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Disturbance in a Generalized Thermoviscoelastic Half-Space with Voids with Microtemperature without Energy Dissipation Due to Thermal Source

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Abstract

In the present paper I investigate the effects of thermal excitations on a generalized thermoviscoelastic half-space with voids with microtemperature without energy dissipation. The exact expressions for displacement components, components of the microtemperature vector, stresses, temperature distribution, change in volume fraction field, and heat flux moment vector are derived using the normal mode analysis approach on non-dimensional field equations. The viscosity effect and effect of microtemperature and voids on field variables has been depicted graphically for temperature gradient boundary.

Keywords: Thermoviscoelasticity with voids, normal-mode analysis, microtemperature.

1. INTRODUCTION

Nunziato and Cowin (1979) were the first to examine the theory of voids in thermoelastic materials. Iesan (1986) investigated the theory of voids in thermoelastic materials. Dhaliwal and Wang (1995) proposed a thermoelasticity theory for elastic materials with voids that incorporates the heat-flux as one of the constitutive variables and assumes a heat-flux evolution equation. Kumar and Rani (2004) examined at how mechanical and thermal stimuli affected the response of a generalised thermoelastic half-space with voids. Kumar and Rani (2007) investigated axisymmetric deformation in a thermoelastic material with voids due to mechanical and thermal causes. The asymptotic spatial behaviour was investigated by Pompei and Scalia (2011).

Viscoelastic materials has applications in the field of Biomechanics and engineering.

Cowin (1985) investigated the viscoelastic behavior of linear elastic materials with voids. Ciarletta and Scalia (1991) studied the integral type linear theory of viscoelastic materials with voids. Martínez and Quintanilla (1998) obtained uniqueness theorem By means of power type function method. De Cicco and Nappa (2003) discussed linear theory of thermoviscoelastic materials with voids. Luo and Li (2007) established some basic principles in dynamic theory of viscoelastic materials with voids. The effect due to time-harmonic normal point source or thermal source in magneto-thermo-viscoelastic half-space has been studied by Abd-Alla and Abo-Dahab (2009) studied. Iesan (2011) established uniqueness, reciprocal and variational theorems in linear theory of thermoviscoelastic materials with voids. Svanadze (2014) studied fundamental solutions and basic properties of equations of the linear thermoviscoelasticity theory with voids. Svanadze (2014) generalized basic results of classical theory of elasticity in the linear theory of viscoelastic materials with voids. Othman and Fekry (2016) investigated the response of magnetic field in a homogeneous, isotropic generalized thermo-viscoelastic diffusion material with voids. Hooda and Deswal (2016) discussed the effect of gravity in generalized thermoviscoelastic solid with voids. Othman and Fekry (2018) studied the influence of gravity and rotation in generalized thermo-viscoelasticity with voids. One-dimensional thermoviscoelastic theory with voids had been explained by Miranville and Quintanilla (2020).

The theory of thermoelasticity for bodies with microstructures shows importance in recent years. Grot invented the concept of microtemperatures and a thermodynamics theory for elastic materials with microstructure (1969). Riha (1975,1977) investigated a heat-conducting micropolar fluid with microtemperatures theory. Casas and Quintanilla (2005) established the exponential stability of solutions of the equation in this theory.

Ieşan (2007) develops a microstretch elastic solids with microtemperatures linear theory. The theory of thermoelasticity with microtemperatures was investigated by Ieşan and Quintanilla (2010). Scalia et al. (2010) investigated basic theorems in thermoelasticity equilibrium theory using microtemperatures. Bitsadze and Jaiani (2013) discussed some basic plane thermoelasticity boundary value problems using microtemperatures. The influence of initial stress on a porous thermoelastic media using micro-temperatures was investigated by Othman et al. (2016). For the half-space, George and Bitsadze (2018) investigated basic thermoelasticity problems with microtemperatures. Marin et al. (2020) discussed microtemperatures and thermoelastic materials with a dipolar structure.

In a homogeneous, isotropic, thermoviscoelastic half-space with voids and microtemperature due to thermal source, the components of displacement, components of the microtemperatures vector, stresses, temperature distribution, change in volume fraction field, and first heat flux moment vector are proposed. The Lord-Shulman, Green-Lindsay, Green and Naghdi theories of types II and III have all been used to demonstrate the concept. To derive exact formulas for physical values, the normal mode analysis is performed. The viscosity effect, influence of microtemperatures, and voids are all depicted visually.

