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Multi Product Dynamic Lot-Sizing with Supplier Selection under Quantity Discount and Budget Constraints

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Abstract

This study's aim is to determine the product purchasing by inventory lot-sizing as a problem of supplier selection with quantity discounts. The objective is to minimize the inventory management costs with the use of mixed integer linear programming. Also, the selection of suppliers with different products will be presented, and the lead-time for suppliers will also be considered, including the conditions of budget. Even so, the results obtained from these tests showed that the determination of the purchasing size and the duration for ordering purchases were correct and were able to be followed through satisfactorily. Finally, this study also tested the efficiency of the determination of lot sizes of purchases by expanding the problem to larger sizes. It was then discovered that the efficiencies in calculations and the results obtained were in good and satisfactory criterions.

Keywords: Lot-Sizing, Supplier Selection, Multi-Product, Multi-Supplier, Quantity discounts

INTRODUCTION

Inventory Lot-Sizing [1] will be the model for purchasing products in lots. Each lot-sizing will have a suitable number of products and are sufficient of use. This method of inventory lot-sizing will keep holding costs low as well as the total cost with the minimal inventory management for the problem of single items through multi-periods with irregular demands in each period, where they did not limit the existing resources under the given conditions[2],[3]. The presentation was done mathematically with limitations in the area of storage space, where the quantities of the inventory combined with the ordered quantities must not exceed the materials storage space[4]. The problems for determining the lot productions were also studied, with a set equation that minimizes the ordering, holding, production, and overtime costs altogether under the condition in which the desired capacity must not exceed normal and overtime production.

Other than the methods for ordering products, the supplier selection is also important, since presently suppliers that sell products are in high demands in the market, which could allow various the organization to have opportunities in purchasing products from the suppliers that are most beneficial. Each supplier will have different product prices and ordering costs. Thus, the model in supplier selection of inventory-lot sizing is highly important, since it will allow an awareness of the quantity of products in each lot-sizing as well as the supplier that was selected for that purchase.

The aim of this model is to minimize the inventory management costs[5], [6]inventory management is also aided by finding economic order quantity, which studies the problems of multi-product manufacturing as well that of multi suppliers and inventory lot-sizing for supplier selection in the form of various products and suppliers with limited storage space and budgets. This was done using linear programming and genetic algorithms. [7]Later on, the model was developed for ordering products with purchase-related quantity discounts of each supplier, also with the aim to minimize total costs. The model had been developed for the problem of supplier selection of inventory lot-sizing in the case of product discounts that considered the purchased quantities and prices during each discount level, [8] and the buyer might find different suppliers as well as different levels of discounts. When discounts are practical for general efficiency with the reduction of product costs per unit, products will be purchased at lower prices per unit. This is done with purchasing excessive quantities of product [9]. Dynamic lot-sizing. For one type of product with respect to the problem of supplier selection, 2 cases of the study was conducted, which was one that did not consider discounts and one that considered both incremental and all-unit discounts[10], and a consideration of models with the use of mixed integer non-linear programming [11] as well as the use of mixed integer linear programming that considered backordering, [12] the problems of lot-sizing with multiple suppliers under quantity discounts, and insufficient products with the Silver-Meal heuristic method [13].

The management of inventory lot-sizing for the problem of supplier selection in the purchasing process consists of 3 activities altogether, which were the decisions for lot-sizing, supplier selection decisions, and holding decisions [14] with the aims of reducing the inventory and costs with product insufficiency by selecting the most appropriate time and lot-sizing. Even so, the lead-time is another important matter to be considered, since with the condition of the industry in the present, most suppliers will have a lead-time for the delivery of raw materials for various processes such as the preparation, finding, pre-delivery quality assurance, time for the delivery of raw materials, etc. Regarding the problem of selection suppliers with various products among many suppliers, the model will consider quantity discounts and lead-time as mean values [15].

