

A Parallel Sporadic Decomposition of Hankel Structured Matrix using Optimal Patch Size for Impulse Noise Removal

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Abstract

In image processing, image denoising is the most essential process since it removes the noises and reconstructs the original image with high quality. As a result, an optimal patch size based sporadic decomposition of Hankel structured matrix in gradient transform domain namely OPS-e4-ALOHA was proposed in which the image patch was considered as a sparse in the gradient domain and the optimal patch size was selected by using a Flower Pollination Algorithm (FPA). However, it uses only one image at a time whereas the computation speed was less while different test images were used simultaneously. Hence in this article, a parallel sporadic decomposition of Hankel structured matrix named Enhanced e4-ALOHA (Ee4-ALOHA) is proposed with optimal patch size selection for removing the impulse noise from the images. In this technique, a multivariate polynomial decomposition problem in low-rank Hankel matrix is considered. To avoid this problem, Quasi-Hankel matrix is introduced based on the coefficients of the polynomial. Therefore, the original decomposition problem is reformulated as a structured low-rank matrix completion. Further, this technique is combined together with the parallel computing framework which has multiple processing cores for increasing the computation speed of matrix construction and denoising processes. Finally, the experimental results show that the effectiveness of the proposed technique.

Keywords: Image denoising, e4-ALOHA, Flower pollination algorithm, Multivariate polynomial decomposition, Quasi-Hankel low-rank matrix.

INTRODUCTION

In general, noise refers an unwanted disturbance in the original images or signals. There are different types of noises occurred during image acquisition or transmission. Among those, the most well-known noise is impulse noise also known as fat-tailed distribution or spike noise. It can arise as a malfunction result of detector pixels in a digital camera or memory component in imaging hardware. This type of noise is further categorized as two types such as salt-and-pepper and Random-Valued Impulse Noise (RVIN). In salt-and-pepper noise case, noisy pixels are simply identified by using an Adaptive Median Filter (AMF) whereas RVIN cannot be simply identified by AMF. Due to the presence of noise in an

image, noise removal or reduction techniques are mostly needed in image processing with the aim of suppressing the noise. In earlier, various image denoising techniques have been designed to remove the impulse noise from the image.

Among different filter techniques, sparse decomposition based noise removal has a better trade-off between the information preservation and the noise suppression. Such sparse representation was improved by a robust Annihilating filter-based low-rank decomposition of Hankel structured matrix namely r-ALOHA [1]. In this technique, a spectrum of a noiseless image was considered as a sparse in Fourier domain whereas the other transform domains were not considered. Therefore, the other transform domains such as log-exponential transform (e1-ALOHA), wavelet transform (e2-ALOHA), radon transform (e3-ALOHA) and gradient transform (e4-ALOHA) were proposed [2]-[3]. In addition to this, an optimal patch size was also selected by using Flower Pollination Algorithm (FPA) that selects a group of similar patches and optimal patch size to model underlying image [4]. However, the speed of the denoising process was less since only one image was considered at a given time.

Hence in this article, a parallel sporadic decomposition of Hankel structured matrix is proposed to improve the impulse noise removal process. In this technique, a multivariate polynomial decomposition problem is considered and overcome by proposing Quasi-Hankel matrix which is constructed from the coefficients of the polynomial. According to this correspondence, the original decomposition problem can be reformulated as structured low-rank matrix completion. This approach is further combined with parallel computing framework to speed up the computation of low-rank method efficiently. In parallel construction, the reconstruction process each test image using Hankel matrices is assigned to the multiple processing cores. Thus, this proposed technique enhances the image denoising in terms of reduced computational complexity and speedup the removal process.

The rest of the article is structured as follows: Section 2 provides the previous researches related to impulse noise removal techniques. Section 3 explains the proposed noise removal technique in brief. Section 4 compares the performance of the proposed technique with the existing technique and Section 5 concludes the research work.

LITERATURE SURVEY

Parallel domain decomposition based algorithm [5] was proposed for large scale color image denoising. In this algorithm, parallel domain decomposition based Newton-Krylov-Schwarz (NKS) method was proposed for numerically solving the total variation minimization model for the color image restoration. Here, an inaccurate Newton method with an analytical Jacobian matrix was used as the nonlinear solver whereas a preconditioned Krylov subspace method was used in every Newton step for solving the Jacobian system and a Restricted Additive Schwarz (RAS) preconditioner was used for accelerating the convergence. However, computational cost was increased significantly.

