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A New Approach for Solving Linear Fractional Programming Problem

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Abstract

In this paper, an algorithm is developed for solving linear fractional programming problems using graphical method. First, the problem will be converted into single linear programming problem and then it can be solved. The solutions which will be obtained by using this algorithm may be bounded, unbounded or have no solution. This algorithm is simple and easy to understand and apply as compare to the earlier existing algorithm. The procedure for the proposed algorithm is illustrated with numerical examples.

Keywords: Linear fractional programming, graphical method.

1. INTRODUCTION

Graphical method of linear fractional programming is used to solve problems by finding the highest or lowest point of intersection between the focus point of objective function and the feasible region on a graph. The graphical method is one of the simplest methods for obtaining the optimal values in linear fractional programming. The linear fractional programming problem will be converted into single linear programming problem and then optimal solution is determined. The algorithm which is developed here is a simple and easy to understand.

The linear fractional programming problem is formulated as:

$$Maximize/Minimize\ Q(x) = \frac{P(x)}{D(x)} = \frac{\sum_{j=1}^{n} p_j x_j + p_0}{\sum_{j=1}^{n} d_j x_j + d_0}$$

Subject to:

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i, \qquad i = 1,2,3 \ldots m_1.$$

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i, \quad i = m_1 + 1, m_1 + 2 \ldots, m_2.$$

$$\sum_{j=1}^{n} a_{ij} x_j = b_i \quad i = m_2 + 1, m_2 + 2 \ldots, m.$$

$$x_j \geq 0 \qquad j = 1,2,3 \ldots n$$

2. ALGORITHM

Step1: Let us take LFP problem with two unknown variables

$$Maximize/\ Minimize\ \mathrm{Q(x)} = \frac{\mathrm{P(x)}}{\mathrm{D(x)}} = \frac{p_1x_1+p_2x_2+p_0}{d_1x_1+d_2x_2+d_0} \quad \ldots (1)$$

Subject to $a_{i1}x_1 + a_{i2}x_2 \leq b_i \quad i = 1,2 \ldots m$

$$x_1 \geq 0 \quad x_2 \geq 0 \qquad \ldots (2)$$

Where x_1 and x_2 are non negative variables.

Step 2: Convert the general form of LFP into the standard form

$$\text{Maximize/Minimize } \mathrm{Q(x)} = \frac{\mathrm{P(x)}}{\mathrm{D(x)}} = \frac{p_1x_1+p_2x_2+p_0}{d_1x_1+d_2x_2+d_0} \quad \ldots (3)$$

$$a_{i1}x_1 + a_{i2}x_2 = b_i \quad i = 1,2 \ldots m \qquad \ldots (4)$$

$$x_1 \geq 0 \quad x_2 \geq 0$$

Step 3: From the standard equation of LFP

Consider P(x)and D(x)in an equality form.

$$\mathrm{P(x)} = p_1x_1 + p_2x_2 = -p_0 \qquad \ldots (5)$$

Similarly for D(x) = $d_1x_1 + d_2x_2 = -d_0$ … (6)

Step 4: Solve the equations (5) and (6), to get focus point (F).

Step 5: Find the extreme points, denoted them by $A(x_1, x_2)$,$B(x_1, x_2)$, $C(x_1, x_2)$.

Step 6: For plotting the graph follow these steps:

(a) By using Step 4 plot the focus point (through the intersection of line P(x) and D(x).

[**NOTE**: If lines P(x) and D(x) intersect each other then the intersection point will be known as focus point, unique solution will be obtained. Then proceed to Step 7. If lines P(x) and D(x) do not intersect each other i.e., if they are parallel to each other. Then there will be no solution].

(b) By using step 5 plot the extreme points i.e., $A(x_1, x_2)$, $B(x_1, x_2)$,$C(x_1, x_2)$.

(c) To find feasible region

Case 1: If constraint has ≤ sign then, region below the line will be shaded.

Case 2: If constraint has ≥ sign then, region above the line will be shaded.

Step 7: Find the objective values by putting the extreme points in Q(x), we will get the values of Q(A),Q(B) and Q(C).

Step 8: From the value of Q(A) Q(B) Q(C), determine that the objective function Q(x) reaches:

Case 1: In maximization problem we will see the highest value among Q(A), Q(B) and Q(C) which will be the maximal value.

Case 2: In minimization problem we will see the lowest value among Q(A), Q(B) and Q(C) which will be the minimal value.

Step 9: Join the focus point to the maximal point, [Q(x) = Q max] and minimal point [Q(x) = Q min]. Whereas Q(x) = K is an arbitrary line which will be drawn from the focus.

Step 10:

(a) Bounded feasible region: If there is a closed common feasible region by the level lines then that is bounded feasible region.

(b)Unbounded feasible region: If there is a no closed common feasible region by the level lines then that is unbounded feasible region.

