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Vulnerability Assessment of Power Systems considering DC and AC Models

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Abstract

This paper presents a comparison of DC and AC models applied to the vulnerability assessment of power systems under intentional attacks. This study is also known as the electric grid interdiction problem (EGIP) and consists of finding the set of most critical transmission assets in terms of the load shedding that would result if these are taken out of service by a malicious agent. The EGIP is modeled as a bilevel attacker-defender problem in which a disruptive agent (attacker) and the system operator (defender) are engaged. The traditional DC approach that only considers active power and voltage angles is contrasted with a more complete AC model that considers the effect of reactive power and voltage profile. Both models were solved using a specialized genetic algorithm. To show the applicability of the models applied to the EGIP several tests were performed using the IEEE 24 bus reliability test system. Results show that while the DC model allows finding faster results, in most cases these are more conservative than those obtained with an AC model.

Keywords*:* Interdiction problem, vulnerability analysis, genetic algorithms.

INTRODUCTION

Power system security is one of the main concerns of power system planners and operators. Currently, power systems are exposed not only to natural occurring phenomena but also to intentional attacks. Some of the most severe effects of a malicious attack on a power system are the load shedding and higher operational costs associated to repairing towers and transmission lines [1]. The traditional approach to assess power system security is based on the well-known N-1 and N-2 security criterion, which consists on guaranteeing the proper operation of the power system after one (N-1) or two (N-2) contingencies have taken place [2]. These type of studies require exhaustive simulation and therefore are associated with high computational burden; on the other hand, only natural occurring outages can be considered. Conversely, the electric grid interdiction problem (EGIP) considers the actions of a malicious agent that aims at maximizing damage to the system.

The EGIP was first formulated in [3] as a max-min attacker-defender model. In such model the malicious agent or terrorist aims at maximizing the total load shedding of the network produced by an interdiction plan; while the defender (system operator) reacts to the interdiction plan by re-dispatching available generation. From the standpoint of game theory, the EGIP can be formulated as a leader-follower Stackelberg game. Both, leader and follower have different objective functions which conflict with each other. The leader moves first anticipating the reaction of the follower aiming at maximizing his gains or minimizing his losses. Once the leader has made a decision on the game, the follower reacts to the leader strategy. The objective function of the leader is computed after the follower has made his choice. In the EGIP the leader is the disruptive agent who decides, considering a set of limited destructive resources, which elements of the system to attack in order to produce maximum damage, measured as load shedding. The disruptive agent must anticipate the reaction of the system operator. Once an interdiction plan is executed, the system operator (follower) reacts by re-dispatching generation or changing the system topology in order to minimize the total load shedding. From the standpoint of mathematical programming, the EGIP can be modeled as a bilevel programming problem in which the leader and follower are positioned in the upper and lower optimization problems, respectively. In [4] and [5] this problem is solved by using duality properties and linearization; in this way the original bilevel programming problem is recast into a single-level optimization problem which is solvable using commercially available software. In [6] a worst-case interdiction analysis of the EGIP is performed by a generalization of Benders decomposition. In [7] the EGIP is formulated considering line switching as a complement or alternative to the system operator reaction. In this case, after an attack plan has taken place, the system operator is able to alter the network topology in order to reduce load shedding. In [8] the author compares two different approaches to model the EGIP: a minimum vulnerability model and a maximum vulnerability model. The former is based on the models presented in [4] and [5] while the later consists on identifying the maximum level of system load shedding that can be attained with a given number of simultaneous line outages.

One common denominator of the aforementioned approaches to the EGIP is the fact that they use a DC model of the network. The DC model allows representing the network in a simplified fashion that only considers angles and active power injections. However, a more detailed representation of the network can be obtained through an AC modelling. In this case, besides active power injections and bus angles; the AC model also considers reactive power and voltage magnitudes.

Using a more detailed modeling of the network involves more computational burden; however, it is compensated by more reliable results. An AC modeling of the EGIP is proposed in [9] and is solved using iterative local search.

This paper presents a comparison of the traditional DC model of the EGIP with an AC model. Both models are solved using a specialized genetic algorithm and results are compared and contrasted. Several tests were performed in the IEEE 24 bus reliability test system showing the applicability of the proposed approach. Results show that when considering a DC model, solutions to the EGIP problem are more conservative than those obtained with the AC model.