A novel Sparsity-Ranking Edge-Preservation Filter (SREPF) [6] was proposed to remove high-density impulse noise in images. The initial process of SREPF according to the sparse matrix representation was used to predict the noisy candidates and decide the processing order of them via a ranking of noise-pixels sparsity in the working window. Then, a modified double Laplacian convolution was applied to confirm the truly noisy pixels and yield a directional mean for recovering them. Conversely, computational complexity must be maintained at a low level for the preservation of edges and removal of impulse noise.

Domain decomposition methods [7] were proposed for nonlocal total variation image restoration. Based on this method, the original problem was decomposed into smaller sub-problems defined on sub-domains. Each sub-problem was effectively solved by using the split Bregman algorithm and Bregmanized operator splitting algorithm. Moreover, all sub-problems defined on sub-domains with same colors were computed in parallel by using coloring technique on the domain decomposition. However, this method requires convergence analysis.

A Patch-based Exponentially Weighted Aggregation (PEWA) [8] was proposed for image denoising. In this method, an image patch was estimated from weakly denoised image patches in the input image. A boosted estimator was obtained by combining weak denoised versions of the input noisy images. Also, a spatial Bayesian prior and a Gibbs energy distribution were used for selecting good candidate patches. Moreover, a dedicated MCMC sampling process was proposed for computing the PEWA estimator efficiently. However, computational complexity was high.

Non-overlapping domain decomposition methods [9] were proposed for dual total variation based image denoising. In this study, both parallel and sequential approaches were proposed for these methods for which convergence to a minimizer of the original problem was established. The associated sub-problems were solved by a semi-smooth Newton method. Moreover, the performance analysis of both sequential and parallel algorithms for image denoising was presented through several numerical experiments. However, the overall computing time of the system was increased due to communication time of the processors.

Image denoising technique [10] was proposed based on parallel k-Singular Value Decomposition (k-SVD). Initially,

partial image blocks were randomly chosen for generating the training signal set. After that, the trained dictionary was generated by alternately iterating between the sparse code phase and the dictionary update phase. Then, the sparse representation of the entire image was performed by the orthogonal matching tracking algorithm that approximates the sparse matrix. Each sparse vector was recorded with a 3-tuple structure for reducing the transmission and computation of data. In the dictionary update phase, the method was updated in a separate way for each atom. However, execution time of this process was not reduced.

PROPOSED METHODOLOGY

In this section, the proposed technique for image denoising model is explained briefly. In this technique, a parallel computing framework is introduced to reduce the reconstruction (computation) time significantly. Initially, the given image with impulse noise M is modeled as a combination of noise-free image X and sparse matrix S by using Hankel matrix \mathcal{H} as follows:

$$\mathcal{H}(M) = X + S \tag{3.1}$$

The image is modeled with the least Total Variations (TV) by considering sparse matrix to obtain the following cost function:

$$\|M - X\|_1 + \lambda TV(X) \tag{3.2}$$

Equation (3.2), $\|\cdot\|_1$ norm denotes the l_1 norm associated with the sum of absolute values of each matrix element to remove the outlier and $TV(X)$ refers the 2D TV penalty in the image modeling. For a given image patch size $x[n]$ and annihilating filter size, the sparse and low-rank decomposition problem using (3.1) requires an additional constraint due to its Hankel structure. Hence, this decomposition problem is reformulated as a structured low-rank matrix completion problem of a quasi-Hankel matrix. The elements of the entire Hankel matrix $\mathcal{H}(M)$ are defined by constructing a submatrix. Consider two ordered sets of multi-indices defined by matrices $\mathcal{X} \in \mathbb{Z}_+^{n \times m_x}$ and $\mathcal{S} \in \mathbb{Z}_+^{n \times m_s}$, where

$$\mathcal{X} = [x^{(1)}, \dots, x^{(m_x)}] \in \mathbb{Z}_+^{n \times m_x} \text{ and } \mathcal{S} = [s^{(1)}, \dots, s^{(m_s)}] \in \mathbb{Z}_+^{n \times m_s} \tag{3.3}$$

