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A Note on Intuitionistic Fuzzy Closure on the Basis of Reference Function

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Abstract

In this article we discuss on intuitionistic fuzzy set on the basis of reference function(IFSRF). We apply the definition of IFSRF on intuitionistic fuzzy closure towards forming A ∪bd(A) = cl(A), for a intuitionistic fuzzy set A.

Keywords: Fuzzy membership function, Fuzzy reference function, Fuzzy membership value.

1. INTRODUCTION

Fuzzy set is defined by Zadeh [10]. After the discovery of fuzzy set, there have been numbers of generalizations of this fundamental concept. Atanassov [1-3] introduced the notion of intuitionistic fuzzy sets. Many authors have given different definitions on fuzzy set. But, in a contrast, it has been accepted that for fuzzy set A and its complement A^C, neither $A \cap A^C$ is empty set nor $A \cup A^C$ is the universal set. Whereas the operations of union and intersection of crisp sets are indeed special cases of the corresponding operation of two fuzzy sets, they end up giving peculiar results while defining $A \cap A^C$ and $A \cup A^C$. Using the notion of intuitionistic fuzzy sets, Coker [5] introduced the notion of intuitionistic fuzzy topological spaces. Many researchers were going on in intuitionistic fuzzy topological spaces and many concepts in fuzzy topology were extended to intuitionistic fuzzy topology. In this article we use intuitionistic fuzzy set on the basis of reference function towards forming on A ∪bd(A) = cl(A), for a intuitionistic fuzzy set A.

2. PRELIMINARY DISCUSSION ON FUZZY SET AND INTUITIONISTIC FUZZY SET

Zadeh [10] discovered the fuzzy set theory. According to the definition of fuzzy set theory, If X be a universal set and x be any particular element of X, then a fuzzy set A, defined on X may be written as a collection of ordered pairs

A = {(x, $\mu_A(x)$; x∈ X}, where $\mu_A(x)$: X →[0, 1], is called the membership function or grade of membership of x in A.

Let us consider two fuzzy sets A and B in the universal set X defined with membership functions

A = {(x, $\mu_A(x)$; x∈ X} and

B = {(x, $\mu_B(x)$; x∈ X}

Then following operations can be observed

(i) Equality: The fuzzy sets A and B are equal denoted by A=B if and only if for every x∈ X, $\mu_A(x)= \mu_B(x)$.

(ii) Subset: Fuzzy set A is subset of fuzzy set B denoted by A⊆ B if for every x∈ X, $\mu_A(x) \leq \mu_B(x)$.

(iii) Complement: The fuzzy set A^C is complement of fuzzy set A if $\mu_{A^C}(x)=1- \mu_A(x)$.

(iv) Union: The union of two fuzzy sets A and B denoted by A∪B is defined by

$$\mu_{A \cup B}(x) = \max \{ \mu_A(x), \mu_B(x) \}$$

(v) Intersection: The intersection of two fuzzy sets A and B denoted by A∩B is defined by $\mu_{A \cap B}(x) = \min \{ \mu_A(x), \mu_B(x) \}$

Now let us observe properties of standard fuzzy set

Let A, B and C be fuzzy sets, then some properties of are shown below

(i) Idempotent: A∩A=A and A∪A=A

(ii) Commutative: A∩B= B∩A and A∪B= B∪A

(iii) Associative: (A∩B)∩A= A∩(B∩C) and (A∪B)∪C = A∪(B∪C)

(iv) Distributive: A∩(B∪C)= (A∩B) ∪ (A∩C) and A∪ (B∩C)= (A∪B) ∩ (A∪C)

(v) Double complement: $(A^C)^C=A$

(vi) DeMorgan's laws: $(A\cap B)^C=A^C \cup B^C$

(vii) Exclusion: $A \cup A^C \neq X$

(viii) Contradiction: $A \cap A^C \neq \phi$

(ix) Identity: A∪ϕ=A and A∩X=A, where ϕ and X are empty and universal set respectively.

Let us now observe on Intuitionistic fuzzy sets:

Let a set X be fixed. An IFS A in X is an object of the following form :

A = $\langle x,\ \mu_A(x), \gamma_A(x)\ ;\ x \epsilon X \rangle$

When $\gamma_A(x)$= 1 - $\mu_A(x)$for all $x\epsilon X$ is ordinary fuzzy set.