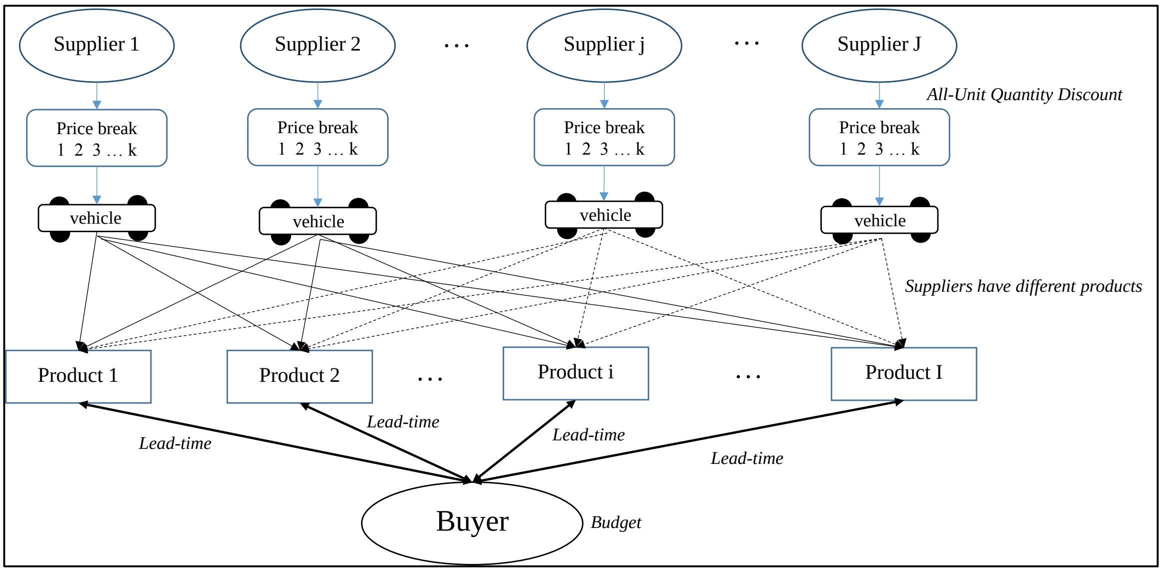

Figure 1: The general diagram of the problem under consideration

Supplier selection is another important matter, since the selection of purchasing products in that period meant that there are different costs along with product lot-sizing and potentially discounts of the model, especially for agencies with demands for different types of products, which results in the generation of higher costs in storage and ordering. These aforementioned problems to date have no studies regarding the selection of suppliers in the case that each supplier is unable to find the same product, with consideration of each of their lead-times.

This study will present mathematical developments with mix integer linear programming and supplier selection for inventory lot-sizing problem with quantity discounts [16] with multi-products through multi-periods with the consideration of supplier lead-times and under the conditions of purchasing budgets. Furthermore, the mathematical model will also consider the selection of suppliers with different products among other differences, where the aims of the study leaned towards minimizing the total costs of inventory management (ordering cost, holding cost, purchasing product cost and transportation cost). The result also alerts us to the determination of ordering lot-sizing and the period for purchasing with the consideration of lead-time as well as being responsive to the demands as a whole.

MODEL DEVELOPMENT

This section will explain supplier selection for inventory lot-sizing with quantity discounts by considering the conditions of supplier lead-times and budgets, where the problems are illustrated as per figure 1 with the following assumptions.

- The demands of each product are independent and made known during periods
- Product shortage or backordering are not allowed
- Each supplier will present all-unit quantity discounts
- The ordering costs will depend on each supplier and are unrelated to product quantities
- The holding costs of each period depend on the product
- The transportation costs depend on the number of vehicles of each supplier
- Lead-time of each supplier is known over a planning horizon
- Inventories are allowed to start responding to demands in the first period (Initial inventory = 0 when the lead-time = 0)
- Each suppliers have different product and cannot make identical product delivers

MODEL PARAMETERS AND DECISION VARIABLES

The notations used in the formulation of the model are as follows:

Indices

i set of index of products (1,…, I)

j set of index of suppliers (1,…, J)

k set of index of price break (1,…, K)

t set of index of time periods (1,…, T)

Parameters

D_{it} the demand of product i in period t

PC_{ijk} the product cost for one unit under the all-unit discount schedule based on the quantity level of product i from supplier j with price break k

