Then G ∪ {b} is not $[\gamma,\gamma']$-preopen in (X, τ).*

Proof. We have, since $\tau_{[\gamma,\gamma']}$-Cl({b}) ⊂ $\tau_{[\gamma,\gamma']}$-Cl(G), so that $\tau_{[\gamma,\gamma']}$-Int($\tau_{[\gamma,\gamma']}$-Cl(G∪{b})) = $\tau_{[\gamma,\gamma']}$-Int($\tau_{[\gamma,\gamma']}$-Cl(G)∪ $\tau_{[\gamma,\gamma']}$-Cl({b})) = $\tau_{[\gamma,\gamma']}$-Int($\tau_{[\gamma,\gamma']}$- Cl(G) = G, and thus G ∪ {b} ⊄ $\tau_{[\gamma,\gamma']}$-Int($\tau_{[\gamma,\gamma']}$-Cl(G ∪ {b})). Hence G ∪ {b} is not $[\gamma,\gamma']$-preopen.

Theorem 3.8. *Let (X, τ) be a topological space. Then every singleton of X is either $[\gamma,\gamma']$-open or $[\gamma,\gamma']$-preclosed.*

Proof. If x} is not $[\gamma,\gamma']$-open, then $\tau_{[\gamma,\gamma']}$-Int({x}) = ϕ . Thus, $\tau_{[\gamma,\gamma']}$- Cl($\tau_{[\gamma,\gamma']}$-Int({x})) = ϕ ; hence {x} is $[\gamma,\gamma']$-preclosed. The proof of the second part is straightforward.

Theorem 3.9. *If A is a $[\gamma,\gamma']$-preopen and $[\gamma,\gamma']$-pg.closed subset of (X, τ), then A is $[\gamma,\gamma']$-preclosed.*

Proof. Since A is $[\gamma,\gamma']$-preopen and $[\gamma,\gamma']$-pg.closed, $\tau_{[\gamma,\gamma']}$-pCl(A) ⊂ A and hence $\tau_{[\gamma,\gamma']}$-pCl(A) = A. This implies that A is $[\gamma,\gamma']$-preclosed by Theorem 2.7 (iv).

Theorem 3.10. *If A is a $[\gamma,\gamma']$-pg.closed subset of (X, τ) such that $A \subset B \subset \tau_{[\gamma,\gamma']}$-pCl(A), then B is also $[\gamma,\gamma']$-pg.closed subset of (X, τ).*

Proof. Let U be a $[\gamma,\gamma']$-preopen set in (X, τ) such that $B \subset U$. Then $A \subset U$. Since A is $[\gamma,\gamma']$-pg.closed, then $\tau_{[\gamma,\gamma']}$-pCl(A) ⊂ U . Now, since $\tau_{[\gamma,\gamma']}$-pCl(A) is $[\gamma,\gamma']$-preclosed, $\tau_{[\gamma,\gamma']}$-pCl(B) ⊂ $\tau_{[\gamma,\gamma']}$-pCl($\tau_{[\gamma,\gamma']}$- pCl(A)) = $\tau_{[\gamma,\gamma']}$-pCl(A) ⊂ U . Therefore, B is also a $[\gamma,\gamma']$-pg.closed.

Theorem 3.11. *A set A in a topological space (X, τ) is $[\gamma,\gamma']$-pg.open if and only if $F \subset \tau_{[\gamma,\gamma']}$-p* Int($A$) *whenever F is $[\gamma,\gamma']$-preclosed in (X, τ) and $F \subset A$.*

Proof. Let A be $[\gamma,\gamma']$-pg.open. Let F be $[\gamma,\gamma']$-

preclosed and $F \subset A$. Then $X \setminus A \subset X \setminus F$, where $X \setminus F$ is $[\gamma,\gamma']$-preopen. $[\gamma,\gamma']$-pg.closedness of $X \setminus A$ implies $\tau_{[\gamma,\gamma']}$-pCl$(X \setminus A) \subset X \setminus F$. By Theorem 2.7, $X \setminus \tau_{[\gamma,\gamma']}$-$p$ Int$(A) \subset X \setminus F$. That is, $F \subset \tau_{[\gamma,\gamma']}$-$p$ Int(A). Conversely, Suppose if F is $[\gamma,\gamma']$-preclosed and $F \subset A$ implies $F \subset \tau_{[\gamma,\gamma']}$-$p$ Int(A). Let $X \setminus A \subset U$ where U is $[\gamma,\gamma']$-preopen. Then $X \setminus U \subset A$ where $X \setminus U$ is $[\gamma,\gamma']$-preclosed. By supposition, $X \setminus U \subset \tau_{[\gamma,\gamma']}$-$p$ Int(A). That is, $X \setminus \tau_{[\gamma,\gamma']}$-$p$ Int$(A) \subset U$. By Theorem 2.7, $\tau_{[\gamma,\gamma']}$-pCl$(X \setminus A) \subset U$. This implies $X \setminus A$ is $[\gamma,\gamma']$-pg.closed and hence A is $[\gamma,\gamma']$-pg.open.

Theorem 3.12. *If $\tau_{[\gamma,\gamma']}$-p Int$(A) \subset B \subset A$ and A is $[\gamma,\gamma']$-pg.open, then B is $[\gamma,\gamma']$-pg.open.*

Proof. Easily follows from Theorems 2.7 and 3.10.

Theorem 3.13. *If a set A is $[\gamma,\gamma']$-pg.open in a topological space (X, τ), then $G = X$ whenever G is $[\gamma,\gamma']$-preopen in (X, τ) and $\tau_{[\gamma,\gamma']}$- p Int$(A) \cup X \setminus A \subset G$.*

Proof. Suppose that G is $[\gamma,\gamma']$-preopen and $\tau_{[\gamma,\gamma']}$-pInt$(A) \cup X \setminus A \subset G$. Now $X \setminus G \subset \tau_{[\gamma,\gamma']}$-$pCl(X \setminus A) \cap A = \tau_{[\gamma,\gamma']}$-$p$ Cl$(X \setminus A) \setminus X \setminus A$. Since $X \setminus G$ is $[\gamma,\gamma']$-preclosed and $X \setminus A$ is $[\gamma,\gamma']$-pg.closed, by Theorem 2.9, $X \setminus G = \emptyset$ and hence $G = X$.

Proposition 3.14. *Let (X, τ) be a topological space and $A, B \subset X$. If B is $[\gamma,\gamma']$-pg.open and if $A \supset \tau_{[\gamma,\gamma']}$-$pInt(B)$, then $A \cap B$ is $[\gamma,\gamma']$- pg.open.*

Proof. Since B is $[\gamma,\gamma']$-pg.open and $A \supset \tau_{[\gamma,\gamma']}$-$pInt(B)$, $\tau_{[\gamma,\gamma']}$-p Int$(B) \subset A \cap B \subset B$. By Theorem 3.12, $A \cap B$ is $[\gamma,\gamma']$-pg.open.