2. **Basic equations**

The constitutive relations for a homogeneous and isotropic thermoelastic medium with voids with microtemperature, according to Iesan and Quintanilla(2000), are

$$t_{ij} = \lambda\ u_{k,k}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + b\varphi\delta_{ij}$$

$$-\beta T(1 + \delta_{2k}\tau_1 \frac{\partial T}{\partial t})\ \delta_{ij},\ (i,j = 1,2,3). \qquad (1)$$

$$q_i = KT_{,i} + k_1 w_1, \qquad (2)$$

$$q_{ij} = -k_4 w_{r,r}\delta_{ij} - k_5 w_{i,j} - k_6 w_{j,i}, \qquad (3)$$

$$\rho\eta = \beta e_{rr} + aT + m^*\varphi, \qquad (4)$$

$$\rho \in_i = -b^\circ w_i,\ \square_i = \alpha\varphi_{,i}, \qquad (5)$$

$a, b^\circ, K_i, (i = 1,2\ldots\ldots,6)$ are constant constitutive coefficients, t_{ij} are components of stress tensor, α, b, ξ_1, m^*, χ material constants due to presence of voids, $\beta = (3\lambda + 2\mu)\alpha_t$, α_t linear thermal expansion, λ, μ Lame's constants, ρ is the reference mass density of the medium, K thermal conductivity, ϕ change in volume fraction field, η is the entropy per unit mass, $\in_i$ are the components of the first moment of the energy vector, q_{ij} are the components of the first heat flux moment vector. q_i are the components of the heat flux vector, u_i are components of displacement vector u, w_i are the components of the microtemperatures vector *w*, T is the temperature change. *T0* is the temperature of the medium in its natural state. *A* comma in the subscript denotes the spatial derivative and δ_{ij} is the Kronecker delta.

Following Iesan and Quintanilla(2000), the constitutive Eqs. (1)-(3), combined with the reduced

Clausius-Duhem inequality imply the following inequalities for the linear theory of thermoelasticity with microtemperatures

$$3K_4 + K_5 + K_6 \geq 0, K_6 + K_5 \geq 0,$$
$$K_6 - K_5 \geq 0, K \geq 0,$$
$$(K_1 + T_0 K_3)^2 - 4T_0 K K_2 \leq 0.$$

According to Iesan and Quintanilla(2000), the linear theory of thermoelasticity with voids with microtemperatures in the absence of body force, heat sources, and extrinsic equilibrated body force has a fundamental set of field equations

(i) Equation of motion for stress

$$t_{ij,j} = \rho\ddot{u}_i, \qquad (6)$$

(ii) Balance energy equation is

$$\rho T_0\dot{\eta} = q_{i,i}, \qquad (7)$$

(iii) First moment of energy balanced equation is

$$\rho\varepsilon_j = q_{ji,i} + q_i, \qquad (8)$$

The temporal derivative is represented by the superposed dot, while the other symbols are as explained previously. Using Eqs (1)-(5) in Eqs (6)-(8), and following Iesan (2011), Lord-Shulman (1967), Green-Lindsay (1972), Green and Naghdi (1993), the equations in a homogeneous, isotropic, thermoviscoelastic medium with voids with microtemperature, in the context of (G-N) theory of type III, in the absence of body force, body couple, equilibrated force,

$$\mu_1\nabla^2\vec{u} + (\lambda_1 + \mu_1)\nabla(\nabla.\vec{u}) + b_1\nabla\varphi\ - \beta\nabla\left(T + \delta_{2k}\tau_1\dot{T}\right)$$
$$= \rho\ddot{\vec{u}}, \qquad (9)$$

$$\alpha_1\nabla^2\varphi - \nu_1(\nabla.\vec{u}) - \xi_1\varphi + (m^* + \tau^*\nabla^2)T = \rho\chi\ddot{\varphi}, \quad (10)$$

$$K\nabla^2 T + K^*\nabla^2\dot{T} - \beta T_0\left(\frac{\partial}{\partial t} + \tau_0\delta_{1k}\frac{\partial^2}{\partial t^2}\right)\nabla.\vec{u}\ -$$
$$(\varsigma\nabla^2 - m^* T_0)\frac{\partial\varphi}{\partial t} + K_1\nabla.\vec{w}\ = \rho c_e(\dot{T} + \tau_0\ddot{T}) +$$
$$aT_0\dot{T}, \qquad (11)$$

$$K_6\nabla^2\vec{w} + (K_4 + K_5)\nabla(\nabla.\vec{w}) - K_3\nabla T - K_2\vec{w}\ = b^\circ\dot{\vec{w}}, \quad (12)$$

where

$$\mu_1 = \mu + \mu^*\frac{\partial}{\partial t}, \lambda_1 = \lambda + \lambda^*\frac{\partial}{\partial t}, b_1 = b + b^*\frac{\partial}{\partial t}, \alpha_1 = \alpha + \alpha^*\frac{\partial}{\partial t}$$
$$\nu_1 = b + \gamma^*\frac{\partial}{\partial t}, \xi_1 = \xi + \xi^*\frac{\partial}{\partial t}, \qquad (13)$$