Q_{ijk} the upper bound quantity of product i from supplier j with price break k

I_{it} the expected ending inventory level of product i in period t

Y_{it} the expected beginning available inventory level of product i in period t

H_i the holding cost of product per period i

O_j the ordering cost of product i from supplier j

S_j the transportation cost per vehicle from supplier j

V_j the maximum transportation (vehicle) from supplier j

II_i the initial inventory

M a large number

L lead-time

B_t the purchasing budget in period t

Decision variables

X_{ijkt} the number of product i ordered from supplier j with price break k in period t

F_{jt-L} a binary variable, set equal to 1 if purchase quantity of product i from supplier j in period t-L, 0 if not

U_{ijkt} a binary variable, set equal to 1 if purchase quantity of product i from supplier j with price break k in period t, 0 if not

MATHEMATICAL FORMULATION

The objective function to minimize total inventory costs, which consist of product costs, ordering costs, inventory holding costs, and transportation costs. It considers products of (i) types from suppliers of (j) sources, and considers the entire period of planning horizon in (t) period in equation (1).

$$Minimize\,TC = \sum_i^I \sum_j^J \sum_t^T O_{ij} F_{jt-L} + \sum_i^I \sum_j^J \sum_k^K \sum_t^T PC_{ijk} X_{ijkt} + \sum_j^J \sum_t^T S_j \left\lceil \frac{\sum_i^I \sum_k^K X_{ijkt}}{V_j} \right\rceil + \sum_i^I \sum_t^T \left[\frac{H_i}{2}\right](2Y_{it} - D_{it}) \quad (1)$$

Subject to

$$II_i \le \sum_{t=1}^{L} D_{it} \qquad \forall_i \quad (2)$$

$$I_{it} = Y_{it} - D_{it} \qquad \forall_i, \forall_t \quad (3)$$

$$Y_{it} = I_{it-1} + \sum_j^J \sum_k^K X_{ijkt} \qquad \forall_i, \forall_t \quad (4)$$

$$\sum_k^K X_{ijkt} \le M \times F_{jt-L} \qquad \forall_i, \forall_j, \forall_t \quad (5)$$

$$Q_{ijk-1} U_{ijkt} < X_{ijkt} \le Q_{ijk} U_{ijkt} \qquad \forall_i, \forall_j, \forall_k, \forall_t \quad (6)$$

$$\sum_i^I \sum_j^J \sum_k^K PC_{ijk} X_{ijkt} \le B_t \qquad \forall_t \quad (7)$$

$$F_{jt-L} = 0 \text{ or } 1 \qquad \forall_i, \forall_j, \forall_t \quad (8)$$

$$U_{ijkt} = 0 \text{ or } 1 \qquad \forall_i, \forall_j, \forall_k, \forall_t \quad (9)$$

Constraint in equation (2), which is an equation that illustrate the number of initial inventory's quantity when the time period is 0, since the product must match the demands in a time when orders are unable to be made.

Constraint in equation (3) is an equation that illustrate the inventory at the end of installment.

Constraint in equation (4) is a conditioned equation that illustrate the inventory at the beginning of the installment.

Constraint in equation (5) is a conditioned equation that illustrate product orders, in which each order must not exceed the maximum number.

Constraint in equation (6) is a conditioned equation that illustrate product orders, in which each order must be done in the correct amount with discounted product levels.

Constraint in equation (7) is a conditioned equation that illustrate the budget for product orders in each period, where the total product costs must not exceed the total budget in each period. Finally, equations (8) - (9) are equations with the binary variables 0 or 1 for use in decision-making for the model.

NUMERICAL EXAMPLE

This section will introduce mathematical problem solving of the model by using LINGO12, which will consider a situation with 3 products and 5 suppliers in a planning horizon with 5 periods, where data was displayed as follows. Product demands and costs are illustrated in table 1. Ordering costs, transportation costs, and the maximum capacity of the transportation vehicles are illustrated in table 2. Table 3 displays the costs of holding the 3 types of products.