Proposition 3.15. *Let the family $[\gamma,\gamma']$-PO(X) of all $[\gamma,\gamma']$-preopensubsets of (X, τ) be closed under finite intersections i.e., let $[\gamma,\gamma']$- PO(X) be the topology on X. If A and B are $[\gamma,\gamma']$-pg.open in (X, τ), then $A \cap B$ is $[\gamma,\gamma']$ -pg.open.*

Proof. Let $X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B) \subset U$, where U is $[\gamma,\gamma']$- preopen. Then $X \setminus A \subset U$ and $X \setminus B \subset U$. Since A and B are $[\gamma,\gamma']$-pg.open, $\tau_{[\gamma,\gamma']}$-pCl$(X \setminus A) \subset U$ and $\tau_{[\gamma,\gamma']}$-pCl$(X \setminus B) \subset U$. By hypothesis, $\tau_{[\gamma,\gamma']}$-pCl$((X \setminus A) \cup (X \setminus B)) = \tau_{[\gamma,\gamma']}$-$pCl(X \setminus A) \cup \tau_{[\gamma,\gamma']}$-$pCl(X \setminus B) \subset U$. That is, $\tau_{[\gamma,\gamma']}$-pCl$(X \setminus (A \cap B)) \subset U$. This shows that $A \cap B$ is $[\gamma,\gamma']$-pg.open.

Theorem 3.16. *If $A \subset X$ is $[\gamma,\gamma']$-pg.closed, then $\tau_{[\gamma,\gamma']}$-pCl$(A) \setminus A$ is $[\gamma,\gamma']$-pg.open.*

Proof. Let A be $[\gamma,\gamma']$-pg.closed. Let F be a $[\gamma,\gamma']$-preclosed set such that $F \subset \tau_{[\gamma,\gamma']}$-$pCl(A) \setminus A$. Then by Theorem 2.9 $F = \emptyset$. So, $F \subset \tau_{[\gamma,\gamma']}$- pInt$(\tau_{[\gamma,\gamma']}$-pCl$(A) \setminus A)$. This shows $\tau_{[\gamma,\gamma']}$-pCl$(A) \setminus A$ is $[\gamma,\gamma']$- pg.open.

Definition 3.17. *A topological space (X, τ) with operations γ and γ' on τ is called $[\gamma,\gamma']$-preregular if for each $[\gamma,\gamma']$-preclosed set F of X not containing x, there exists disjoint $[\gamma,\gamma']$-preopen sets U and V such that $x \in U$ and $F \subset V$.*

Theorem 3.18. *The following are equivalent for a topological space*

(X, τ) with operations γ and γ' on τ:

(i) *X is $[\gamma,\gamma']$-preregular.*

(ii) *For each $x \in X$ and each $U \in [\gamma,\gamma']$-PO(X, x), there exists a $V \in [\gamma,\gamma']$-PO(X, x) such that $x \in V \subset \tau_{[\gamma,\gamma']}$-$pCl(V) \subset U$.*

(iii) *For each $[\gamma,\gamma']$-preclosed set F of X, $\cap\{\tau_{[\gamma,\gamma']}$-$p$ Cl$(V) : F \subset V, V \in [\gamma,\gamma']$-PO(X)$\} = F$*

(iv) *For each A subset of X and each $U \in [\gamma,\gamma']$-PO(X) with $A \cap U \neq \emptyset$, there exists a $V \in [\gamma,\gamma']$-PO(X) such that $A \cap U \neq \emptyset$ and $\tau_{[\gamma,\gamma']}$-pCl$(V) \subset U$.*

(v) *For each nonempty subset A of X and each $[\gamma,\gamma']$-preclosed subset F of X with $A \cap F = \emptyset$, there exists $V, W \in [\gamma,\gamma']$-PO(X) such that $A \cap V \neq \emptyset$,*

$F \subset W$ and $W \cap V = \emptyset$

(vi) *For each $[\gamma,\gamma']$-preclosed set F and $x \notin F$, there exists $U \in [\gamma,\gamma']$-PO(X) and a $[\gamma,\gamma']$-pg.open set V such that $x \in U$, $F \subset V$ and $U \cap V = \emptyset$.*

(vii) *For each $A \subset X$ and each $[\gamma,\gamma']$-preclosed set F with $A \cap F = \emptyset$, there exists $U \in [\gamma,\gamma']$-PO(X) and a $[\gamma,\gamma']$-pg.open set V such that $A \cap U \neq \emptyset$, $F \subset V$ and $U \cap V = \emptyset$.*

(viii) *For each $[\gamma,\gamma']$-preclosed set F of X, $F = \cap\{\tau_{[\gamma,\gamma']}$-pCl(V) : $F \subset V$, V is $[\gamma,\gamma']$-pg.open$\}$*

