

# Spatial Analysis of Robotic Arm with Constrained Joints

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## Abstract

Robotic arms are still a dream to be implemented for domestic purposes because of their complexity in operation and construction. Even though a lot of years in research, robots are not yet ready to meet the versatility of various tasks and functions. To develop low-cost robotic arms, the links which can be joined by servo motors may have a constrained angular rotation limits which are not considered in the computation of conventional inverse kinematics, involving huge efforts in programming and developing spatial control algorithms. In this paper, an approach is made to analyze the workspace of a 3 link chain which has a constrained motion of 180 degrees at joints, further the relations between the joint parameters and the end-effector position is also observed and compared.

**Keywords:** Robotic Arms, Kinematics, Constrained joint, Unconstrained joint, End-effector, Redundancy.

## 1. INTRODUCTION

Robots are always known for their high repeatability and efficiency. Yet they are not ready enough to meet the real world needs which can entirely replace human beings in basic functions and repetitive tasks. Modern world machines are developed to work side by side interacting with human beings. The cost of robots increases with precision and the accuracy they can handle. The use of robots in the industry is wide since they accomplish tasks that are dangerous or monotonous for humans, this is the case of an industrial robot used for cleaning in electrical substations, which works in the high voltage area.

Robot kinematics method is the study of the motion (kinematics) of robots. In a kinematic analysis, the positions of all the links are calculated without considering the forces that cause this motion. In the kinematic analysis of the manipulator position, there are two separate problems to solve: direct kinematics and inverse kinematics [1]. It is also an important consideration in the design of robot manipulators since it involves calculating relative positions between the coordinate system attached to the moving parts causing a possible increase in the uncertainty and the accumulated error in the transformations as a consequence of this it would affect the positioning accuracy and tracking of trajectories of the manipulator [2]. The Geometric Design Problem has been studied extensively for planar mechanisms and robotic systems and has recently drawn much attention to researchers for spatial multi-articulated systems [3]. For spatial mechanisms and manipulators, a few of them had been solved using algebraic methods [4, 5]. Even though algebraic methods were shown to be very efficient in solving several geometric design problems for spatial mechanical systems,

the complexity of the design equations had limited their usage and there exist many types of robotic and mechanical systems that are used frequently, such as the 3R, 4R, and 5R manipulators, for which the algebraic solutions of the geometric design problem has not yet been discovered.

Working space of robotic arm is the reference point at the end of the robot arm to achieve the set point of space and is discussed from the geometric aspects of the work to be performed. Analytical work is also used to determine the spatial configuration of the robot arm. Working space represents the range of activities, which is a parameter of the ability to work the robot arm kinematics, and is also an important indicator [6]. Bin Liao et al. carried out an optimal design of a three-degree-of-freedom planar revolute-chain parallel manipulator with a view to improving its operational velocity and accuracy for pick-and-place tasks.[7] The advantages of the articulated robot arm are higher reach from the base, useful in continuous path generation, applied to weld operation and reaching the congested small openings without interference[1].

The configuration of the robot arm in this journal is similar to the articulated robot arm which has four revolute joints with an irregular workspace due to joint constraints. In this paper, the workspace of a 4-DOF manipulator is studied and the effect of joint parameters on various end-effector positions is observed, with and without the effect of constraints on joint motion.

## Nomenclature:

$\theta_1, \theta_2, \theta_3, \theta_4$	Angles of links relative to horizontal (Degree)
$L_1, L_2, L_3$	Link lengths (cm)
H	Height of the base of the robot (cm)
$(x_1, y_1, z_1)$	Coordinates of point of origin of the chain
$(x_2, y_2, z_2)$	Coordinates of joint 2
$(x_3, y_3, z_3)$	Coordinates of joint 3
$(x_4, y_4, z_4)$	Coordinates of end-effector point