τ_0, τ_1 are thermal relaxation times, For L-S theory, τ_1= 0, δ_{1k}=1 and for G-L theory τ_1>0, δ_{1k}= 0 (i.e., k=1 for L-S theory and k = 2 for G-L theory). The thermal relaxations $\tau_0\ and\ \tau_1$ satisfy the inequality $\tau_1 \geq \tau_0 > 0$ for the G-L theory only, $\nabla = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ and other symbols have their usual meaning. $\mu^*, \lambda^*, b^*, \alpha^*, \gamma^*, \xi^*$ are constitutive coefficient. c_e is specific heat at constant strain. K^* is the material constant characteristic of the theory, When $K^*\to 0$ then (11) reduces to the heat conduction equation in (G-N) theory (of type II).

3. Formulation and Solution of the problem

In the undeformed state at uniform temperature, we study a homogeneous, isotropic, thermally conducting generalised thermoviscoelastic half-space with voids with microtemperature. To. The rectangular Cartesian co-ordinate system (x,y,z) is introduced, with the origin at z = 0 and the z-axis pointing normally into the medium. At the origin of the rectangular Cartesian co-ordinates, a thermal source is supposed to be acting.

All quantities studied in the two-dimensional issue are functions of the time variable t and the coordinates x and z.

As a result, the displacement vector u and the microtemperature vector w are used.

$u=(u_1(x,z,t),0,u_3(x,z,t))$, $w=(w_1(x,z,t),0,w_3(x,z,t))$

To simplify the algebra, only problems with zero initial conditions are considered.

Introducing dimensionless quantities

$$x' = \frac{\omega_1^*}{c_2}x, \qquad z' = \frac{\omega_1^*}{c_2}z,$$

$$t' = = \omega_1^* t, \ \ (u_1', u_3') = \frac{\omega_1^*}{c_2}(u_1, u_3),$$

$$(w_1', w_3') = \frac{\omega_1^*}{c_2}(w_1, w_3), \ \ T' = \frac{T}{T_0},$$

$$\varphi' = \frac{\omega_1^{*2}\chi}{c_2^2}\varphi, \qquad \epsilon_1 = \frac{\beta c_2^2}{K\omega_1^*} \ , \ \tau_0' = \omega_1^*\tau_0 \ ,$$

$$\tau_1' = \omega_1^*\tau_1 \ , a' = \frac{\omega_1^*}{c_2}a,$$

$$q_{ij}' = \frac{\omega_1^*}{\beta T_0 c_2^2} q_{ij} \qquad (14)$$

$$t_{zz}' = \frac{t_{zz}}{\beta T_0}, \qquad t_{zx}' = \frac{t_{zx}}{\beta T_0},$$

$$\square' = \frac{\square c_2}{\omega_1^*}, \qquad (15)$$

where

$c_2 = \left(\frac{\mu}{\rho}\right)^{\frac{1}{2}}$ and $\omega_1^* = \frac{\rho c_e c_2^2}{K}$.

After suppressing the primes, equations (9) – (12) can be recast into the dimensionless form as:

$$\left(1+\gamma_1\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u_1}{\partial x^2}+\frac{\partial^2 u_1}{\partial z^2}\right) + \left(\gamma_2+\gamma_3\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u_1}{\partial x^2}+\frac{\partial^2 u_3}{\partial x\partial z}\right) + \left(\gamma_4+\gamma_5\frac{\partial}{\partial t}\right)\frac{\partial\varphi}{\partial x} - \gamma_6\left(1+\tau_1\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} = \gamma_7\frac{\partial^2 u_1}{\partial t^2}, \qquad (16)$$

$$\left(1+\gamma_1\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u_3}{\partial x^2}+\frac{\partial^2 u_3}{\partial z^2}\right) + \left(\gamma_2+\gamma_3\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u_1}{\partial z\partial x}+\frac{\partial^2 u_3}{\partial z^2}\right) + \left(\gamma_4+\gamma_5\frac{\partial}{\partial t}\right)\frac{\partial\varphi}{\partial z} - \gamma_6\left(1+\tau_1\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z} = \gamma_7\frac{\partial^2 u_3}{\partial t^2}, \qquad (17)$$