3. NUMERICAL EXAMPLES

Case I: Problem with bounded solution

$$Maximize\,Z(x)=\frac{3x_1+5x_2+15}{2x_1+4x_2+16}$$

$$Subject\,to: \quad 4x_1-6x_2\geq 36$$
$$5x_1+7x_2\leq 35$$
$$x_1\geq 0 \; and \; x_2\geq 0$$

Standard form is

$$Maximize\,Z(x)=\frac{3x_1+5x_2+15}{2x_1+4x_2+16}$$
$$Subject\,to: \quad 4x_1-6x_2=36$$
$$5x_1+7x_2=35$$
$$x_1\geq 0\,and\,x_2\geq 0$$

For focus points numerator and denominator of the objective function Z(x), are solved separately,

$$P(x)=3x_1+5x_2+15$$
$$D(x)=2x_1+4x_2+16$$

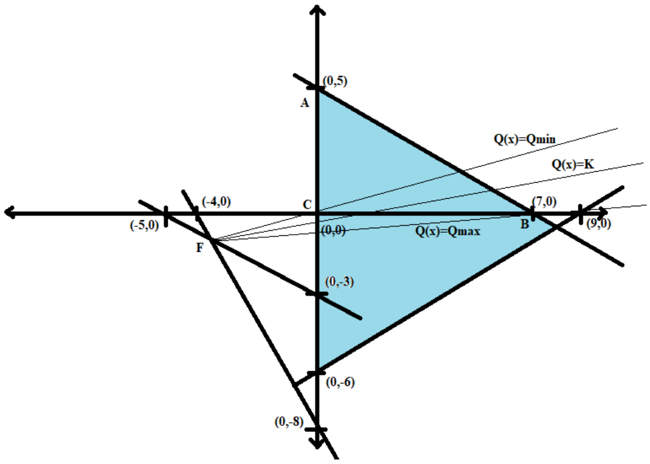
Which gives; for P(x) it is (0,-3) and (-5, 0) and for D(x) (0,-8) and (-4, 0)

For calculating the extreme points, solving the constraints

$4x_1-6x_2=36$, gives the points (0, -6) and (9, 0)

$5x_1+7x_2=35$, gives the points (0, 5) and (7, 0)

Solving the numerator and denominator for the focus point F. The following graph will be plotted



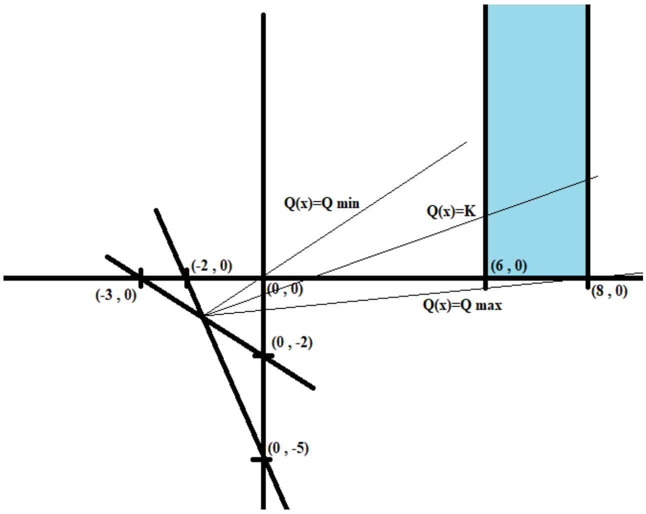
Feasible region of the above graph is ABC with A(0,5), B(7, 0) and C(0, 0); and Z_{max}=1.2 at B.

Result: The problem has ***Bounded Solution*.**

Case II: Problem with unbounded solution

$$Maximize\,Z(x)=\frac{3x_1+2x_2+6}{5x_1+3x_2+15}$$
$$Subject\,to: x_1\leq 8, x_1\leq 6$$
$$and\;\; x_1\geq 0\,and\,x_2\geq 0$$

Solving this example by using the newly developed algorithm, the nature of graph will be,



Feasible region of the above graph is shown by the shaded region, which is unbounded. Hence the problem has ***Unbounded Solution*.**

Case III: Problem with no solution

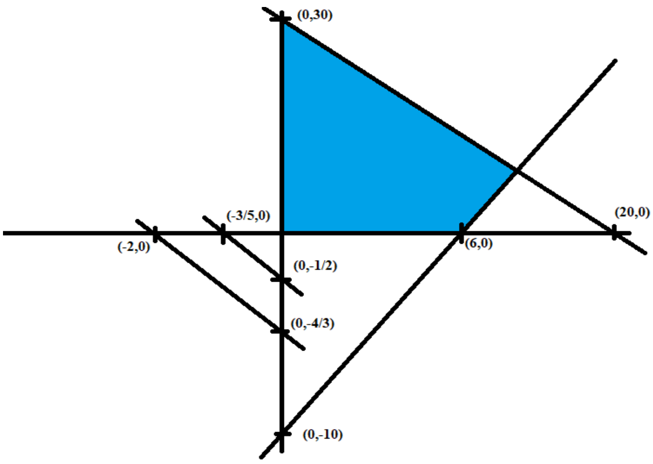
$$Maximize\, Q(x) = \frac{2x_1 + 3x_2 + 4}{5x_1 + 6x_2 + 3}$$

$$Subject\ to: \quad 5x_1 - 3x_2 \geq 30$$

$$3x_1 + 2x_2 \leq 60$$

$$and \quad x_1 \geq 0\ ,\ x_2 \geq 0$$

Similarly, we get the following graph.



Since, there is no Focus point, the problem has ***No Solution.***

4. CONCLUSION

Linear fractional programming has attracted many minds which resulted in its continuous growth. In this paper, an algorithm has been developed for solving linear fractional programming problem by graphical method. This algorithm is more efficient and easy to implement, comparative to the other available methods. It has been validated by solving numerical problems which have bounded unbounded and no solutions.

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