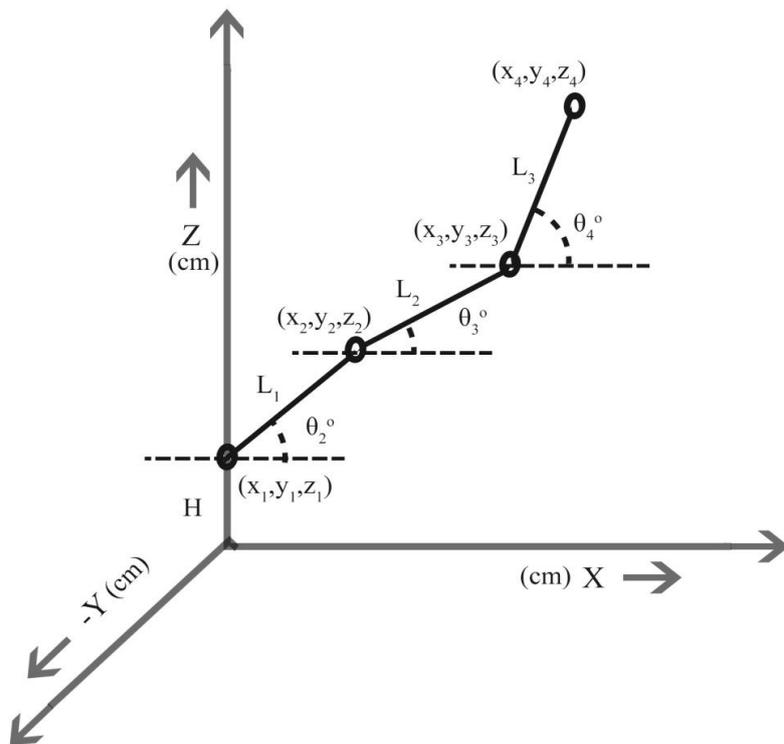
## 2. METHODOLOGY

The methodology involves the technology utilized for performing the designing and analysis of the object.

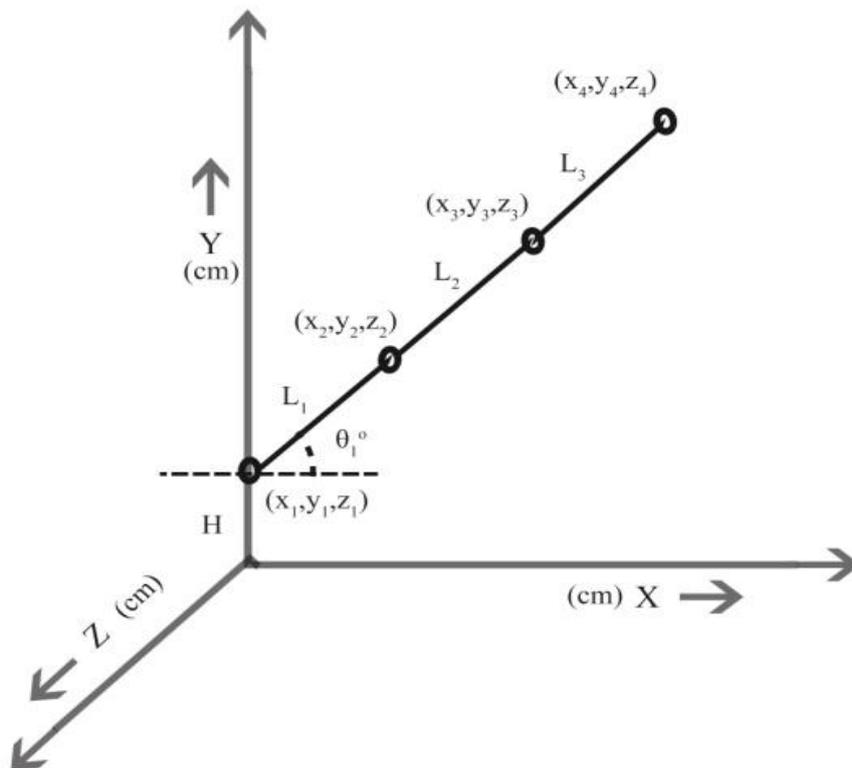
- Design of a virtual model of robotic arm
- Computation
- Analysis of workspace
- Analysis of joint parameters

### 2.1 Design of virtual model:

Consider the figures Fig.1, Fig. 2 which shows the robot configuration form side view and top view respectively. From the top view, the chain always lies in a plane which can be rotated by angle  $\theta_1$  through X-Z plane.