MATHEMATICAL MODELING

In this section the DC and AC models of the EGIP are presented and explained in detail. The DC model is the one proposed in [8] while the AC model is an adaptation of the one presented in [9].

DC interdiction model

This model is given by equations (1) to (9). The objective function consists on maximizing the sum of load shedding in every bus given by P_{DS_n}. In this case N is the number of load buses and δ^{Lin} is a binary interdiction vector that indicates whether a line is on service (one) or out of service (zero). The objective function is subject to the limits of destructive resources M expressed in (2). In this case l is the line index and L is the set of lines. Note that the binary nature of the interdiction vector is given in (3). Equations (4) to (9) represent the reaction of the power system operator. The objective of this agent is to minimize the cost of operation and load shedding as indicated in (4). The first term of the objective function corresponds to operative costs. In the case P_g^{Gen} is the power supplied by generator g and c_g is its cost. The second term is the cost of load shedding. In this case c_{DS_n} is the cost of non-supplied demand in bus *n*. The vector of decision variables x, in the lower level optimization problem includes bus angles (represented by θ_n), active power generation, load shedding and active power flows in lines given by P_l^{Lin}. The lower level optimization problem (reaction of the system operator) is subject to the definition of active power flows given by (5). Note that active power flows are multiplied by δ_l^{Lin} meaning that there are no power flows on faulted lines. The active power balance constraints are given by (6), and limits on generation, bus angles and load shedding are given by (7), (8) and (9), respectively. In this case Z_l is the impedance of line *l*, A_{nl} is the incidence matrix, g is the generator index, G is the set of generators and P_{D_n} is the active power demand in bus *n*.

$$\max_{\delta^{Lin}} \sum_n P_{DS_n}; \quad \forall n \in N \tag{1}$$

Subject to:

$$\sum_l \left(1-\delta_l^{Lin}\right) \le M; \quad \forall l \in l \tag{2}$$

$$\delta_l^{Lin} \in \{0,1\}; \quad \forall l \in L \tag{3}$$

$$\min_{\boldsymbol{x}} \sum_g c_g P_g^{Gen} + \sum_n c_{DS_n} P_{DS_n}; \tag{4}$$

$$\boldsymbol{x} = \left[\theta_n, P_g^{Gen}, P_{DS_n}, P_l^{Lin}\right]$$

Subject to:

$$P_l^{Lin} = \delta_l^{Lin} * \frac{1}{Z_l} \sum_{n\in N} A_{nl}\theta_n; \quad \forall l \in L \tag{5}$$

$$\sum_{g\in G} P_g^{Gen} - \sum_{l\in L} A_{nl} P_l^{Lin} + P_{DS_n} = P_{D_n}; \quad \forall n \in N \tag{6}$$

$$P_g^{min} \le P_g^{Gen} \le P_g^{max}; \quad \forall g \in G \tag{7}$$

$$\theta_n^{min} \le \theta_n \le \theta_n^{max}; \quad \forall n \in N \tag{8}$$

$$0 \le P_{DS_n} \le P_{D_n}; \quad \forall n \in N \tag{9}$$

AC interdiction model

This model is given by equations (10) to (28). Note that the upper level optimization problem (leader) given by (10)-(12) is the same as the one described for the DC model (equations (1)-(4)). The main difference is in the lower level optimization problem given by (13)-(28) which considers a more detailed modeling of the network. The objective function given by (13) is the same as the one of the DC model (equation (4)); however, the vector of decision variables also includes voltage magnitudes V_n, reactive power generation Q_g^{Gen}, reactive power flow in lines Q_l^{Lin} and reactive load shedding Q_{DS_n}. Equations (14) to (17) indicate the limits on voltage magnitudes and angles in buses, as well as limits on active and reactive power generation, respectively. Equations (18) to (20) indicate the limits of apparent power low in lines S_l^{Lin}, as well as limits on active and reactive load shedding, respectively. Note that equations (19) and (20) indicate that the load shedding must not exceed the demand of the bus. Equations (21) and (22) represent the net active and reactive power injected in bus n, respectively; g_{mn} and b_{mn} are real and imaginary entries of m,n position of the admitance matrix, respectively; θ_{mn} represents the angular difference between nodes *m* and *n*. Equation (23) expresses the active and reactive components of the apparent power flow. Equations (24) and (25) are the mathematical expressions defining active and reactive power flow, respectively. Note that (24) and (25) are multiplied by δ_l^{Lin} meaning that there are no power flows on faulted lines. Equations (26) and (27) indicate the active and reactive power balance constraints in very node; finally, equation (28) indicates that the reference angle θ_{ref} must be zero.

