International Journal of Applied Engineering Research ISSN 0973-4562 Volume 14, Number 16 (2019) pp. 3489-3493

Number of Symmetrical objects in n-dimensional space with different layers

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Abstract

Though there are numerous results on most efficient way to pack symmetrical objects (circles, spheres etc.) with calculation of packing factors, there is no formula to count number of such objects in symmetrical configuration in n-dimensional space. We present here an empirical formula for total number of symmetrical objects in n-dimensional space with different layers. The formula is tested for one, two and three dimensional space with experiment.

Keywords: Hex number, Rhombic dodecahedral numbers, Magic squares, Nexus numbers

1 INTRODUCTION

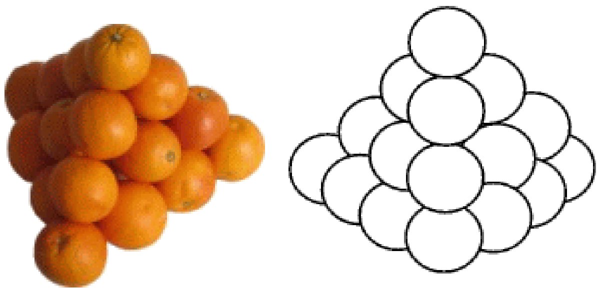
The problem of finding the most efficient way to pack spheres has a long history, dating back to the crystalline arrays conjectured by Kepler (Conway, John H.; Neil J.A. Sloane 1999) and the random geometries explored by Bernal (Brass, Peter; Moser, W. O. J.; Pach, János 2005). Apart from its mathematical interest, the problem has practical relevance in a wide range of fields from granular processing to fruit packing (O. R. Musin 2003). There are currently numerous experiments showing that the loosest way to pack spheres (random loose packing) gives a density of ~55 per cent (Pfender, Florian; Ziegler, Günter M. September 2004); Levenshtein, Vladimir I. 1979; Odlyzko, A. M., Sloane, N. J. A., 1979). On the other hand, the most compact way to pack spheres (random close packing) results in a maximum density of ~64 per cent (Brass, Peter; Moser, W. O. J.; Pach, János 2005; Pfender, Florian; Ziegler, Günter M. 2004). Although these values seem to be robust, they do not provide exact number of objects in a symmetrical configuration which stable and there is as yet no formula for counting number of symmetrical objects in such close packing.

Here we present an empirical formula which gives exact number of symmetrical objects in symmetrical configuration. We describe layer of arrangement in following manner.

In one dimension layer one ($\boldsymbol{L=1}$) means only one point, layer two ($\boldsymbol{L=2}$) means two another points touching the point in layer one, layer ($\boldsymbol{L=3}$) three means another two points in boundary touching the inner point in layer two and so on.

In two dimensions the arrangement is similar to arrangement of carom board. In $\boldsymbol{L=1}$ there is only one object. In $\boldsymbol{L=2}$ there are objects in the whole boundary of the central object in layer one.

In three dimension the layer $\boldsymbol{L=1}$ the central one object. Second layer $\boldsymbol{L=2}$ means the objects which can be attached around the first with self balance in the space with the help of boundary at bottom and the force of gravity. The common example of fruit arrangement is shown below.



The above orange stack is close packing rested on floor.

2. EMPIRICAL FORMULA

Here we present an empirical formula which gives exact number of centrally symmetrical objects in a self balanced symmetrical configuration.

$$N = L^{(n+1)} - (L-1)^{(n+1)}$$

Here N represents total number of symmetrical objects in n-dimesional space up to L layer. The formula has been derived by physical experimentation in one, two and three dimensions.

We have generated N for 1, 2, 3, 4, and 5-dimensional space with different layers.