Also for each IFS A in X, if $\pi_A(x)$=1- $\mu_A(x)$ - $\gamma_A(x)$, Then

$\pi_A(x)$ is called the degree of indeterminacy of x to A, or called the degree of hesitancy of x to A.

Some basic operation on Intuitionistic Fuzzy sets

i. A ≤ B iff $\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$

ii. A =B iff A ≤B and B ≥A

iii. $A^C = \langle x, \gamma_A(x), \mu_A(x); x \epsilon X \rangle$

iv. A ∪ B = < x , Max$\{\mu_A(x), \mu_B(x)\}$, Min$\{\gamma_A(x), \gamma_B(x)\}$ >

v. A ∩ B = < x , Min$\{\mu_A(x), \mu_B(x)\}$, Max$\{\gamma_A(x), \gamma_B(x)\}$ >

Now let us observe some properties on intuitionistic fuzzy set

Let A, B and C be three intuitionistic fuzzy sets, then some properties of are shown below

(i) Idempotent: A∩A=A and A∪A=A

(ii) Commutative: A∩B= B∩A and A∪B= B∪A

(iii) Associative: (A∩B)∩A= A∩(B∩C) and (A∪B)∪C = A∪(B∪C)

(iv) Distributive: A∩(B∪C)= (A∩B) ∪ (A∩C) and A∪ (B∩C)= (A∪B) ∩ (A∪C)

(v) Double complement: $(A^C)^C$=A

(vi) DeMorgan's laws: $(A \cap B)^C = A^C \cup B^C$

(vii) Identity: A∪ϕ=A and A∩X=A, where ϕ and X are empty and universal set respectively.

Baruah [4] has given the definition of fuzzy set in a new way. According to this definition, to define a fuzzy set, two functions namely fuzzy membership function and fuzzy reference function are necessary. Fuzzy membership value is the difference between fuzzy membership function and fuzzy reference function.

Let $\mu_1(x)$ and $\mu_2(x)$ be two functions such that $0 \le \mu_2(x) \le \mu_1(x) \le 1$. For fuzzy number denoted by {x, $\mu_1(x)$, $\mu_2(x)$;x϶ X}, we call $\mu_1(x)$ as fuzzy membership function and $\mu_2(x)$ a reference function such that $(\mu_1(x) - \mu_2(x))$ is the fuzzy membership value for any x in X.

Some basic operations on fuzzy sets on the basis of reference function

Let A={x, $\mu_1(x)$, $\mu_2(x)$; x϶ X} and B={x, $\mu_3(x)$, $\mu_4(x)$; x϶ X} be two fuzzy sets defined over the same universe X.

a. A ⊆B iff $\mu_1(x) \le \mu_3(x)$ and $\mu_4(x) \le \mu_2(x)$ for all x϶ X.

b. A⋃ B ={x, max($\mu_1(x)$, $\mu_3(x)$), min($\mu_2(x)$, $\mu_4(x)$)} for all x϶ X.

c. A⋂ B ={x, min($\mu_1(x)$, $\mu_3(x)$), max($\mu_2(x)$, $\mu_4(x)$)} for all x϶ X.

If for some x϶ U, min($\mu_1(x)$, $\mu_3(x)$) ≤ max($\mu_2(x)$, $\mu_4(x)$)}, then our conclusion will be $A \cap B = \phi$.

d. A^C={x, $\mu_1(x)$, $\mu_2(x)$; x϶ X}^{C} ={x, $\mu_2(x)$, 0; x϶ X}⋃ {x, 1, $\mu_1(x)$; x϶ X}

e. If D = {x, μ(x), 0; x϶ X} then D^C={x, 1, μ(x); x϶ X} for all x϶ X.

Baruah [4] solved out the problem on fuzzy set for excluded middle laws. According to his theory on fuzzy set the following properties can be seen.