$$\max_{\delta^{Lin}} \sum_{n} P_{DS_n}; \quad \forall n \in N \tag{10}$$

Subject to:

$$\sum_{l} (1-\delta_l^{Lin}) \leq M; \quad \forall l \in L \tag{11}$$

$$\delta_l^{Lin} \in \{0,1\}; \quad \forall l \in L \tag{12}$$

$$\min_{\boldsymbol{x}} \sum_{g} c_g P_g^{Gen} + \sum_{n} c_{DS_n} P_{DS_n}; \qquad \boldsymbol{x} = \begin{bmatrix} \theta_n, V_n, P_g^{Gen}, Q_g^{Gen}, P_l^{Lin}, Q_l^{Lin}, \\ P_{DS_n}, Q_{DS_n} \end{bmatrix} \tag{13}$$

Subject to:

$$\theta_n^{min} \leq \theta_n \leq \theta_n^{max}; \quad \forall n \in N \tag{14}$$

$$V_n^{min} \leq V_n \leq V_n^{max}; \quad \forall n \in N \tag{15}$$

$$P_g^{min} \leq P_g^{Gen} \leq P_g^{max}; \quad \forall g \in G \tag{16}$$

$$Q_g^{min} \leq Q_g^{Gen} \leq Q_g^{max}; \quad \forall g \in G \tag{17}$$

$$S_l^{min} \leq S_l^{Lin} \leq S_l^{max}; \quad \forall l \in L \tag{18}$$

$$0 \leq P_{DS_n} \leq P_{D_n}; \quad \forall n \in N \tag{19}$$

$$0 \leq Q_{DS_n} \leq Q_{D_n}; \quad \forall n \in N \tag{20}$$

$$P_n = V_n \sum_{n} V_m [g_{mn} \cos(\theta_{mn}) + b_{mn} \sin(\theta_{mn})]; \quad \forall n \in N \tag{21}$$

$$Q_n = V_n \sum_{n} V_m [g_{mn} \sin(\theta_{mn}) + b_{mn} \cos(\theta_{mn})]; \quad \forall n \in N \tag{22}$$

$$(S_l^{Lin})^2 = (P_l^{Lin})^2 + (Q_l^{Lin})^2; \quad \forall l \in L \tag{23}$$

$$P_l^{Lin} = \delta_l^{Lin} \cdot [g_{mn} V_n^2 + g_{mn} V_m V_n \cos(\theta_{mn}) - b_{mn} V_m V_n \sin(\theta_{mn})]; \quad \forall l \in L \tag{24}$$

$$Q_l^{Lin} = \delta_l^{Lin} \cdot [-b_{mn} V_n^2 + b_{mn} V_m V_n \cos(\theta_{mn}) - b_{mn} V_m V_n \sin(\theta_{mn})]; \quad \forall l \in L \tag{25}$$

$$P_g^{Gen} - P_{D_n} + P_{DS_n} = P_n; \quad \forall n \in N \tag{26}$$

$$Q_g^{Gen} - Q_{D_n} + Q_{DS_n} = Q_n; \forall n \in N \tag{27}$$

$$\theta_{ref} = 0 \tag{28}$$

SOLUTION APPARAOCH

Both DC and AC models of the EGIP are challenging problems since even when the objective functions and constraints are linear, a bilevel programming problem is intrinsically nonconvex and nonlinear [8]. These type of problems are better handled by metaheuristics than by classical optimization techniques. In this case, a specialized genetic algorithm was developed to solve both models.

Problem codification

Every solution candidate of the EGIP corresponds to a binary vector δ^{Lin} that indicates which lines are out of service due to the execution of an attack plan. Figure 1 illustrates a 9 bus power system with 13 lines. In this case lines L1, L5, L10 and L12 are under attack. Note that the attack plan is indicated with a binary vector with zeros in those lines that are out of service.