$$\left\{\left(1+\gamma_8\frac{\partial}{\partial t}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right) - \gamma_{14}\frac{\partial^2}{\partial t^2} - \gamma_{11}\right\}\varphi - \left(\gamma_9+\gamma_{10}\frac{\partial}{\partial t}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\varphi_1 + \left(\gamma_{12}+\gamma_{13}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\right)T = 0, \qquad (18)$$

$$(1+\gamma_{15}\frac{\partial}{\partial t})\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right) - \epsilon_1\left(\frac{\partial}{\partial t}+\tau_0\delta_{1k}\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_3}{\partial z}\right) - \left\{\gamma_{16}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right) - \gamma_{17}\right\}\frac{\partial\varphi}{\partial t} + \gamma_{18}\left(\frac{\partial w_1}{\partial x}+\frac{\partial w_3}{\partial z}\right) = \gamma_{19}\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T + \gamma_{20}\frac{\partial T}{\partial t}, \qquad (19)$$

$$\left(\frac{\partial^2 w_1}{\partial x^2}+\frac{\partial^2 w_1}{\partial z^2}\right) + \gamma_{21}\left(\frac{\partial^2 w_1}{\partial x^2}+\frac{\partial^2 w_3}{\partial x\partial z}\right) - \gamma_{22}\frac{\partial T}{\partial x} - \gamma_{23}w_1 = \gamma_{24}\frac{\partial w_1}{\partial t}, \qquad (20)$$

$$\left(\frac{\partial^2 w_3}{\partial x^2}+\frac{\partial^2 w_3}{\partial z^2}\right) + \gamma_{21}\left(\frac{\partial^2 w_1}{\partial x\partial z}+\frac{\partial^2 w_3}{\partial z^2}\right) - \gamma_{22}\frac{\partial T}{\partial z} - \gamma_{23}w_3 = \gamma_{24}\frac{\partial w_3}{\partial t}, \qquad (21)$$

where

$$\gamma_1 = \frac{\mu^*\omega_1^*}{\mu}, \gamma_2 = \frac{\lambda+\mu}{\mu}\frac{\omega_1^*}{c_2},$$

$$\gamma_3 = \frac{\lambda^*+\mu^*}{\mu}\frac{\omega_1^{*2}}{c_2}, \gamma_4 = \frac{bc_2^2}{\omega_1^*\mu\chi},$$

$$\gamma_5 = \frac{b^*c_2}{\mu\chi}, \quad \gamma_6 = \frac{\beta T_0}{\mu}, \gamma_7 = \frac{\rho\omega_1^{*2}}{\mu},$$

$$\gamma_8 = \frac{\alpha^*\omega_1^*}{\alpha}, \gamma_9 = \frac{b\chi}{\alpha}, \gamma_{10} = \frac{\gamma^*\chi\omega_1^*}{\alpha},$$

$$\gamma_{11} = \frac{\xi c_2^2}{\omega_1^{*2}\alpha}, \gamma_{12} = \frac{\xi^* c_2^2}{\omega_1^{*2}\alpha}, \quad ,$$

$$\gamma_{13} = \frac{m^*\chi T_0}{\alpha} \gamma_{14} = \frac{\omega_1 c_2^2}{\alpha\omega_1^*},$$

$$\gamma_{15} = \frac{K^*\omega_1^*}{K}, \gamma_{16} = \frac{\varsigma c_2^2}{\omega_1^* K T_0 \chi},$$

$$\gamma_{17} = \frac{m^* c_2^4}{\omega_1^{*3} K\chi}, \gamma_{18} = \frac{K_1 c_2^2}{\omega_1^{*2} K T_0},$$

$$\gamma_{19} = \frac{\rho c_e c_2^2}{\omega_1^* K}, \gamma_{20} = \frac{a T_0 c_2^2}{\omega_1^* K},$$

$$\gamma_{21} = \frac{K_4+K_5}{K_6}, \gamma_{22} = \frac{K_3 T_0 c_2^2}{K_6\omega_1^{*2}}, \gamma_{23} = \frac{K_2 c_2^2}{K_6\omega_1^{*2}},$$

$$\gamma_{24} = \frac{b^\circ c_2^2}{K_6\omega_1^*}. \in_1 = \frac{\beta c_2^2}{K\omega_1^*},$$