Equation (3.3), \mathbb{Z}_+ denotes the set of nonnegative integers. Then, the quasi-Hankel matrix $\mathcal{H}_{\mathcal{X}, \mathcal{S}}(M) \in \mathbb{X}^{m_x \times m_s}$ is defined as follows:

$$\left(\mathcal{H}_{\mathcal{X}, \mathcal{S}}(M)\right)_{i,j} := M_{x^{(i)}+s^{(j)}} \tag{3.4}$$

The quasi-Hankel matrix is a submatrix of $\mathcal{H}(M)$ corresponding to the sets $\{x^{(1)}, \dots, x^{(m_x)}\}$ and $\{s^{(1)}, \dots, s^{(m_s)}\}$ of row and column indices. Here, $\mathcal{H}_{\mathcal{X}, \mathcal{S}}$ takes into account only the multi-indices from the Minkowski sum $\mathcal{X} + \mathcal{S}$. Once the quasi-Hankel matrix is constructed, the minimal rank

completion is achieved for the unknown entries M_{x+s} for $x + s \in \Delta^{(n-1,2p)} \setminus \Delta^{(n-1,p)}$ where n refers the number of variate polynomials and p refers the degree of polynomial. Based on this, a minimal rank is determined. Then, the decomposition (DC) named affine decomposition is defined as follows:

$$x_x^{(DC)} = \sum_{k=1}^r h_k(x) \lambda_k^c, \forall x \in \Delta^{(n-1,p)} \quad (3.5)$$

$$s_s^{(DC)} = \sum_{k=1}^r h_k(s) \lambda_k^d, \forall s \in \Delta^{(n-1,p)} \quad (3.6)$$

Where $\{s_s^{(DC)}\}_{s \in \Delta^{(n-1,p)}}$ and $\{x_x^{(DC)}\}_{x \in \Delta^{(n-1,p)}}$ are the dehomogenized coefficients obtained from the normalized coefficients, r is the waring rank of $\mathcal{H}_{x,s}(M)$ and h_k are the polynomials of \mathbb{X} . Finally, the exponents λ_k are reconstructed from the image of the matrix $\mathcal{H}_{x,s}(M)$. The coefficients of h_k are obtained by solving the linear systems (3.5) and (3.6). Here, the image patch size is selected based on the FPA and the image patches are sparse in the gradient domain. In this technique, the reconstruction is performed separately on each test image. Therefore, a parallel framework i.e., multiple processing cores on computer is proposed to efficiently accelerate the reconstruction. The reconstruction process each test image using Hankel matrices is assigned to the multiple processing cores. Through the parallel processing, the reconstruction time is reduced efficiently.

RESULTS AND DISCUSSIONS

In this section, the performance effectiveness of the proposed technique named OPS-Ee4-ALOHA is evaluated and compared with the existing technique i.e., OPS-e4-ALOHA by using the simulation tool such as MATLAB 2018a. The comparison is made in terms of Peak Signal-to-Noise Ratio (PSNR), Structural Similarity Index Metric (SSIM) and reconstruction/computation time for different images such as Lena, Barbara, balloon and cameraman at noise density level is 25%. Since these test images has a number of regions with high information and allow easy comparison between different methods to provide better outcomes.

Peak Signal-to-Noise Ratio (PSNR)

It defines the fraction of maximum possible signal power to the corrupting noise power. Generally, it is computed by using Mean Squared Error (MSE) as:

$$PSNR = 10 \log_{10} \frac{255^2}{MSE} \quad (4.1)$$

$$MSE = \frac{1}{m \cdot n} \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} (M_{ij} - I_{ij})^2 \quad (4.2)$$