Number of objects *N* in n-dimensional space with *L* layers					
n=1		n=2	n=3	n=4	n=5
N=L^2-(L-1)^2		N=L^3-(L-1)^3	N=L^4-(L-1)^4	N=L^5-(L-1)^5	N=L^6-(L-1)^6
L	N	N	N	N	N
1	1	1	1	1	1
2	3	7	15	31	63
3	5	19	65	211	665
4	7	37	175	781	3367
5	9	61	369	2101	11529
6	11	91	671	4651	31031
7	13	127	1105	9031	70993
8	15	169	1695	15961	144495
9	17	217	2465	26281	269297
10	19	271	3439	40951	468559
11	21	331	4641	61051	771561
12	23	397	6095	87781	1214423
13	25	469	7825	122461	1840825
14	27	547	9855	166531	2702727
15	29	631	12209	221551	3861089
16	31	721	14911	289201	5386591
17	33	817	17985	371281	7360353
18	35	919	21455	469711	9874655
19	37	1027	25345	586531	13033657
20	39	1141	29679	723901	16954119
21	41	1261	34481	884101	21766121
22	43	1387	39775	1069531	27613783
23	45	1519	45585	1282711	34655985
24	47	1657	51935	1526281	43067087
25	49	1801	58849	1803001	53037649
26	51	1951	66351	2115751	64775151
27	53	2107	74465	2467531	78504713
28	55	2269	83215	2861461	94469815
29	57	2437	92625	3300781	112933017
30	59	2611	102719	3788851	134176679
31	61	2791	113521	4329151	158503681
32	63	2977	125055	4925281	186238143
33	65	3169	137345	5580961	217726145
34	67	3367	150415	6300031	253336447
35	69	3571	164289	7086451	293461209
36	71	3781	178991	7944301	338516711
37	73	3997	194545	8877781	388944073
38	75	4219	210975	9891211	445209975
39	77	4447	228305	10989031	507807377
40	79	4681	246559	12175801	577256239

Number of objects N in n-dimensional space with L layers					
n=1		n=2	n=3	n=4	n=5
N=L^2-(L-1)^2		N=L^3-(L-1)^3	N=L^4-(L-1)^4	N=L^5-(L-1)^5	N=L^6-(L-1)^6
41	81	4921	265761	13456201	654104241
42	83	5167	285935	14835031	738927503
43	85	5419	307105	16317211	832331305
44	87	5677	329295	17907781	934950807
45	89	5941	352529	19611901	1047451769
46	91	6211	376831	21434851	1170531271
47	93	6487	402225	23382031	1304918433
48	95	6769	428735	25458961	1451375135
49	97	7057	456385	27671281	1610696737
50	99	7351	485199	30024751	1783712799
51	101	7651	515201	32525251	1971287801
52	103	7957	546415	35178781	2174321863
53	105	8269	578865	37991461	2393751465
54	107	8587	612575	40969531	2630550167
55	109	8911	647569	44119351	2885729329
56	111	9241	683871	47447401	3160338831
57	113	9577	721505	50960281	3455467793
58	115	9919	760495	54664711	3772245295
59	117	10267	800865	58567531	4111841097
60	119	10621	842639	62675701	4475466359
61	121	10981	885841	66996301	4864374361
62	123	11347	930495	71536531	5279861223
63	125	11719	976625	76303711	5723266625
64	127	12097	1024255	81305281	6195974527
65	129	12481	1073409	86548801	6699413889
66	131	12871	1124111	92041951	7235059391
67	133	13267	1176385	97792531	7804432153
68	135	13669	1230255	103808461	8409100455
69	137	14077	1285745	110097781	9050680457
70	139	14491	1342879	116668651	9730836919
71	141	14911	1401681	123529351	10451283921
72	143	15337	1462175	130688281	11213785583
73	145	15769	1524385	138153961	12020156785
74	147	16207	1588335	145935031	12872263887
75	149	16651	1654049	154040251	13772025449
76	151	17101	1721551	162478501	14721412951
77	153	17557	1790865	171258781	15722451513
78	155	18019	1862015	180390211	16777220615
79	157	18487	1935025	189882031	17887854817
80	159	18961	2009919	199743601	19056544479
81	161	19441	2086721	209984401	20285536481