Let A, B and C be three intuitionistic fuzzy sets, then some properties of are shown below

(i) Idempotent: A∩A=A and A∪A=A

(ii) Commutative: A∩B= B∩A and A∪B= B∪A

(iii) Associative: (A∩B)∩A= A∩(B∩C) and (A∪B)∪C = A∪(B∪C)

(iv) Distributive: A∩(B∪C)= (A∩B) ∪ (A∩C) and A∪ (B∩C)= (A∪B) ∩ (A∪C)

(v) Double complement: $(A^C)^C$=A

(vi) DeMorgan's laws: $(A \cap B)^C = A^C \cup B^C$

(vii) Exclusion: $A \cup A^C = X$

(viii) Contradiction: $A \cap A^C = \phi$

(ix) Identity: A∪ϕ=A and A∩X=A, where ϕ and X are empty and universal set respectively.

3. FUZZY TOPOLOGY

A fuzzy topology on a nonempty set X is a family τ of fuzzy set in X satisfying the following axioms

(T1) $0_X, 1_X \in \tau$

(T2) $G_1 \bigcap G_2 \in \tau$, for any $G_1, G_2 \in \tau$

(T3) $\bigcup G_i \in \tau$, for any arbitrary family $\{G_i : G_i \in \tau, i \in I\}$.

In this case the pair (X,τ) is called a fuzzy topological space and any fuzzy set in τ is known as fuzzy open set in X and clearly every element of τ^C is said to be fuzzy closed set.

3.1 Interior of a fuzzy set

Let (X, τ) be fuzzy topology also A={x, $\mu_A(x)$, $\gamma_A(x)$} be fuzzy set on X.

Then interior of a fuzzy set A is defined as union of all open subsets contained in A, denoted it as int(A) and is defined as follows

Int(A)=⋃ {P : P is open set in X and P⊆A }

3.2 Closure of fuzzy set

Let (X, τ) be fuzzy topology and A={x, μ(x), γ(x); x϶ X} be fuzzy set in X. Then fuzzy closure of A are defined by

Cl(A)=⋂ {G: G is fuzzy closed set in X and A⊆G}

3.3 Fuzzy Boundary: Let A be a fuzzy set in fuzzy topological space X. Then the fuzzy boundary of A is defined as Bd(A)=cl(A)∩cl(A^C).

4 MAIN WORK

It is seen that in fuzzy topology for a fuzzy set A, A⋃bd(A)⊆cl(A) but equality does not hold. Whereas in classical topology for an arbitrary set A of a topological space X, A ⋃bd(A)= cl(A). Many papers has been published in fuzzy boundary with remarks that in fuzzy topology A⋃bd(A)⊆cl(A), but converse does not hold. So in this article we would like to focus in this matter with example.

First we observe with standard definition of fuzzy set

Let suppose X={a, b} and let A={(a, 0.3), (b, 0.6)} be a fuzzy set on X, then by existing definition of fuzzy set the collection τ={0_X, 1_X, A} is fuzzy topology on X.

Let B={(a, 0.6), (b, 0.7)} be fuzzy set on X.

Then cl(B)=1_X and cl(B^C)=A^C.

Now bd(B)= cl(B)∩cl(B^C)=A^C.

Therefore B⋃bd(B)= {(a, 0.6), (b, 0.7)} ⋃ A^C={(a, 0.7), (b, 0.7)}.

Hence cl(B)≠ B⋃bd(B).

Now, let us expressed same problem with intuitionistic fuzzy set.

X={ a, b} and A = $\{< a,\ 0.3,\ 0.7 >, < b,\ 0.6,\ 0.4 >,\}$ be intuitionistic fuzzy set on X. Then τ={0_X, 1_X, A} is intuitionistic fuzzy topology on X.

Let B = $\{< a,\ 0.6,\ 0.4 >, < b,\ 0.7,\ 0.3 >\}$

Then Cl(B) = 1_X and Cl(B^C) = A^C

Now bd(B)= cl(B)∩cl(B^C)=A^C

Therefore B⋃bd(B) ≠1_X

Hence cl(B)≠ B⋃bd(B).

It is seen that cl(B)≠ B⋃bd(B), in both the cases.

Now let us do the same problem by applying intuitionistic fuzzy set on the basis of reference function.

Before that let us go through some basic things of intuitionistic fuzzy set on the basis of reference function.