Proof. (i)⇒ (ii) Let $x \notin X \setminus U$, where $U \in [\gamma,\gamma']$-*PO*(*X, x*). Thenthere exists *G, V* $\in [\gamma,\gamma']$-*PO*(*X*) such that $(X \setminus U) \subset G$, $x \in V$ and $G \cap V = \emptyset$. Therefore $V \subset (X \setminus G)$ and so $x \in V \subset \tau_{[\gamma,\gamma']}$-*p*Cl($V$) $\subset (X \setminus G) \subset U$. (ii)⇒ (iii) Let $(X \setminus F) \in [\gamma,\gamma']$-*PO*(*X, x*). Then by(2) there exists an $U \in [\gamma,\gamma']$-*PO*(*X, x*) such that $x \in U \subset \tau_{[\gamma,\gamma']}$-*p*Cl($U$) $\subset (X \setminus F)$. So, $F \subset X \setminus \tau_{[\gamma,\gamma']}$-*p*Cl($U$) $= V$, $V \in [\gamma,\gamma']$-*PO*(*X*) and $V \cap U = \emptyset$.Then by Theorem 2.7, $x \notin \tau_{[\gamma,\gamma']}$-*p*Cl($V$). Thus $F \supset \cap\{\tau_{[\gamma,\gamma']}$-*p*Cl(*V*): $F \subset V$, $V \in [\gamma,\gamma']$-*PO*(*X*)$\}$. (iii)⇒ (iv) Let $U \in [\gamma,\gamma']$-*PO*(*X*) with $x \in U \cap A$. Then $x \notin (X \setminus U)$ and hence by(iii) there exists a $[\gamma,\gamma']$-preopen set W such that $(X \setminus U) \subset W$ and $x \notin \tau_{[\gamma,\gamma']}$-*p*Cl($W$). We put $V = X \setminus \tau_{[\gamma,\gamma']}$-*p*Cl($W$), which is a $[\gamma,\gamma']$-preopen set containing x and hence $V \cap U \neq \emptyset$. Now $V \subset (X \setminus W)$ and so $\tau_{[\gamma,\gamma']}$-*p*Cl(V) $\subset (X \setminus W) \subset U$ (iv)⇒ (v) Let F be a set as in hypothesis of (v). Then $(X \setminus F)$ is $[\gamma,\gamma']$-preopen and $(X \setminus F) \cap A \neq \emptyset$. Then there exists $V \in [\gamma,\gamma']$-*PO*(*X*) such that $A \cap V \neq \emptyset$ and $\tau_{[\gamma,\gamma']}$- *p*Cl(V) $\subset (X \setminus F)$. If we put $W = X \setminus \tau_{[\gamma,\gamma']}$-*p*Cl($V$), then $F \subset W$ and $W \cap V = \emptyset$. (v)⇒ (i) Let F be a $[\gamma,\gamma']$-preclosed set not containing x. Then by (v), there exist *W, V* $\in [\gamma,\gamma']$-*PO*(*X*) such that $F \subset W$ and $x \in V$ and $W \cap V = \emptyset$. (i)⇒ (vi) Obvious. (vi)⇒ (vii) For $a \in A$, $a \notin F$ and hence by (vi) there exists $U \in [\gamma,\gamma']$-*PO*(*X*) anda $[\gamma,\gamma']$-pg.open set V such that $a \in U$, $F \subset V$ and $U \cap V = \emptyset$. So, $A \cap U \neq \emptyset$. (vii)⇒ (i) Let $x \notin F$, where F is $[\gamma,\gamma']$-pg.closed. Since $\{x\} \cap F = \emptyset$, by (vii) there exists $U \in [\gamma,\gamma']$-*PO*(*X*) and $[\gamma,\gamma']$-pg.open set W such that $x \in U$, $F \subset W$ and $U \cap W = \emptyset$. Now put $V = \tau_{[\gamma,\gamma']}$-*p*Int($W$). Using definition of $[\gamma,\gamma']$-pg.open sets we get $F \subset V$ and $V \cap U = \emptyset$. (iii)⇒ (viii) We have $F \subset \cap\{\tau_{[\gamma,\gamma']}$-*p*Cl($V$): $F \subset V$ and V is $[\gamma,\gamma']$-pg.open$\}$ $\subset \cap\{\tau_{[\gamma,\gamma']}$-*p*Cl($V$): $F \subset V$ and V is $[\gamma,\gamma']$-preopen$\}= F$. (viii)⇒ (i) Let F be a $[\gamma,\gamma']$-preclosed set in X not containing x. Then by (viii) there exists a $[\gamma,\gamma']$-pg.open set W such that $F \subset W$ and $x \in X \setminus \tau_{[\gamma,\gamma']}$-*p*Cl($W$). Since F is $[\gamma,\gamma']$-preclosed and W is $[\gamma,\gamma']$- pg.open , $F \subset \tau_{[\gamma,\gamma']}$-*p*Int($W$). Take $V = \tau_{[\gamma,\gamma']}$-*p*Int($W$). Then $F \subset V$, $x \in U = X \setminus \tau_{[\gamma,\gamma']}$-*p*Cl($V$) and $U \cap V = \emptyset$.

Definition 3.19. *A topological space (X, τ) with operations γ and γ' on τ is called $[\gamma,\gamma']$-prenormal if for any pair of disjoint $[\gamma,\gamma']$-preclosed sets A and B of X, there exist disjoint $[\gamma,\gamma']$-preopen sets U and V such that $A \subset U$ and $F \subset V$.*

Theorem 3.20. *For a topological space (X, τ) with operations γ and γ'* on τ, the following are equivalent:

(i) *X is $[\gamma,\gamma']$-prenormal.*

(ii) *For each pair of disjoint $[\gamma,\gamma']$-preclosed sets A and B of X, there exist disjoint $[\gamma,\gamma']$-pg.open sets U and V such that $A \subset U$ and $B \subset V$.*

(iii) *For each $[\gamma,\gamma']$-preclosed set A and any $[\gamma,\gamma']$-preopen set V containing A, there exists a $[\gamma,\gamma']$-pg.open set U such that $A \subset U \subset \tau_{[\gamma,\gamma']}$*-*p*Cl($U$) $\subset V$.

(iv) *For each $[\gamma,\gamma']$-preclosed set A and any $[\gamma,\gamma']$-pg.open set B containing A, there exists a $[\gamma,\gamma']$-pg.open set U such that $A \subset U \subset \tau_{[\gamma,\gamma']}$*-*p*Cl($U$) $\subset \tau_{[\gamma,\gamma']}$-*p*Int($B$)

(v) *For each $[\gamma,\gamma']$-preclosed set A and any $[\gamma,\gamma']$-pg.open set B containing A, there exists a $[\gamma,\gamma']$-preopen set G such that $A \subset G \subset \tau_{[\gamma,\gamma']}$*-*p*Cl($G$) $\subset \tau_{[\gamma,\gamma']}$-*p* Int($B$).

(vi) *For each $[\gamma,\gamma']$-pg.closed set A and any $[\gamma,\gamma']$-*

preopen set B containing A, there exists a $[\gamma,\gamma']$-*preopen set U such that* $\tau_{[\gamma,\gamma']}$-*p*Cl(*A*) $\subset U \subset \tau_{[\gamma,\gamma']}$-*p*Cl(*U*) $\subset B$.

(vii) *For each* $[\gamma,\gamma']$-*pg.closed set A and any* $[\gamma,\gamma']$-*preopen set B containing A, there exists a* $[\gamma,\gamma']$-*pg.open set G such that* $\tau_{[\gamma,\gamma']}$-*p*Cl(*A*) $\subset G \subset \tau_{[\gamma,\gamma']}$-*p*Cl(*G*) $\subset B$.

Proof. $(i) \Rightarrow (ii)$: Follows from the fact that every $[\gamma,\gamma']$-preopen set is $[\gamma,\gamma']$-pg.open. $(ii) \Rightarrow (iii)$: let A be a $[\gamma,\gamma']$-closed set and V any $[\gamma,\gamma']$-preopen set containing A. Since A and $(X \setminus V)$ are disjoint $[\gamma,\gamma']$-preclosed sets, there exist $[\gamma,\gamma']$-pg.open sets U and W such that $\subset U$, $(X \setminus V) \subset W$ and $U \cap W = \emptyset$. By Theorem 3.11, we get $(X \setminus V) \subset \tau_{[\gamma,\gamma']}$-*p*Int($W$). Since $U \cap \tau_{[\gamma,\gamma']}$-*p*Int($W$) = $\emptyset$, we have $\tau_{[\gamma,\gamma']}$-*p*Cl(U) $\cap$ $\tau_{[\gamma,\gamma']}$-*p*Int(W) = $\emptyset$, and hence $\tau_{[\gamma,\gamma']}$-*p* Cl(U) $\subset X \setminus \tau_{[\gamma,\gamma']}$-*p*Int($W$) $\subset V$. Therefore, A $\subset U \subset \tau_{[\gamma,\gamma']}$-*p*Cl($U$) $\subset V$. $(iii) \Rightarrow (i)$: Let A and B *be* any disjoint $[\gamma,\gamma']$-preclosed sets of X. Since $(X \setminus B)$ is an $[\gamma,\gamma']$-preopen set containing A, there exists a $[\gamma,\gamma']$-pg.open set G *such* that A $\subset G \subset \tau_{[\gamma,\gamma']}$-*p* Cl($G$) $\subset X \setminus B$. Since G is a $[\gamma,\gamma']$- pg.open set, using Theorem 3.11, we have $A \subset \tau_{[\gamma,\gamma']}$-*p* Int($G$). Taking