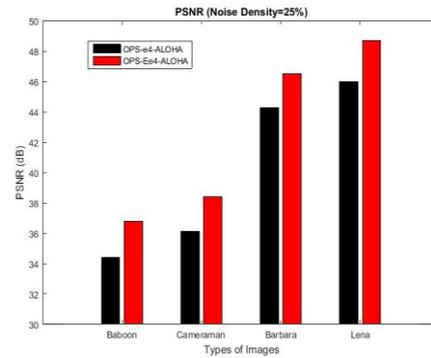


Figure 1: Comparison of PSNR

Figure 1 shows the comparison of proposed and existing techniques by considering the images such as baboon, cameraman, Barbara and Lena in terms of PSNR (dB) at noise density level is 25%. For example, when Lena image is considered, the PSNR value of proposed OPS-Ee4-ALOHA is 5.89% higher than OPS-e4-ALOHA. From the analysis, it is observed that the OPS-Ee4-ALOHA has better PSNR than the OPS-e4-ALOHA.

Reconstruction Time

It defines the time taken to reconstruct the noiseless images from noisy images.

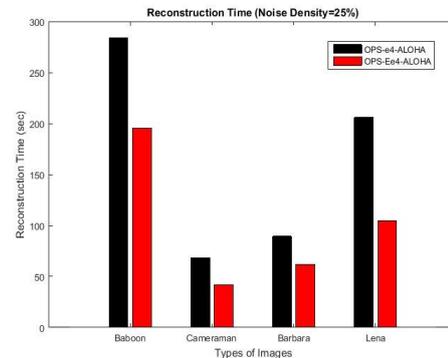


Figure 2: Comparison of Reconstruction Time

Figure 2 demonstrates the comparison of reconstruction time (seconds) for proposed and existing techniques by considering various images like baboon, cameraman, Barbara and Lena at noise density level is 25%. For example, when Barbara image is considered, the reconstruction time of proposed OPS-Ee4-ALOHA is 30.84% less than OPS-e4-ALOHA. Through the analysis, it is that noticed that OPS-Ee4-ALOHA has reduced reconstruction (computation) time than the OPS-e4-ALOHA.

Structural Similarity Index Matrix (SSIM)

It defines the similarity value between the original and denoised images. It is computed as:

$$SSIM(x, y) = \frac{(2\mu_M \mu_I + c_1)(2\sigma_{MI} + c_2)}{(\mu_M^2 + \mu_I^2 + c_1)(\sigma_M^2 + \sigma_I^2 + c_2)} \quad (4.3)$$

Here, μ_M, μ_I are averages and σ_M^2, σ_I^2 are variances of original image (M) and noiseless image (I) respectively. Also, c_1, c_2 are constants and σ_{MI} is the covariance of M and I .

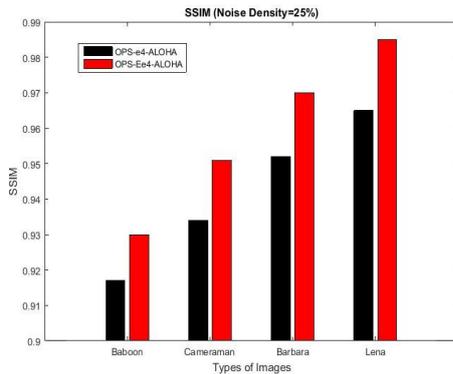


Figure 3: Comparison of SSIM

Figure 3 demonstrates the comparison of SSIM for proposed and existing techniques by considering various images like baboon, cameraman, Barbara and Lena at noise density level is 25%. For example, when cameraman image is considered, the SSIM value of proposed OPS-Ee4-ALOHA is 1.82% higher than OPS-e4-ALOHA. Through the analysis, it is concluded that OPS-Ee4-ALOHA has improved SSIM than the OPS-e4-ALOHA.

CONCLUSION

In this article, a parallel sporadic decomposition of Hankel structured matrix is proposed to improve the impulse noise removal process. In this technique, a multivariate polynomial decomposition problem is considered and overcome by proposing Quasi-Hankel matrix which is constructed from the coefficients of the polynomial. According to this correspondence, the original decomposition problem can be reformulated as structured low-rank matrix completion. This approach is further combined with parallel computing framework to speed up the computation of low-rank method efficiently. Thus, this proposed technique enhances the image denoising in terms of reduced computational complexity and speedup the removal process. Finally, the experimental results are proved that the proposed technique has better denoising performance than the existing technique in terms of PSNR, SSIM and reconstruction time.

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