4.1 Definition

Let X be a fixed set. An intuitionistic fuzzy set A on the basis of reference function (IFSRF) is a form

A=$\langle x,\ (\mu_A(x),\ 0),\ (1,\ \mu_A(x))\ ; x□X\rangle$, Where $0 \le \mu_A(x) \le 1$. Here, $\mu_A(x) - 0$ is membership value which is taken from 0 to $\mu_A(x)$ and $\mu_A(x)$ is known as membership function and 0 is reference function of the membership value and similarly $1-\mu_A(x)$is non membership value which is taken from $\mu_A(x)$ to 1. Membership function and reference function are clearly explained in Baruah(2011)

Now some basic operation on IFSRF

Let A=$\langle x,\ (\mu_A(x),\ 0),\ (1,\ \mu_A(x))\ ; x\epsilon X\rangle$ and B=$\langle x,\ (\mu_B(x),\ 0),\ (1,\ \mu_B(x))\ ; x\epsilon X\rangle$

I. A⊆B if and only if $\mu_A(x) - 0 \le \mu_B x - 0$ and $1-\mu_A(x) \le 1-\mu_B(x)$

II. A ∩ B = $\langle x, \min\{ (\mu_A(x),\ 0),\ (\mu_B(x),\ 0)\}, \max\{ (1,\ \mu_A), (1,\ \mu_B(x))\}\ ; x\epsilon X\rangle$

Here $\min\{ (\mu_A(x),\ 0),\ (\mu_B(x),\ 0)\}$ ={ $\min(\mu_A(x), \mu_B(x))$, max (0, 0)} and $\max\{ (1,\ \mu_A), (1,\ \mu_B(x))\}$ = { $\max(1, 1)$, $\min(\mu_A(x), \mu_B(x))$}

III. A ⋃ B = $\langle x, max(\mu_A(x),\ 0),\ (\mu_B(x),\ 0)\}, min\ (1,\ \mu_A), (1,\ \mu_B(x))\}\ ; x\epsilon X\rangle$

Here $\max\{ (\mu_A(x),\ 0),\ (\mu_B(x),\ 0)\}$ ={ $\max(\mu_A(x), \mu_B(x))$, min (0, 0)} and $\min\{ (1,\ \mu_A), (1,\ \mu_B(x))\}$ = {min(1, 1), $\max(\mu_A(x), \mu_B(x))$}

IV. A^C=$\langle x,\ (1,\ \mu_A(x))\ (\mu_A(x),\ 0,\ ; x\epsilon X\rangle$.

Note: In IFSRF A=$\langle x,\ (\mu_1(x),\ \mu_2(x)),\ (\gamma_1(x), \gamma_2(x))\ ; x\epsilon X\rangle$ if $\mu_1(x) - \mu_2(x)$ <0 then we assume that $\mu_1(x) - \mu_2(x) = 0$ and similarly if $\gamma_1(x) - \gamma_2(x) < 0$ then $\gamma_1(x) - \gamma_2(x) = 0$.

Now, let us expressed earlier problem in IFSRF form.

X={ a, b} and A = $\{< a,\ (0.3, 0), (1, 0.3) >, < b,\ (0.6, 0), (1, 0.6) >\}$ be intuitionistic fuzzy set on X. Then τ={0_X, 1_X, A} is intuitionistic fuzzy topology on X on the basis of reference function.

Let B = $\{< a,\ (0.6, 0),\ (1, 0.6) >, < b,\ (0.7, 0),\ (1, 0.7) >\}$

Then Cl(B) = 1_X and Cl(B^C) = A^C

Now bd(B)= cl(B)∩cl(B^C)=A^C

Therefore

B⋃bd(B)

= $\{< a,\ (0.6, 0),\ (1, 0.6) >, < b,\ (0.7, 0),\ (1, 0.7) >\}$⋃ A^C

=$\{< a,\ (1, 0),\ (0, 0) >, < b,\ (1, 0),\ (0, 0) >\}$

=1_X

Hence Cl(B) = B⋃bd(B)

5. CONCLUSION

In standard fuzzy topology and intuitionistic fuzzy topology it is observed that A⋃ bd(A)⊆cl(A), for a fuzzy set A. Whereas in crisp topology it is seen that A⋃ bd(A) = cl(A). In this article we use new definition of intuitionistic fuzzy set on the basis of reference function and clearly we found that A⋃

bd(A) = cl(A), for an intuitionistic fuzzy set A. We hope the definition of IFSRF will give some contribution on development of intuitionistic fuzzy set and fuzzy topology.

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