**Figure 1.** Side view of the robotic arm



**Figure 2.** Top view of the robotic arm

Also considering that the robotic arm is placed on a platform of height 'H'. Without considering  $\theta_1$ , the other three degrees of freedom  $\theta_2, \theta_3, \theta_4$  can be observed in Fig 1 at joints 1, 2, 3 which are lying in the same plane.

The angles are measured with respect to horizontal x-axis and angles in anti-clockwise direction are taken in positive and clockwise directions are taken as negative.

**Table 1. Ranges of angles**

Angle	Constrained range	Unconstrained range
$\theta_1$	[-90 90]	[-180 180]
$\theta_2$	[0 180]	[-180 180]
$\theta_3$	$[\theta_2-90 \theta_2+90]$	[-180 180]
$\theta_4$	$[\theta_3-90 \theta_3+90]$	[-180 180]

## 2.2 Computation

The program has been developed in Matlab software. For computation lengths of all links are taken as 9 cm with a base height 'H' of 10 cm. End-effector position in the X-Z plane is calculated considering  $\theta_1=0^\circ$  constant and using the forward kinematic equations given below.

$$z_4=L_1 \times \text{Cos}(\theta_2)+L_2 \times \text{Cos}(\theta_3)+L_3 \times \text{Cos}(\theta_4) \quad (1)$$

$$x_4=L_1 \times \text{Sin}(\theta_2)+L_2 \times \text{Sin}(\theta_3)+L_3 \times \text{Sin}(\theta_4) \quad (2)$$

$y_4=0$ ; as  $\theta_1=0^\circ$ .

The program involves in a looping process tree where the three angles are incremented in the step of 5 degrees in the case of constrained joints and a step of 10 degrees in the case of unconstrained joints obtaining 50,653 end-effector points each in the workspace and plotting the point cloud as

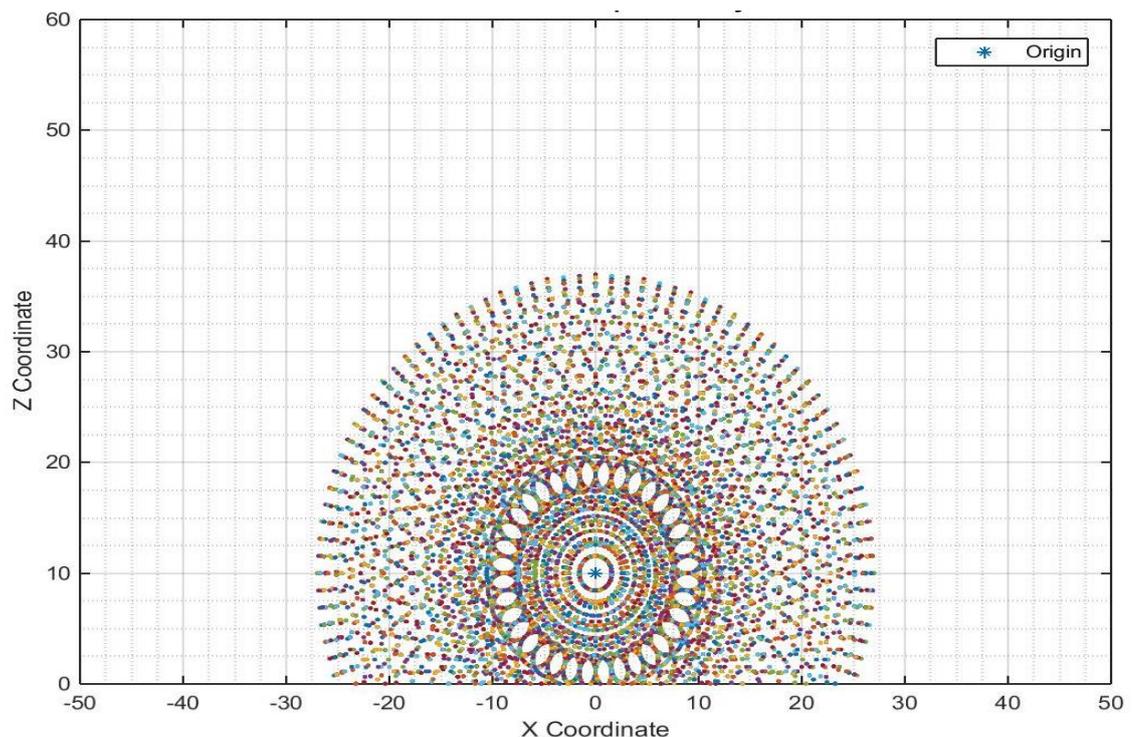
well as joint parameters in their respective plots. This step increment is taken keeping in mind the efficiency of computation and the limits of data handling in the software. The chain must completely lie above the reference surface which is Z-Y plane, therefore  $z_1 > 0$ ,  $z_2 > 0$  and  $z_3 > 0$ .