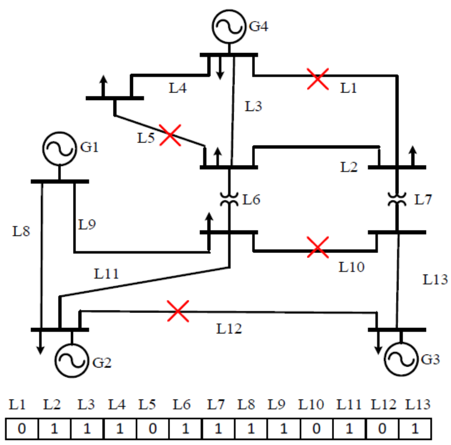

Figure 1. Illustration of an interdiction vector in a power system.

Implemented Genetic Algorithm

Genetic algorithms (GAs) are adaptive heuristic search procedures based on the principles of Darwinian evolution. GAs have been widely used to approach non-convex optimization problems such as the ones described in this paper. Figure 2 depicts the flowchart of the proposed GA. It starts with an initial population of interdiction vectors. Given a limit on destructive resources the disruptive agent determines the number of lines to render out of service. With this number, a series of possible interdiction plans (as illustrated in Figure 1) are randomly generated. Then, depending on the model to be solved, a DC or AC optimal power flow is run to account for the reaction of the system operator. The optimal power flow is run in the software Matpower [10] and indicates the load shedding produced by every interdiction plan. This stage is called fitness evaluation. The fitness of a given interdiction plan (individual) represents the load shedding that the disruptive agent obtains with it. The process of selection is based on tournament and favors the best individuals (those interdiction vectors associated with high load shedding) to be recombined with other of similar fitness. The recombination is performed at a single point randomly selected. Every pair of individuals generate two new ones; in this stage two times the initial population is created. The mutation makes a minimal change in some of the individuals, with a given probability, and is performed with every bit of the interdiction vectors. If mutation or recombination leads to non-feasible candidates, such candidates are penalized in the objective function. Finally, to keep the number of candidate solutions constant, only the best solutions are kept in each iteration. The process stops when a maximum number of iterations has been reached.

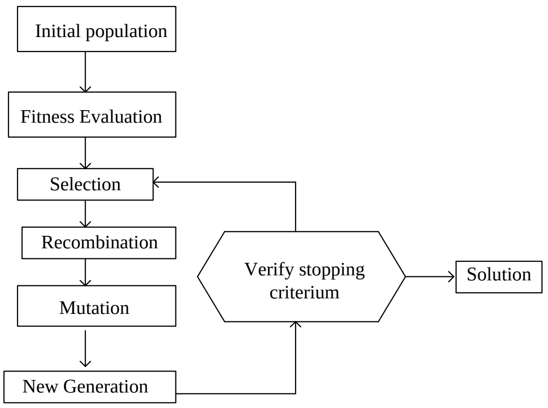

Figure 2. Flowchart of the proposed genetic algorithm.

TEST AND RESULTS

Several tests were performed with the DC and AC models using the IEEE 24 bus reliability test system which data can be consulted in [11]. This system comprises 24 buses, 38 lines, 32 generators and 17 loads. The load profile selected for this study corresponds to a winter weekday at 6:00 pm (2850 MW). The tests were performed considering an increasing number of destroyed lines, which can be seen as an increase of destructive resources (M). For simplicity it is considered that the cost of attacking each line is the same and is equal to one monetary unit; being M the number of monetary units available to the disruptive agent in each case. Table 1 shows the worst combination of destroyed lines and the corresponding load shedding for this system. The GA was set with an initial population of 20 individuals, a recombination rate of 50%, a mutation rate of 5% and a maximum of 100 iterations. Note that the solutions found with the AC model correspond to higher load shedding in almost all situations; however, the solutions found with the AC model took (in average) twice the computational time of those found with the DC model.

For M=2 the best attack plan consists on isolating bus 14. In this bus the load is higher than the local generation and the non-served load is 194MW for both models. For M=4 the best attack plan is destroying lines 3-24, 12-23, 13-23 and 14-16. This attack aims at isolating the upper part of the system from the lower one in which most demand is located. A similar strategy is repeated when the value of M increases. Figure 3 depicts several interdiction plans. It was found that some lines are common for different attack plans; for example, line 14-16 is considered in interdiction plans with M=2, 4 and 6. Also, the corridors 20-23 and 15-21 are both considered for M=8 and 10.

Table 1. Worst combination of destroyed lines with different destructive resources.