$U = \tau_{[\gamma,\gamma']}$-*p* Int($G$) and $V = X \setminus \tau_{[\gamma,\gamma']}$-*p* Cl($G$), we have two disjoint $[\gamma,\gamma']$-preopen sets U and V such that $A \subset U$ and $B \subset V$. Hence X is $[\gamma,\gamma']$-prenormal. $(v) \Rightarrow (iii)$: let A be a $[\gamma,\gamma']$-closed set and V

any $[\gamma,\gamma']$-preopen set containing A. Since every $[\gamma,\gamma']$-preopen set is $[\gamma,\gamma']$-pg.open, there exists a $[\gamma,\gamma']$-preopen set G such that $A \subset G \subset \tau_{[\gamma,\gamma']}$-*p*Cl($G$) $\subset \tau_{[\gamma,\gamma']}$-*p*Int($V$). Also. we have a $[\gamma,\gamma']$- pg.open

set G such that $A \subset G \subset \tau_{[\gamma,\gamma']}$-*p*Cl($G$) $\subset \tau_{[\gamma,\gamma']}$-*p*Int($V$) $\subset V$. $(iii) \Rightarrow (v)$: Let A be a $[\gamma,\gamma']$-closed set and B aby $[\gamma,\gamma']$-pg.open set containing A. Using Theorem 3.11 of a $[\gamma,\gamma']$-pg.open set we get $A \subset \tau_{[\gamma,\gamma']}$-*p*Int($B$) = V, say. Then applying (iii), we get a $[\gamma,\gamma']$- pg.open set U such that $A = \tau_{[\gamma,\gamma']}$-*p*Cl($A$) $\subset U \subset \tau_{[\gamma,\gamma']}$-*p*Cl($U$) $\subset V$.Again, using the same Theorem 3.11 we get $A \subset \tau_{[\gamma,\gamma']}$-*p*Int($U$), and hence $A \subset \tau_{[\gamma,\gamma']}$-*p*Int($U$) $\subset U \subset \tau_{[\gamma,\gamma']}$-*p*Cl($U$) $\subset V$; which implies $A \subset \tau_{[\gamma,\gamma']}$-*p*Int($U$) $\subset \tau_{[\gamma,\gamma']}$-*p*Cl($\tau_{[\gamma,\gamma']}$-*p*Int($U$)) $\subset \tau_{[\gamma,\gamma']}$-*p*Cl($U$) $\subset V$,that is, $A \subset G \subset \tau_{[\gamma,\gamma']}$-*p*Cl($G$) $\subset \tau_{[\gamma,\gamma']}$-*p*Int($B$), where $G = \tau_{[\gamma,\gamma']}$- *p*Int($U$). $(iii) \Rightarrow (vii)$: Let A be a $[\gamma,\gamma']$-pg.closed set abd B any $[\gamma,\gamma']$-preopen set containing A. Since A is a $[\gamma,\gamma']$-pg.closed set, we have $\tau_{[\gamma,\gamma']}$ -*p*Cl(A) $\subset B$, therefore, we can find a $[\gamma,\gamma']$-pg.open set U such that $\tau_{[\gamma,\gamma']}$-*p*Cl(A) $\subset U \subset \tau_{[\gamma,\gamma']}$-*p*Cl($U$) $\subset B$. $(vii) \Rightarrow (vi)$: Let A be a $[\gamma,\gamma']$-pg.closed set and B any $[\gamma,\gamma']$-preopen set containing A, then by (vii) there exists a $[\gamma,\gamma']$-pg.open set G such that τ_β- Cl(A) $\subset G \subset \tau_{[\gamma,\gamma']}$-*p*Cl($G$) $\subset B$. Since G is a $[\gamma,\gamma']$-pg.open set, then by Theorem 3.11, we get $\tau_{[\gamma,\gamma']}$-*p*Cl(A) $\subset \tau_{[\gamma,\gamma']}$-*p*Int($G$). If we take $U = \tau_{[\gamma,\gamma']}$-*p*Int($G$), the proof follows.

Definition 3.21. *Let* (X, τ) *be a topological space and* $A \subset X$.

(i) *The set denoted by* $\tau_{[\gamma,\gamma']}$-*pD*(*A*) *and defined by* {*x* : *for every* $[\gamma,\gamma']$-*preopen set U containing x*, $U \cap (A \setminus \{x\}) \neq \emptyset$ } *is called the* $\tau_{[\gamma,\gamma']}$-*prederived set of A.*

(ii) *The* $\tau_{[\gamma,\gamma']}$-*prefrontier of A, denoted by* $\tau_{[\gamma,\gamma']}$-*pFr*(*A*) *is defined as* $\tau_{[\gamma,\gamma']}$-*p*Cl(A) $\cap$ $\tau_{[\gamma,\gamma']}$-*p*Cl($X \setminus A$).

Proposition 3.22. *Let* (X, τ) *be a topological space and* $A \subset X$. *Then the following propeties hold:*

(i) $\tau_{[\gamma,\gamma']}$-*p*Int(A) = $A \setminus (\tau_{[\gamma,\gamma']}$-*pD*($X \setminus A$).

(ii) $\tau_{[\gamma,\gamma']}$-*p*Cl(A) = $A \cup \setminus \tau_{[\gamma,\gamma']}$-*pD*($A$).

Proof. Clear.

4. $[\gamma,\gamma']$-$[\beta,\beta']$-PRECONTINUOUS FUNCTIONS

Definition 4.1. *A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be $[\gamma,\gamma']$-$[\beta,\beta']$-precontinuous* [2] *at a point $x \in X$ if for each $[\beta,\beta']$-preopen subset V in Y containing $f(x)$, there exists a $[\gamma,\gamma']$-preopen subset U of X containing x such that $f(U) \subset V$.*

Theorem 4.2. *For a function $f : (X, \tau) \rightarrow (Y, \sigma)$, the following statements are equivalent:*

(i) *f is $[\gamma,\gamma']$-$[\beta,\beta']$-precontinuous;*

(ii) *The inverse image of each $[\beta,\beta']$-preopen set in Y is $[\gamma,\gamma']$- preopen in X;*

(iii) *For each subset B of Y, $\tau_{[\gamma,\gamma']}$-pCl$(f^{-1}(B)) \subset f^{-1}(\sigma_{[\beta,\beta']}$-$pCl(B))$;*

(iv) *For each subset A of X, $f(\tau_{[\gamma,\gamma']}$-$pCl(A)) \subset \sigma_{[\beta,\beta']}$-$pCl(f(A))$;*

(v) *For each subset A of X, $f(\tau_{[\gamma,\gamma']}$-$pD(A)) \subset \sigma_{[\beta,\beta']}$-$pCl(f(A))$;*

(vi) *For any subset B of Y, $f^{-1}(\sigma_{[\beta,\beta']}$-$pInt(B)) \subset \tau_{[\gamma,\gamma']}$-$pInt(f^{-1}(A))$;*

(vii) *For each subset C of Y, $\tau_{[\gamma,\gamma']}$-$pFr(f^{-1}(C)) \subset f^{-1}(\sigma_{[\beta,\beta']}$-$pFr(B))$.*

Proof. Easy proof and hence omitted.