Table 1: Demands of three products over a planning horizon of five periods and budget

	Planning Horizon (Five Periods)				
Periods(t)	1	2	3	4	5
Product 1	230	1750	650	1410	2950
Product 2	465	1510	2410	515	1850
Product 3	500	700	300	800	1000
Budget(B_t)	5000	12000	9000	14500	10000

Table 2: The ordering cost (O_j), transportation cost (S_j) and maximum transportation (V_j) of each supplier

	Supplier				
	1	2	3	4	5
O_j	250	220	235	210	195
S_j	21	22	30	23	22
V_j	25	25	25	25	23

Table 3: The holding cost (H_i) of three products

	products		
	1	2	3
H_i	0.11	0.11	0.15

The quantity discounts of each supplier will be displayed as all-unit discounts. This model will present the selection of suppliers in the case that each supplier has different products. This study will use the 4 ($k=4$) as the discount level for each supplier, where table 4 displays the products of each supplier, the discount levels, and product prices.

Table 4: The quantity discount is all-unit of three product and product cost

Product	Supplier	Price break	products Quantity level	Unit product cost
Product 1	Supplier 1	1	0-2000	2.99
		2	2001-3899	2.85
		3	3900 or more	2.74
	Supplier 3	1	0-1500	2.90
		2	1501-2500	2.85
		3	2501-4000	2.75
		4	4001 or more	2.60
	Supplier 4	1	0-1800	2.88
		2	1801-2600	2.78
		3	2601-4000	2.65
		4	4000 or more	2.50
Product 2	Supplier 1	1	0-900	3.05
		2	901-1800	2.96
		3	1801 or more	2.83
	Supplier 2	1	0-999	2.98
		2	1000-2599	2.82
		3	2600-4099	2.79
		4	4100 or more	2.76
	Supplier 5	1	0-1100	3.00
		2	1101-2000	2.90
		3	2001-3200	2.82
		4	3201 or more	2.75
Product 3	Supplier 2	1	0-1100	3.00
		2	1101-2200	2.93
		3	2201-3400	2.82
		4	3401 or more	2.75
	Supplier 3	1	0-1300	3.25
		2	1301-2500	3.15
		3	2501 or more	3.00
	Supplier 4	1	0-1500	3.10
		2	1501-2500	2.95
		3	2501-3500	2.90
		4	3501 or more	2.83

COMPUTATIONAL RESULTS

This section will introduce mathematical problem solving of the model with mixed integer linear programming by using LINGO12 on a computer with the specifications of Intel Core i5 2.50 GHz and memory/RAM 4 GB. The results of the problem solving concerns the variables of the quantity (i) of 3 types of products, 5 suppliers (j), 4 quantity discount levels (k), through 5 planning periods (t).

Regarding the results of the ordering of 3 products, this modelling test will label the lead-time of suppliers as 1 ($L=1$), where the beginning of the model's process must first decide the starting quantity of the product inventory to respond to demands in the first time period (II_1= 230, II_2=465 and II_3=500). Then, the system will begin to consider making orders according to demands in the next time period. Figure 2 displays the level of product 2 inventory, which displays the behavior of the inventory levels at the start and the filling of product in the beginning of the time periods in which there are demands.

Table 5 displays the results of calculations with LINGO12 for the problem with dimensions 3×5×4×5. The results show that the ordering process begins at period 1 since this model considers supplier lead-times. Thus, the ordering of the product must always be done in the F_{t-L} period in order for product to be filled in the inventory in the beginning of the time periods in which there are demands.

This will illustrate the results of product 1 ($i=1$, where the results will determine that the starting inventory (II_1) has 230 units, since the lead-time is equal to 1 period. thus, 2029 units of product were bought from the supplier 4 ($j=4$) at the 2 discount level ($k=2$) in the period 2 ($t=2$), along with 371 units from supplier 1 ($j=1$) at the 1 discount level ($k=1$) in the period 3 ($t=3$), 4297 units from supplier 4 ($j=4$) at the 4 discount level ($k=4$) in the period 4 ($t=4$), and finally 63 units from supplier 1 ($j=1$) at the 1 discount level ($k=1$) in the period 5 ($t=5$)