## 3. RESULTS

The results with constraints and without constraints are shown separately below; the workspace is shown in 2D plane i.e., the cross-section of volume at X-Z plane. All lengths are measured in (cm) and angles are measured in (Degrees).

### 3.1 Without Constraints

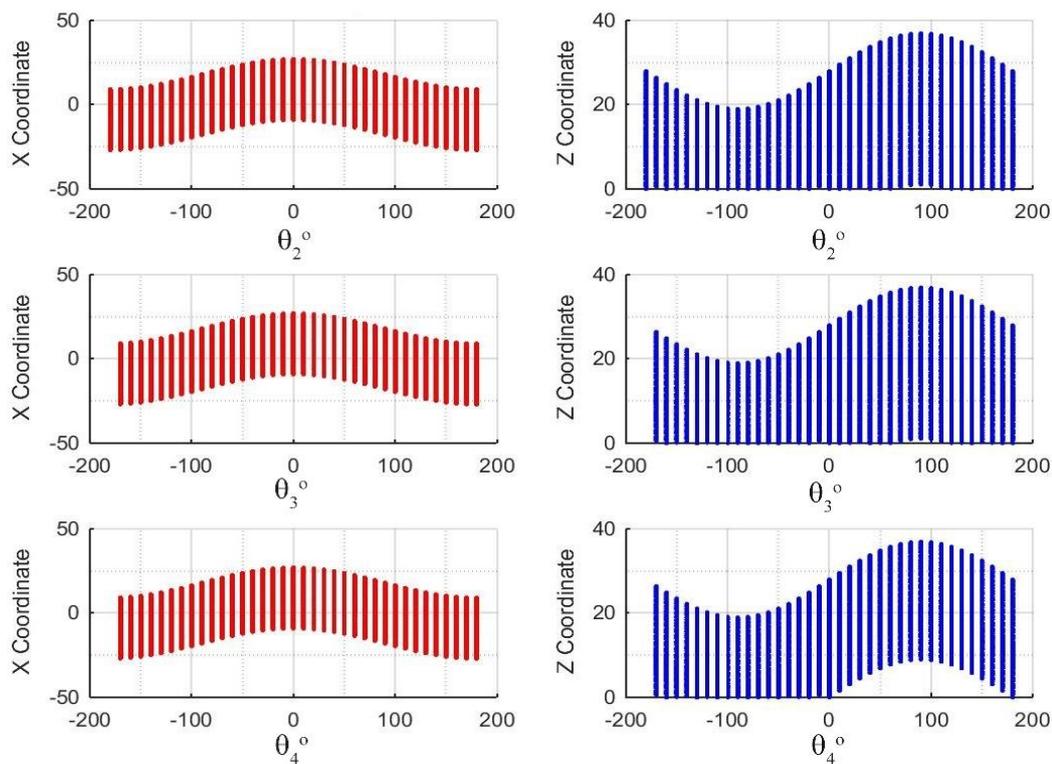
After plotting the point cloud in this case the workspace thus obtained is shown in Fig 3, which is a cross-section of the workspace in X-Z plane.