Theorem 4.3. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a $[\gamma,\gamma']$-$[\beta,\beta']$-precontinuous function. Then for each subset V of Y, $f^{-1}(\sigma_{[\beta,\beta']}$-$pInt(V)) \subset \tau_{[\gamma,\gamma']}$- pCl$(\tau_{[\gamma,\gamma']}$-pInt$(f^{-1}(V)))$.*

Proof. Let V be any subset of Y. Then $\sigma_{[\beta,\beta']}$-p Int(V) is $[\beta,\beta']$-preopen in Y and so $f^{-1}(\sigma_{[\beta,\beta']}$-$pInt(V))$ is $[\gamma,\gamma']$-preopen in X. Hence $f^{-1}(\sigma_{[\beta,\beta']}$-$pInt(V)) \subset \tau_{[\gamma,\gamma']}$-$p$Cl $(\tau_{[\gamma,\gamma']}$-p Int$(f^{-1}(\sigma_{[\beta,\beta']}$-$p$ Int$(V)))) \subset \tau_{[\gamma,\gamma']}$-$pCl(\tau_{[\gamma,\gamma']}$-$p$Int $(f^{-1}(V)))$.

Corollary 4.4. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a $[\gamma,\gamma']$-$[\beta,\beta']$-precontinuous function. Then for each subset V of Y,* $\tau_{[\gamma,\gamma']}$-pInt $(\tau_{[\gamma,\gamma']}$-pCl $(f^{-1}(V))) \subset f^{-1}(\sigma_{[\beta,\beta']}$-$p$Cl $(V))$.

Theorem 4.5. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a bijection. Then f is $[\gamma,\gamma']$-$[\beta,\beta']$-precontinuous if and only if $\sigma_{[\beta,\beta']}$-pInt $(f(U)) \subset f(\tau_{[\gamma,\gamma']}$- pInt $(U))$ for each subset U of X.*

Proof. Let U be any subset of X. Then by Theorem 4.2, $f^{-1}(\sigma_{[\beta,\beta']}$- pInt $(f(U))) \subset \tau_{[\gamma,\gamma']}$-$p$Int $(f^{-1}(f(U)))$. Since f is bijection, $\sigma_{[\beta,\beta']}$-pInt $(f(U)) = f(f^{-1}(\tau_{[\gamma,\gamma']}$-$p$Int $(f(U)))$ $\subset f(\tau_{[\gamma,\gamma']}$-$p$Int $(U))$. Conversely, let V be any subset of Y. Then $\sigma_{[\beta,\beta']}$-pInt $(f(f^{-1}(V))) \subset f(\tau_{[\gamma,\gamma']}$-$p$Int $(f^{-1}(V)))$. Since f is bijection, $\sigma_{[\beta,\beta']}$-pInt $(V) = \sigma_{[\beta,\beta']}$-$p$Int $(f(f^{-1}(V))) \subset f(\tau_{[\gamma,\gamma']}$-$p$Int $(f^{-1}(V)))$; hence $f^{-1}(\sigma_{[\beta,\beta']}$-$p$Int $(V) \subset \tau_{[\gamma,\gamma']}$-$p$Int $(f^{-1}(V))$. Therefore, by Theorem 4.2, f is $[\gamma,\gamma']$-$[\beta,\beta']$-precontinuous.

Theorem 4.6. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a function. Then $X \setminus \tau_{[\gamma,\gamma']}$-$pC(f) = \cup\{\tau_{[\gamma,\gamma']}$-$pFr\ (f^{-1}(V)) : V \in [\beta,\beta']$-$PO(Y), f(x) \in V, x \in X\}$, where $\tau_{[\gamma,\gamma']}$-$pC(f)$ denotes the set of points at which f is $[\gamma,\gamma']$-$[\beta,\beta']$*-precontinuous.

Proof. Let $x \in X \setminus \tau_{[\gamma,\gamma']}$-$pC(f)$. Then there exists $V \in [\beta,\beta']$-$PO(Y)$ containing $f(x)$ such that $f(U)$ is not a subset of V, for every $[\gamma,\gamma']$- preopen set U containing x. Hence $U \cap (X \setminus f^{-1}(V)) \neq \emptyset$ for every $[\gamma,\gamma']$-preopen set U containing x. Then, $x \in \tau_{[\gamma,\gamma']}$-$p$Cl $(X \setminus f^{-1}(V))$. Then $x \in f^{-1}(V) \cap \tau_{[\gamma,\gamma']}$-$p$Cl $(X \setminus f^{-1}(V)) \subset \tau_{[\gamma,\gamma']}$-$pFr\ (f^{-1}(V))$. So, $X \setminus \tau_{[\gamma,\gamma']} - pC(f) \subset \cup\{\tau_{[\gamma,\gamma']} - pFr(f^{-1}(V)) : V \in [\beta,\beta'] - PO(Y), f(x) \in V, x \in X\}$. Conversely, let $x \notin X \setminus \tau_{[\gamma,\gamma']}$-

pC(*f*). Then for each $V \in [\beta, \beta']$-*PO*(*Y*) containing *f* (*x*), $f^{-1}(V)$ is a $[\gamma, \gamma']$-preopen set *U* containing *x*. Thus, $x \in \tau_{[\gamma,\gamma']}$-*p*Int ($f^{-1}(V)$) and hence $x \notin \tau_{[\gamma,\gamma']}$-*pFr*($f^{-1}(V)$), for every $V \in \beta SO(Y)$ containing *f* (*x*). Therefore, $X \setminus \tau_{[\gamma,\gamma']}$ –*pC*(*f*) ⊃ ∪{$\tau_{[\gamma,\gamma']}$–*pFr*($f^{-1}(V)$) : $V \in [\beta, \beta']PO(Y)$, *f* (*x*) ∈*V*, *x* ∈*X*}.