M	Destroyed lines	Load Shedding	
		DC	AC
2	11-14, 14-16	194	194
4	3-24, 12-23, 13-23, 14-16	516	559.8
6	3-24, 7-8, 9-12, 10-12, 11-13, 14-16	1017	1022.9
8	9-12, 10-12, 11-13, 15-21, 15-21, 16-17, 20-23, 20-23	1198	1206.5
10	7-8, 9-12, 10-12, 11-13, 15-21, 15-21, 16-17, 20-23, 20-23	1468	1476.8

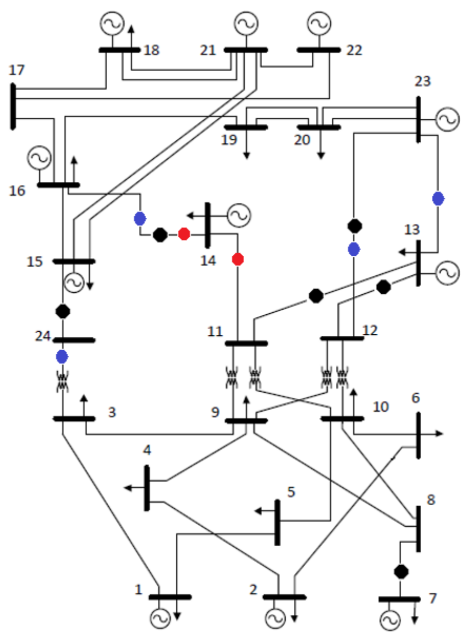

Figure 3. Illustration of interdiction plans for M=2 (red dots), M=4 (blue dots), and M=6 (black dots).

The criticality of different lines might differ according to the load profile. To take this into account, several tests were performed considering high, medium and low demand scenarios. Table 2 presents the results for these scenarios considering M=4 and Figure 3 depicts the corresponding attack plans. Note that the destroyed lines considered by the disruptive agent are not the same in all scenarios.

It was found that the difference in results with the DC and AC model, as with the previous tests, are not very high. In general, the load shedding reported with the DC model is slightly lower than that of the AC model; however, differences tend to increase in the high demand scenario.

Table 2. Worst combination of destroyed lines for different load profiles

Demand	Load Shedding		Destroyed lines
	DC	AC	
High	516	559.8	3-24, 12-23, 13-23, 14-16
Medium	254	256.47	15-24, 16-19, 20-23, 20-23
Low	194	194.7	2-6, 6-10, 11-14, 16-14

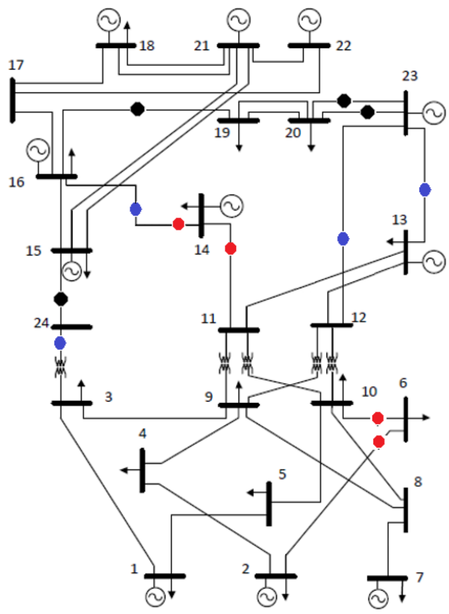

Figure 4. Illustration of interdiction plans for different load profiles: high (blue dots), medium (black dots) and low (red dots).

CONCLUSIONS

This paper presented a comparison of two interdiction models for the vulnerability assessment of power systems. In this case the interaction of a disruptive agent and the system operator was modeled as a bilevel programming problem, and solved with a genetic algorithm. Results allow to conclude that the DC model often reports conservative solutions; that is, solutions with lower load shedding than those found with the AC model. This situation does not seem critical since, difference in results are small although they are higher when considering a high demand scenario.

The main advantage of using a GA for solving the EGIP is the possibility of having a set of high quality solutions instead of a single one. This gives the system operator more information about the most vulnerable elements and provides signals for future reinforcements of the network or stricter surveillance on critical elements.

With the use of a GA instead of a classical mathematical programming approach, both DC and AC models can be solved without resorting to linearization schemes to transform the EGIP into a single-level optimization problem. Future work will consider the effect of distributed energy resources in the vulnerability assessment of power systems.

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