**Figure 3.** Workspace of unconstrained joints

We can also observe the point cloud spread throughout the space within the area enclosed by a circle of radius 27 cm from the origin which is the maximum reachable distance of

the arm. The relations between the joint angles and the end-effector coordinates can be observed in Fig 4.

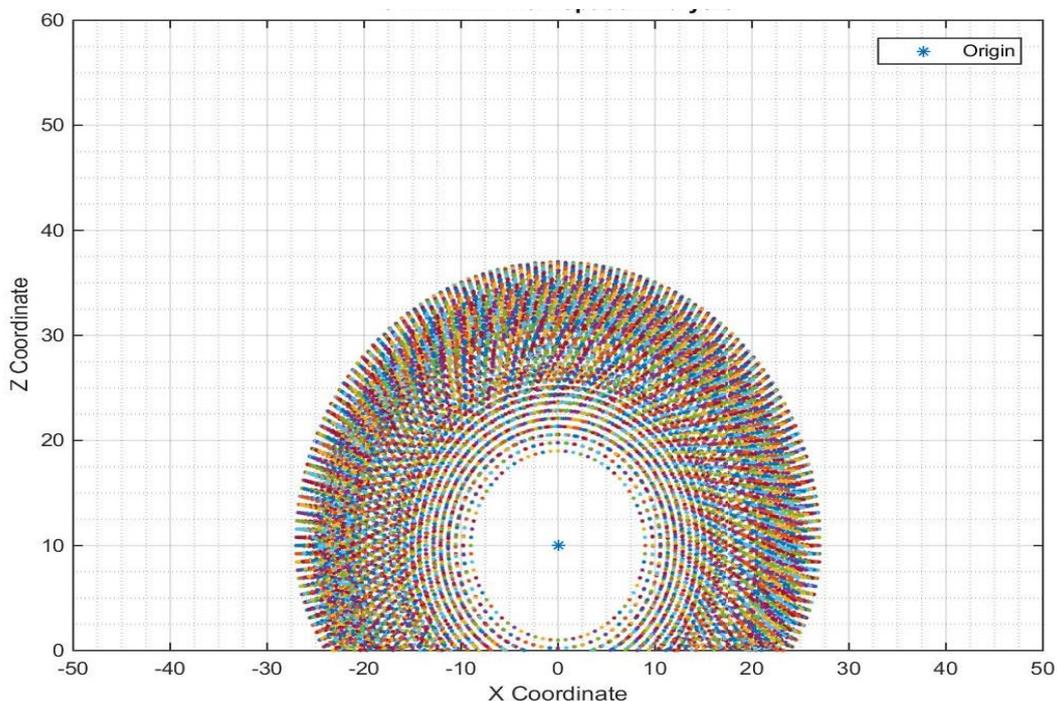


**Figure 4.** Joint parameters of unconstrained joints

The angles are spread across the ranges defined and also show the redundancy parameter, having multiple positions for a particular angle of any joint. The discontinuities in the plots in Fig 4 are due to step computation of  $10^\circ$  and leaving the intermediate angles. When are computed will fill the graphs and follow a similar pattern and structure.

### 3.2 With Constraints

After plotting the point cloud in this case the workspace thus obtained is shown in Fig 5, which is a cross-section of the workspace in X-Z plane.



**Figure 5.** Workspace of constrained joints

We can observe the point cloud spread throughout the space within the area enclosed by a circle of radius 27 cm from origin as same as in the previous case but the robotic arm does not have any end-effector points in the circle of radius 9 cm from origin, which means the work volume of the robot is decreased. The relations between the joint angles

and the end-effector coordinates can be observed in Fig 6, in the first two plots between end-effector coordinates and angle  $\theta_2$ , the x-axis ranges from [0 180] only as defined in Table 1. On comparing the effect of joint parameters on end-effector coordinates in Fig 4 and Fig 6 the loss of redundancy and reachability can be observed.