Theorem 4.7. *Let f : (X, τ) → (Y, σ) be a* $[\gamma, \gamma']$-$[\beta, \beta']$-*precontinuous*, $[\gamma, \gamma']$-$[\beta, \beta']$-*preclosed*, $[\gamma, \gamma']$-$[\beta, \beta']$-*preopen surjective function on (X, τ). If X is* $[\gamma, \gamma']$-*preregular, then Y is* $[\beta, \beta']$-*preregular.*

***Proof*.** Let *K* be a $[\beta, \beta']$-preclosed in *Y* and *y* ∈ *K*. Since *f* is $[\gamma, \gamma']$-$[\beta, \beta']$-precontinuous and *X* is $[\gamma, \gamma']$-preregular for each point $x \in f^{-1}(y)$, there exist disjoint *V*, *W* ∈ $[\gamma, \gamma']$-*PO*(*X*) such that *x* ∈ *V* and $f^{-1}(K) \subset W$. Now, since *f* is $[\gamma, \gamma']$-$[\beta, \beta']$-preclosed, there exists a $[\beta, \beta']$-preopen set *U* containing *K* such that $f^{-1}(U) \subset W$. As *f* is a $[\gamma, \gamma']$-$[\beta, \beta']$-preopen function, we have $y = f(x) \in f(V)$ and $f(V)$ is $[\beta, \beta']$-preopen in *Y*. Now, $f^{-1}(U) \cap V = \emptyset$; hence $U \cap f(V) = \emptyset$. Therefore, *Y* is $[\beta, \beta']$-preregular.

Theorem 4.8. *If* $f : (X, \tau) \to (Y, \sigma)$ *is a* $[\gamma, \gamma']$-$[\beta, \beta']$ *-precontinuous,* $[\gamma, \gamma']$-$[\beta, \beta']$-*preclosed surjective function and X is* $[\gamma, \gamma']$-*prenormal, then Y is* $[\beta, \beta']$-*prenormal.*

***Proof*.** Let *A* and *B* be two disjoint $[\beta, \beta']$-preclosed sets in *Y*. Since *f* is $[\gamma, \gamma']$-$[\beta, \beta']$-precontinuous, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint $[\gamma, \gamma']$- preclosed sets in *X*. Now as *X* is $[\gamma, \gamma']$-prenormal, there exist disjoint $[\gamma, \gamma']$-preopen sets *V* and *W* such that $f^{-1}(A) \subset V$ and $f^{-1}(B) \subset W$.

Since *f* is $[\gamma, \gamma']$-$[\beta, \beta']$-preclosed, there exist $[\beta, \beta']$-preopen sets *M* and *N* such that $A \subset M$, $B \subset N$, $f^{-1}(M) \subset V$ and $f^{-1}(N) \subset W$. Since $V \cap W = \emptyset$, we have $M \cap N = \emptyset$; hence *Y* is $[\beta, \beta']$-prenormal.

Definition 4.9. *A function* $f : (X, \tau) \to (Y, \sigma)$ *is said to be:*

(i) $[\gamma, \gamma']$-$[\beta, \beta']$-*preopen if f (U) is a* $[\beta, \beta']$-*preopen set of Y for every* $[\gamma, \gamma']$-*preopen set U of X.*

(ii) $[\gamma, \gamma']$-$[\beta, \beta']$-*preclosed [2] if f (U) is a* $[\beta, \beta']$ *-preclosed set of Y for every* $[\gamma, \gamma']$-*preclosed set U of X.*

Theorem 4.10. *For a bijective function* $f : (X, \tau) \to (Y, \sigma)$, *the following statements are equivalent:*

(i) *f is* $[\gamma, \gamma']$-$[\beta, \beta']$-*preopen;*

(ii) *f is* $[\gamma, \gamma']$-$[\beta, \beta']$-*preclosed;*

(iii) *f is* $[\gamma, \gamma']$-$[\beta, \beta']$-*precontinuous.*

***Proof*.** The proof is clear.

Theorem 4.11. *For a function* $f : (X, \tau) \to (Y, \sigma)$, *the followingstatements are equivalent:*

(i) *f is* $[\gamma, \gamma']$-$[\beta, \beta']$-*preopen;*

(ii) $f(\tau_{[\gamma,\gamma']}\text{-}p\mathrm{Int}(U)) \subset \sigma_{(\beta,\beta')}\text{-}p\mathrm{Int}(f(U))$ *for each subset U of X;*

(iii) $\tau_{[\gamma,\gamma']}\text{-}p\mathrm{Int}(f^{-1}(V)) \subset f^{-1}(\sigma_{(\beta,\beta')}\text{-}p\mathrm{Int}(V))$ *for each subset V of Y.*

***Proof*.** $(i) \Rightarrow (ii)$: Let *U* be any subset of *X*. Then $\tau_{[\gamma,\gamma']}$-*p*Int (*U*) is a $[\gamma, \gamma']$-preopen set of *X*. Then $f(\tau_{[\gamma,\gamma']}\text{-}p\mathrm{Int}(U))$ is a $[\beta, \beta']$-preopen set of *Y*. Since $f(\tau_{[\gamma,\gamma']}\text{-}p\mathrm{Int}(U)) \subset f(U)$, $f(\tau_{[\gamma,\gamma']}\text{-}p\mathrm{Int}(U)) = \sigma_{[\beta,\beta']}$-*p*Int $(f(\tau_{[\gamma,\gamma']}\text{-}p\mathrm{Int}(U))) \subset \sigma_{[\beta,\beta']}\text{-}p\mathrm{Int}(f(U))$. (*ii*) ⇒ (*iii*): Let *V* be any subset of *Y*. Then $f^{-1}(V)$ is a subset of *X*. Hence $f(\tau_{[\gamma,\gamma']}\text{-}p\mathrm{Int}(f^{-1}(V))) \subset \sigma_{[\beta,\beta']}$-*p*Int $(f(f^{-1}(V))) \subset \sigma_{[\beta,\beta']}\text{-}p\mathrm{Int}(V)$). Then $\tau_{[\gamma,\gamma']}$-*p*Int $(f^{-1}(V)) \subset f^{-1}(f(\tau_{[\gamma,\gamma']}\text{-}p\mathrm{Int}(f^{-1}(V)))) \subset f^{-1}(\sigma_{[\beta,\beta']}\text{-}p\mathrm{Int}(V))$.(*iii*) ⇒ (*i*): Let *U* be any $[\gamma, \gamma']$-preopen set of

X.Then $\tau_{[\gamma,\gamma']}$-pInt $(U) = U$ and $f(U)$ is a subset of Y. Now, $V = \tau_{[\gamma,\gamma']}$-$p$Int $(V) \subset \tau_{[\gamma,\gamma']}$-$p$Int $(f^{-1}(f(V))) \subset f^{-1}(\sigma_{[\beta,\beta']}$-$p$Int $(f(V)))$. Then $f(V) \subset f(f^{-1}(\sigma_{[\beta,\beta']}$-$p$Int $(f(V))))$ $\subset \sigma_{[\beta,\beta']}$-$p$Int $(f(V))$ and $\sigma_{[\beta,\beta']}$-pInt $(f(V)) \subset f(V)$. Hence $f(V)$ is a $[\beta,\beta']$-preopen set of Y; hence f is $[\gamma,\gamma']$-$[\beta,\beta']$-preopen.