Number of objects N in n-dimensional space with L layers					
n=1		n=2	n=3	n=4	n=5
N=L^2-(L-1)^2		N=L^3-(L-1)^3	N=L^4-(L-1)^4	N=L^5-(L-1)^5	N=L^6-(L-1)^6
82	163	19927	2165455	220614031	21577134943
83	165	20419	2246145	231642211	22933701945
84	167	20917	2328815	243078781	24357658247
85	169	21421	2413489	254933701	25851484009
86	171	21931	2500191	267217051	27417719511
87	173	22447	2588945	279939031	29058965873
88	175	22969	2679775	293109961	30777885775
89	177	23497	2772705	306740281	32577204177
90	179	24031	2867759	320840551	34459709039
91	181	24571	2964961	335421451	36428252041
92	183	25117	3064335	350493781	38485749303
93	185	25669	3165905	366068461	40635182105
94	187	26227	3269695	382156531	42879597607
95	189	26791	3375729	398769151	45222109569
96	191	27361	3484031	415917601	47665899071
97	193	27937	3594625	433613281	50214215233
98	195	28519	3707535	451867711	52870375935
99	197	29107	3822785	470692531	55637768537
100	199	29701	3940399	490099501	58519850599

3. ANALYSIS:

The series yielded for n=2 and different values of L is the Hex numbers or centred hexagonal numbers : 1,7, 19, 37, 61, 91, 127, 169, 217, 271, 331, 397, 469, 547, 631, 721, 817, 919, 1027, 1141, 1261, 1387, 1519, 1657, 1801, 1951, 2107, 2269, 2437, 2611, 2791, 2977, 3169, 3367, 3571, 3781, 3997, 4219, 4447, 4681, 4921, 5167, 5419, 5677, 5941, 6211, 6487 …..

The structure is similar to the arrangement of carrom board with discs in different layers. The hexagonal lattice is the familiar 2-dimensional lattice in which each point has 6 neighbors. This is sometimes called the triangular lattice. Final digits of Hex numbers (hex(N) mod 10) are periodic with palindromic period of length 5 {1, 7, 9, 7, 1}. Last two digits of Hex numbers (hex(N) mod 100) are periodic with palindromic period of length 100 (Alexander Adamchuk, Aug 11 2006).

The number of cubes in the stack must always be the same number for width, length, height (at true regular cubic stack) and the maximum number of visible cubes can always be found by taking any cubic number and subtracting the number of the cube that is one less.

Examples: 125 - 64 = 61, 64 - 27 = 37, 27 - 8 = 19.

The sequence of digital roots of the a(n) is period 3: repeat [1,7,1] (Ant King, Jun 17 2012).

The average of the first n (n>0) centered hexagonal numbers is the n-th square (Philippe Deléham, Feb 04 2013). Therefore our formula leads to generation of Hex numbers (with n=2 and L=1,2,3,…. in our main formula)

$$N_{Hex} = L^3 - (L-1)^3$$

For n=3 and different L values we get the Rhombic dodecahedral numbers 1, 15, 65, 175, 369, 671, 1105, 1695, 2465…etc. The physical arrangement is similar to orange stack. Final digits of N, i.e., N mod 10, are repeated periodically with period of length 5 {1,5,5,5,9}. There is symmetry in this list since the sum of two numbers equally distant from the ends is equal to 10 = 1 + 9 = 5 + 5 = 2*5. Last two digits of a(n), i.e., a(n) mod 100, are repeated periodically with period of length 50 (Alexander Adamchuk, Aug 11 2006). The numbers are the constant number found in magic squares of order n, where n is an odd number. A Magic Square of side 1 is 1; 3 is 15; 5 is 65 and so on (David Quentin Dauthier, Nov 07 2008). Thus our formula leads to generation of Rhombic dodecahedral numbers (with n=3 and L=1,2,3,…. in our main formula)

$$N_{dod} = L^4 - (L-1)^4$$

For n=4, we get the beautiful nexus numbers 1, 31, 211, 781, 2101, 4651, 9031, 15961, 26281, 40951, 61051, 87781, 122461, 166531, 221551, 289201, 371281, 469711, 586531, 723901, 884101, 1069531,….. with last digit being always 1 and the series can be summaries as

$$N_{nexus} = L^5 - (L-1)^5$$

As seen from above our formula for number of symmetrical objects in a self balanced configuration yields some very important beautiful mathematical series. More research is in progress to investigate the physical meaning of such series.

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