Table 5: The results of the buyer order quantities of product 1, 2 and 3

	Product		
	1	2	3
Procurement Lot-size	X_{1422}=2029	X_{2222}=1510	X_{3212}=700
	X_{1113}=371	X_{2133}=2470	X_{3213}=300
	X_{1444}=4297	X_{2214}=455	X_{3214}=800
	X_{1115}=63	X_{2135}=1850	X_{3215}=1000
Ordering cost	1800		
Holding cost	1358.06		
Product cost	43920.48		
Transportation cost	14006.48		
Total cost	61085.02		

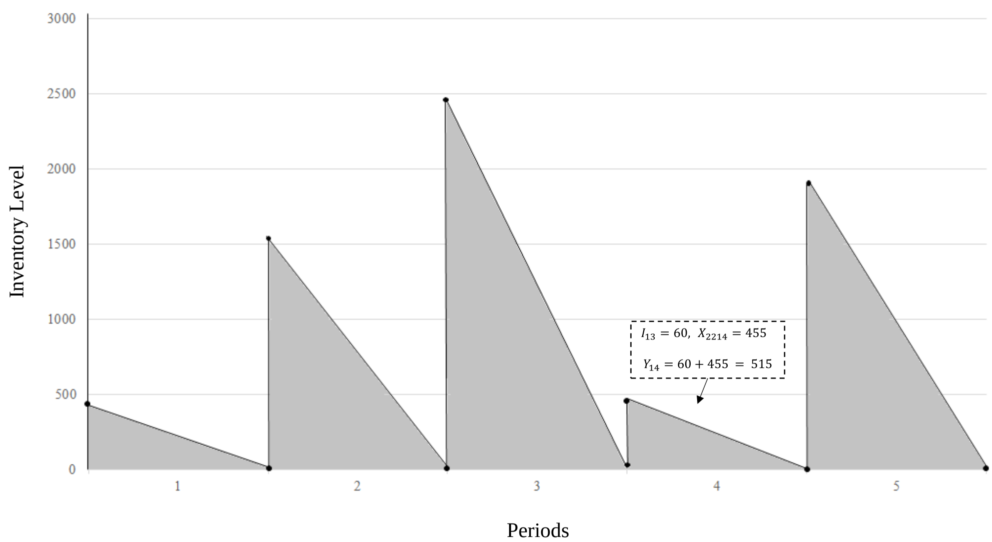

Figure 2: The example inventory level of product 2

Regarding product 2, the results will determine that the starting inventory (II_2) has 465 units, since the lead-time is equal to 1 period. thus, 1510 units of product were bought from the supplier 2 ($j=2$) at the 2 discount level ($k=2$) in the period 2 ($t=2$), along with 2470 units from supplier 1 ($j=1$) at the 3 discount level ($k=3$) in the period 3 ($t=3$), 455 units from supplier 2 ($j=2$) at the 1 discount level ($k=1$) in the period 4 ($t=4$), and finally 1850 units from supplier 1 ($j=1$) at the 3 discount level ($k=3$) in the period 5 ($t=5$).

For product 3, the results will determine that the starting inventory (II_3) has 500 units, since the lead-time is equal to 1 period. Thus, 700 units of product were bought from the supplier 2 ($j=2$) at the 1 discount level ($k=1$) in the period 2 ($t=2$), along with 300 units from supplier 2 ($j=2$) at the 1 discount level ($k=1$) in the period 4 ($t=4$). There are also purchases of 800 and 1000 units in the 4 and 5 time periods respectively (t=4, 5), both form the 2 supplier ($j=2$) at the 1 discount level ($k=1$)

ANALYSIS PROBLEMS SIZE

The model of inventory lot-sizing for this matter of supplier selection was allocated 120 minutes of time for problem solving calculations [5] with indices i, j, k, and t, where i = product, j = supplier, k = discount level, and t = period.

Regarding each discount level (k), this study will use a discount level of 4 ($k=4$) for each problem in analyzing the sensitivity of problem solving time testing. This analysis will begin with the smallest problems (3,3,4,5) and then the largest problem (5,5,4,50), where the results obtained from the usage of LINGO12 in problem solving (time limited at 120 minutes) discovered that the larger problem cannot be answered in the given time. Thus, equation (10), which is an equation used to study the error percentage from the large problem, was used to calculate the answer of the total cost, in which the larger problem will obtain 2 answers, which are upper and lower limits.