Corollary 4.12. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then f is $[\gamma,\gamma']$-$[\beta,\beta']$-preclosed and $[\gamma,\gamma']$-$[\beta,\beta']$-precontinuous if and only if $f(\tau_{[\gamma,\gamma']}$-p*Cl *$(V)) = \sigma_{(\beta,\beta')}$-p*Cl*$(f(V)))$ for evey subset V of X.*

Corollary 4.13. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then f is $[\gamma,\gamma']$-$[\beta,\beta']$-preopen and $[\gamma,\gamma']$-$[\beta,\beta']$-precontinuous if and only if $f^{-1}(\sigma_{(\beta,\beta')}$-p*Cl *$(V))) = \tau_{[\gamma,\gamma']}$-p*Cl *$(f^{-1}((V)))$ for evey subset V of Y.*

Theorem 4.14. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then f is a $[\gamma,\gamma']$-$[\beta,\beta']$-preclosed function if and only if for each subset V of X, $\sigma_{[\beta,\beta']}$-p*Cl *$(f(V)) \subset f(\tau_{[\gamma,\gamma']}$-p*Cl *$(V))$.*

Proof. Let f be a $[\gamma,\gamma']$-$[\beta,\beta']$-preclosed function and V any subset of X .Then $f(V) \subset f(\tau_{[\gamma,\gamma']}$-$pCl(V))$ and $f(\tau_{[\gamma,\gamma']}$-$pCl(V))$ is a $[\beta,\beta']$-preclosed set of Y. We have $\sigma_{[\beta,\beta']}$-pCl $(f(V)) \subset \sigma_{[\beta,\beta']}$-$p$Cl $(f(\tau_{[\gamma,\gamma']}$-$p$Cl $(V))) = f(\tau_{[\gamma,\gamma']}$-$p$Cl $(V))$. Conversely, let V be a $[\gamma,\gamma']$-preopen set of X. Then $f(V) \subset \sigma_{[\beta,\beta']}$-$p$Cl $(f(V))$ $\subset f(\tau_{[\gamma,\gamma']}$-$p$Cl $(V)) = f(V)$; hence $f(V)$ is a $[\beta,\beta']$-preclosed subset of Y. Therefore, f is a $[\gamma,\gamma']$- $[\beta,\beta']$-preclosed function.

Theorem 4.15. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then f is a $[\gamma,\gamma']$-$[\beta,\beta']$-preclosed function if and only if for each subset V of Y, $f^{-1}(\sigma_{[\beta,\beta']}$-p*Cl *$(V)) \subset \tau_{[\gamma,\gamma']}$-p*Cl *$(f^{-1}(V))$.*

Proof. Let V be any subset of Y. Then by Theorem 4.14, $\sigma_{[\beta,\beta']}$-pCl $(V) \subset f(\tau_{[\gamma,\gamma']}$-$p$Cl $(f^{-1}(V)))$. Since f is bijection, $f^{-1}(\sigma_{[\beta,\beta']}$-$p$Cl $(V)) = f^{-1}(\sigma_{[\beta,\beta']}$-$p$Cl $(f(f^{-1}(V)))) \subset$ $f^{-1}(f(\tau_{[\gamma,\gamma']}$-$p$Cl $(f^{-1}(V)))) = \sigma_{[\beta,\beta']}$-$p$Cl $(f^{-1}(V))$. Conversely, let U be any subset of X. Since f is bijection, $\sigma_{[\beta,\beta']}$-pCl $(f(U)) = f(f^{-1}(\sigma_{[\beta,\beta']}$-$p$Cl $(f(U))) \subset f(\tau_{[\gamma,\gamma']}$-$p$Cl $(f^{-1}(f(U)))) = f(\tau_{[\gamma,\gamma']}$-$p$Cl $(U))$. Therefore, by Theorem 4.14, f is a $[\gamma,\gamma']$-$[\beta,\beta']$- preclosed function.

Theorem 4.16. *Let $f : (X, \tau) \to (Y, \sigma)$ be a $[\gamma,\gamma']$-$[\beta,\beta']$-preopen function. If V is a subset of Y and U is a $[\gamma,\gamma']$-preclosed subset of X containing $f^{-1}(V)$, then there exists a $[\beta,\beta']$-preclosed set F of Y containing V such that $f^{-1}(F) \subset U$.*

Proof. Let V be any subset of Y and U a $[\gamma,\gamma']$-preclosed subset of X containing $f^{-1}(V)$, and let $F = Y \setminus (f(X \setminus V))$. Then $f(X \setminus V) \subset f(f^{-1}(X \setminus V)) \subset X \setminus V$ and $X \setminus U$ is a $[\gamma,\gamma']$-preopen set of X. Since f is $[\gamma,\gamma']$-$[\beta,\beta']$-preopen, $f(X \setminus U)$ is a $[\beta,\beta']$-preopen set of Y. Hence F is a $[\beta,\beta']$-preclosed set of Y and $f^{-1}(F) = f^{-1}(Y \setminus (f(X \setminus U)) \subset U$.

Theorem 4.17. *Let $f : (X, \tau) \to (Y, \sigma)$ be a $[\gamma,\gamma']$-$[\beta,\beta']$-preclosed function. If V is a subset of Y and U is a $[\gamma,\gamma']$-preopen subset of X containing $f^{-1}(V)$, then there exists $[\beta,\beta']$-preopen set F of Y containing V such that $f^{-1}(F) \subset U$.*

Proof. The proof is similar to the Theorem 4.16.

Theorem 4.18. *Let $f : (X, \tau) \to (Y, \sigma)$ be a $[\gamma,\gamma']$-$[\beta,\beta']$-preopen function. Then for each subset V of Y, $f^{-1}(\sigma_{[\beta,\beta']}$-p*Int *$(\sigma_{[\beta,\beta']}$-p*Cl *$(V)) \subset \tau_{[\gamma,\gamma']}$-p*Cl *$(f^{-1}(V)$.*

Proof. Let V be any subset of Y. Then $\tau_{[\gamma,\gamma']}$-pCl ($f^{-1}(V)$ is a $[\gamma,\gamma']$- preclosed set of X containing $f^{-1}(V)$. Since f is $[\gamma,\gamma']$-$[\beta,\beta']$-preopen, by Theorem 4.16, there is a $[\beta,\beta']$-preopen set F of Y containing V such that $f^{-1}(\sigma_{[\beta,\beta']}$-$p$Int ($\sigma_{[\beta,\beta']}$-$p$Cl ($V$)) $\subset \tau_{[\gamma,\gamma']}$-$p$Int ($\tau_{[\gamma,\gamma']}$-$p$Cl ($F$)) $\subset f^{-1}(F) \subset \tau_{[\gamma,\gamma']}$-$p$Cl ($f^{-1}(V)$).