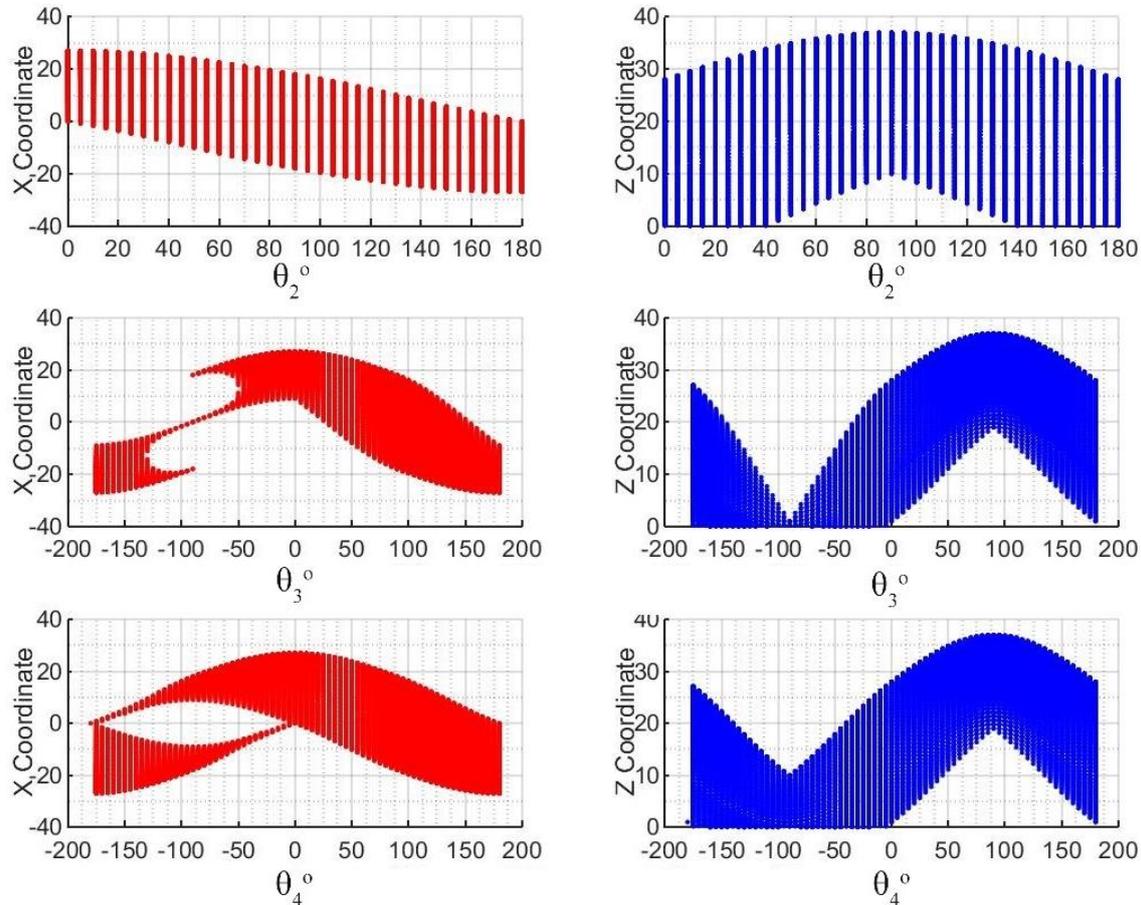


Figure 6. Joint parameters of constrained joints

#### 4. CONCLUSION

The figures shown above reveals the workspace and joint parameters of the robot with constrained joints. Also, the results are compared with a similar robot which does not have constrained joints. The workspace of the robotic arm with constrained joints is reduced allot but can still reach a number of points through the workspace this is due to the redundancy of the considered kinematic chain. Further, this work can be used to define integral functions of area enclosures in the plots for solving complicated kinematic problems rather than complex mathematical equations for developing high efficient spatial control programs. These kinds of studies give us a proper understanding of workspace interactions of various robot configurations with constraints and redundancy.

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