Percentage error of LINGO12

$$= \left[\frac{\text{Upper bound - Lower bound}}{\text{Upper bound}}\right] \times 100 \qquad (10)$$

Table 6 illustrate problem sizes, solution times, total costs, and error percentages. It can be seen that the larger problem will result in a higher solution times and the answer cannot be displayed (Limit = 120 minutes). The result was the total cost in the form of upper bound and lower bound.

Table 6: Computational results by LINGO12

Problem size	Solution Time (minute)	Total cost	% Error	Problem size	Solution Time (minute)	Total cost	% Error
3,3,4,5	0.06	62757.22	0	3,3,4,30	120	373300.9[a],373279.0[b]	0.0059
4,3,4,5	0.04	65618.49	0	4,3,4,30	120	380471.4[a], 380248.8[b]	0.059
5,3,4,5	0.06	90201.37	0	5,3,4,30	120	544890.3[a], 543154.1[b]	0.319
3,4,4,5	0.03	61085.02	0	3,4,4,30	19.24	355158	0
4,4,4,5	0.06	64843.12	0	4,4,4,30	120	380987.9	0
5,4,4,5	0.15	88942.81	0	5.4.4.30	120	666323.9[a], 660383.4[b]	0.892
3,5,4,5	0.03	61085.02	0	3,5,4,30	38.35	355147.6	0
4,5,4,5	0.08	64843.12	0	4,5,4,30	120	499786.2[a], 496968[b]	0.564
5,5,4,5	0.14	88938.65	0	5,5,4,30	120	546291.6[a], 542904.6[b]	0.619
3,3,4,10	0.27	144344.3	0	3,3,4,40	71.22	481940.2	0
4,3,4,10	0.25	150672.1	0	4,3,4,40	120	645644.9[a], 640988.5[b]	0.721
5,3,4,10	120	202467.9[a], 201756.5[b]	0.351	5,3,4,40	120	866601.2[a], 858343.2[b]	0.953
3,4,4,10	0.26	140709	0	3,4,4,40	120	481042.2	0
4,4,4,10	0.40	148461.7	0	4,4,4,40	120	636579.9[a], 630560.6[b]	0.946
5,4,4,10	95.25	199999.7	0	5,4,4,40	120	735668.9[a], 730339.5[b]	0.724

3,5,4,10	0.32	140709	0	3,5,4,40	120	605626.5[a], 600534.6[b]	0.841
4,5,4,10	1.00	148461.7	0	4,5,4,40	120	515137.8[a], 512708.3[b]	0.472
5,5,4,10	120	199799.3[a], 199796.1[b]	0.002	5,5,4,40	120	735218.9[a], 729977.7[b]	0.713
3,3,4,20	74.54	315350.3	0	3,3,4,50	120	611690.9, 610338[b]	0.221
4,3,4,20	120	327970.7[a], 327567.3[b]	0.123	4,3,4,50	120	787694.1[a], 780101.9[b]	0.964
5,3,4,20	120	330210.7[a], 330209.6[b]	0.001	5,3,4,50	120	932712.9[a], 923947.4[b]	0.940
3,4,4,20	120	307846.7[a], 306441.8[b]	0.456	3,4,4,50	120	732663[a], 726404.5[b]	0.854
4,4,4,20	120	322856[a], 321184.3[b]	0.518	4,4,4,50	120	653949.8[a], 648007[b]	0.909
5,4,4,20	12.04	330360	0	5,4,4,50	120	927283.1[a], 920517.1[b]	0.730
3,5,4,20	120	307409.7[a], 306783.8[b]	0.204	3,5,4,50	120	608633[a], 607256.4[b]	0.226
4,5,4,20	14.51	221420.9	0	4,5,4,50	120	653092.3[a], 648161.1[b]	0.755
5,5,4,20	120	330232.7[a], 330087[b]	0.044	5,5,4,50	120	1052280[a], 1037861[b]	1.370