Theorem 4.19. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be a bijection such that foreach subset* V *of* Y, $f^{-1}(\sigma_{[\beta,\beta']}$-$p$Int ($\sigma_{[\beta,\beta']}$-$p$Cl ($V$)) $\subset \tau_{[\gamma,\gamma']}$-$p$Cl ($f^{-1}(V)$). *Then* f *is a* $[\gamma,\gamma']$-$[\beta,\beta']$-*preopen function*

Proof. Let U be a $[\gamma,\gamma']$-preopen subset of X. Then $f(X \setminus U)$ is a subset of Y and $f^{-1}(\sigma_{[\beta,\beta']}$-$p$Int ($\sigma_{[\beta,\beta']}$-$p$Cl ($f(X \setminus U)$))) $\subset \tau_{[\gamma,\gamma']}$ -pCl ($f^{-1}(f(X \setminus U))$) = $\tau_{[\gamma,\gamma']}$-pCl ($X \setminus U$) = $X \setminus U$, and so $\sigma_{[\beta,\beta']}$-pInt ($\sigma_{[\beta,\beta']}$-pCl ($f(X \setminus U)$)) $\subset f(X \setminus U)$. Hence $f(X \setminus U)$ is a $[\beta,\beta']$-preclosed set of Y and $f(U) = X \setminus (f(X \setminus U))$ is a $[\beta,\beta']$-preopen set of Y. Therefore, f is a $[\gamma,\gamma']$- $[\beta,\beta']$-preopen function.

Definition 4.20. [2] *A function* $f : (X, \tau) \to (Y, \sigma)$ *is said to be* $[\gamma,\gamma']$-$[\beta,\beta']$-prehomeomorphism if f and $f^{-1}$are $[\gamma,\gamma']$-$[\beta,\beta']$-precontinuous.

Theorem 4.21. *Let* $f: (X, \tau) \to (Y, \sigma)$ *be a bijection.Then the following statements are equivalent:*

(i) f *is* $[\gamma,\gamma']$-$[\beta,\beta']$-*prehomeomorphism;*

(ii) f^{-1}*is* $[\gamma,\gamma']$-$[\beta,\beta']$-*prehomeomorphism;*

(iii) f *and* f^{-1}*are* $[\gamma,\gamma']$-$[\beta,\beta']$-*preopen* ($[\gamma,\gamma']$-$[\beta,\beta']$-*preclosed);*

(iv) f *is* $[\gamma,\gamma']$-$[\beta,\beta']$-*precontinuous and* $[\gamma,\gamma']$-$[\beta,\beta']$-*preopen* ($[\gamma,\gamma']$-$[\beta,\beta']$-*preclosed);*

(v) $f(\tau_{[\gamma,\gamma']}$-$p$Cl ($V$)) = $\sigma_{[\beta,\beta']}$-pCl ($f(V)$) *for each subset* V *of* X*;*

(vi) $f(\tau_{[\gamma,\gamma']}$-$p$Int ($V$)) = $\sigma_{[\beta,\beta']}$-pInt ($f(V)$) *for each subset* V *of* X*;*

(vii) $f^{-1}(\sigma_{[\beta,\beta']}$-pInt (V)) = $\tau_{[\gamma,\gamma']}$-pInt (f^{-1}(V)) for each subset V of Y ;

(viii) $\tau_{[\gamma,\gamma']}$-pCl (f^{-1}(V)) = $f^{-1}(\sigma_{[\beta,\beta']}$-pCl (V)) for each subset V of Y .

Proof. (i) ⇒ (ii): It follows immediately from the definition of a $[\gamma,\gamma']$- $[\beta,\beta']$-prehomeomorphism. (ii) ⇒ (iii) ⇒ (iv): It follows from Theorem 4.10. (iv) ⇒ (v): It follows from Theorem 4.15 and Corollary 4.12.

(v) ⇒ (vi): Let U be a subset of X. Then by Theorem 2.7, $f(\tau_{[\gamma,\gamma']}$- pInt (U)) = $X \setminus f(\tau_{[\gamma,\gamma']}$-$p$Cl ($X \setminus U$)) = $X \setminus \sigma_{[\beta,\beta']}$-$p$Cl ($f(X \setminus U)$) = $\sigma_{[\beta,\beta']}$- pInt ($f(U)$). (vi) ⇒ (vii): Let V be a subset of Y. Then $f(\tau_{[\gamma,\gamma']}$- pInt ($f^{-1}(V)$)) = $\sigma_{[\beta,\beta']}$-pInt ($f(f^{-1}(V))$) = $\sigma_{[\beta,\beta']}$-pInt ($f(V)$). Hence $f^{-1}(f(\tau_{[\gamma,\gamma']}$-$p$Int ($f^{-1}(V)$))) = $f^{-1}(\sigma_{[\beta,\beta']}$-$p$Int ($V$)). Therefore, $f^{-1}(\sigma_{[\beta,\beta']}$- pInt (V)) = $\tau_{[\gamma,\gamma']}$-pInt ($f^{-1}(V)$). (vii) ⇒ ($viii$): Let V be a subset of Y.Then by Theorem 2.7, $\tau_{[\gamma,\gamma']}$-pCl ($f^{-1}(V)$) = $X \setminus (f^{-1}(\sigma_{[\beta,\beta']}$- pInt ($Y \setminus V$))) = $X \setminus (\tau_{[\gamma,\gamma']}$- pInt ($f^{-1}((X \setminus V))$)) = $f^{-1}(\sigma_{[\beta,\beta']}$-$p$Cl ($V$)).(viii)⇒ ($i$): It follows from Theorem 4.15 and Corollary 4.13.

Theorem 4.22. *Every topological space* (X, τ) *with operations* γ *and* γ' on τ is $[\gamma,\gamma']$-pre-$T_{1/2}$.

Proof. Let $x \in X$. Wr prove (X, τ) is $[\gamma,\gamma']$-pre-$T_{1/2}$, it is sufficient to show that $\{x\}$ is $[\gamma,\gamma']$-preopen or $[\gamma,\gamma']$-preclosed. Now, if $\{x\}$ is $[\gamma,\gamma']$-open, then it is obviously $[\gamma,\gamma']$-preopen. If $\{x\}$ is not $[\gamma,\gamma']$- open, then $\tau_{[\gamma,\gamma']}$-Int($\{x\}$) = $\emptyset$; hence $\tau_{[\gamma,\gamma']}$-Cl $\tau_{[\gamma,\gamma']}$-Int($\{x\}$)) = $\emptyset \subset \{x\}$. Therefore, $\{x\}$ is $[\gamma,\gamma']$-preclosed.

REFERENCES

[1] S.Kasahara, Operation-compact spaces, Math. Japonica 24 (1979), 97 -105.

[2] C. Carpintero, N. Rajesh and E. Rosas, On $[\gamma,\gamma']$-preopen sets, Demon. Math., XLVI (3) (2013), 617-629.

[3] S. Kousalyadevi and P. Komalavalli, Bioperation-via Regular open sets (sub- mited).

[4] H. Maki and T. Noiri, Bioperations and some separation axioms, Scientiae Math. Japonicae, 53(1)(2001), 165-180.