[a]Upper bound, [b]Lower bound by LINGO12

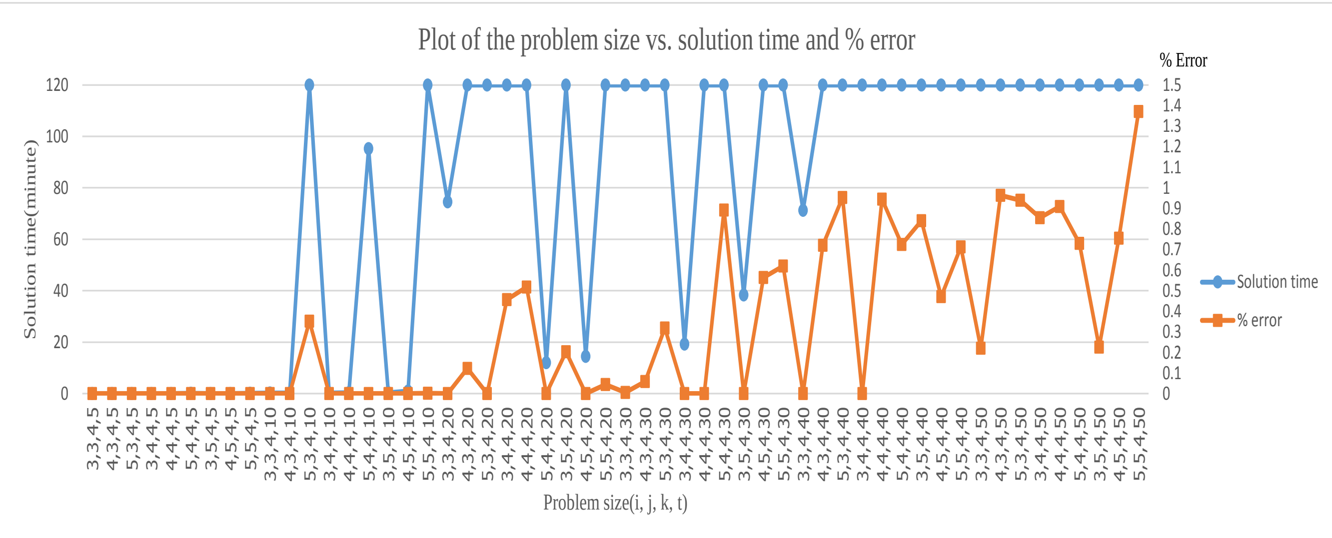

Figure 3: Plot of the problem size vs. solution time and % error

Thus, the researcher used equation (10) to find the error percentage between the lower and upper limits to find the degree of error. It was discovered that the highest error percentage was 5,5,4,50 which was equal to 1.370%, which was the largest problem of this study (5 product, 5 suppliers, 4 discount levels, 50 periods).

Figure 3 displays the relationship between problem sizes, solution times, and percentage error obtained from results evaluation with LINGO12. It was discovered that the smallest problem could be solved in a short time (less than 120 minutes), which is the problem with 5 product types, 5 suppliers, and 20 periods. ($i=5, j=5, k=4, t=20$) It can be seen that the solving of the smallest problem (3x4x4x5) took only 0.03 minutes.

The larger problems with 20 periods onwards will mostly have higher solution times and cannot find the appropriate answer in the allotted time. Thus, their error percentages were studied. Figure 3 shows that after 20 periods, the error percentage becomes higher than that of smaller problems, where the problem with parameters 5x5x4x50 had the highest error percentage.

CONCLUSIONS

This paper presented the determination ordering sizes for inventory lot-sizing with problems of supplier selection in cases with product discounts by considering the lead-times of each supplier and each suppliers have different product and cannot make identical product deliver. There were also conditions of budgets for product ordering, cases of multi-products, and multi-period. An objective function was used in the study to obtain the lowest total cost, which consisted of the ordering cost, holding cost, product cost, and transportation costs. Results illustrated that this model was sufficient to calculate and determine ordering lot-sizing accurately to respond to demands satisfactorily. Finally, the problem sizes were expanded in order to test abilities in finding answers, which discovered that the determination of ordering lot-sizing were best within the first 20 time periods. In the case of larger problems when tested with percentage errors, their answers were still found to be satisfactory and acceptable.

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