Using the dimensional form of the expression relating displacement components and microtemperature components $u(x,z,t)$ and $w(x,z,t)$ as well as the scalar potential function $\varphi_1(x,z,t)$,$\psi_1(x,z,t)$,$\varphi_2(x,z,t)$ and $\psi_2(x,z,t)$

$$u_1 = \frac{\partial\varphi_1}{\partial x} - \frac{\partial\psi_1}{\partial z}, \quad \mathrm{u}_3 = \frac{\partial\varphi_1}{\partial z} + \frac{\partial\psi_1}{\partial x},$$

$$w_1 = \frac{\partial\varphi_2}{\partial x} - \frac{\partial\psi_2}{\partial z},$$
$$\mathrm{w}_3 = \frac{\partial\varphi_2}{\partial z} + \frac{\partial\psi_2}{\partial x}. \quad (22)$$

in equations (16) – (21), we obtain

$$\left[(1+\gamma_2) + \frac{\partial}{\partial t}(\gamma_1+\gamma_3) - \gamma_7\frac{\partial^2}{\partial t^2}\right]\nabla^2\varphi_1 + \left(\gamma_4 + \gamma_5\frac{\partial}{\partial t}\right)\varphi$$
$$-\gamma_6(1+\tau_1\delta_{2k}\frac{\partial}{\partial t})T = 0\,, \quad (23)$$

$$[(1+\gamma_{15}\frac{\partial}{\partial t})\nabla^2 - \gamma_{19}(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}) - \gamma_{20}\frac{\partial}{\partial t}]T$$
$$-\in_1 (\frac{\partial}{\partial t} + \tau_0\delta_{1k}\frac{\partial^2}{\partial t^2})\nabla^2\varphi_1 - (\gamma_{16}\nabla^2 - \gamma_{17})\frac{\partial\varphi}{\partial t}$$
$$+\gamma_{18}\nabla^2\varphi_2 = 0 \quad (24)$$

$$(1+\gamma_8\frac{\partial}{\partial t})\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) - \gamma_{14}\frac{\partial^2}{\partial t^2} - \gamma_{11}\}\varphi$$
$$-(\gamma_9 + \gamma_{10}\frac{\partial}{\partial t})\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\varphi_1$$
$$+(\gamma_{12} + \gamma_{13}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right))T$$
$$= 0, \quad (25)$$

$$\{\nabla^2(1+\gamma_{21}) - \gamma_{23} - \gamma_{24}\frac{\partial}{\partial t}\}\varphi_2 - \gamma_{22}T = 0, \quad (26)$$

$$\{\nabla^2 - \gamma_{23} - \gamma_{24}\frac{\partial}{\partial t}\}\psi_2 = 0, \quad (27)$$

$$[(1+\gamma_1\frac{\partial}{\partial t})\nabla^2 - \gamma_7\frac{\partial^2}{\partial t^2}]\psi_1 = 0, \quad (28)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2},$$

4. Normal mode analysis

The following form can be used to decompose the solution of the considered physical variable in terms of normal modes

$$(u_1, w_1, \varphi_1, \psi_1, u_3, w_3, \varphi_2, \psi_2, \varphi, \sigma_{ij}, q_{ij}, T)(x,z,t)$$
$$= (u_1 *, w_1 *, \varphi_1 *, \psi_1 *, u_3 *, w_3 *, \varphi_2 *, \psi_2 *, \varphi, * \sigma_{ij} *, q_{ij} *, T *)(z)\, exp(\omega t + imx), \quad (29)$$

where $(u_1 *, w_1 *, \varphi_1 *, \psi_1 *, u_3 *, w_3 *, \varphi_2 *, \psi_2 *, \varphi, * \sigma_{ij} *, q_{ij} *, T *)$ are the magnitude of the functions, The complex time constant is ω and The wave number in the x –direction is denoted by a.

In equations (23)-(28), we use normal mode analysis technique and obtain

$$\left(\gamma_{25}\frac{\partial^2}{\partial z^2} - \gamma_{26}\right)\varphi_1^* + \gamma_{27}\varphi^* - \gamma_{28}T *= 0$$
$$= \quad (30)$$

$$(\gamma_{29}\frac{\partial^2}{\partial z^2} - \gamma_{30})\varphi^* - \gamma_{31}(\frac{\partial^2}{\partial z^2} - m^2)\varphi_1^* + (\gamma_{13}\frac{\partial^2}{\partial z^2} + \gamma_{32})T *$$
$$= 0, \quad (31)$$

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$$-\gamma_{33}(\frac{\partial^2}{\partial z^2}-m^2)\varphi_1^* + \gamma_{18}(\frac{\partial^2}{\partial z^2}-m^2)\varphi_2^* - (\gamma_{16}\frac{\partial^2}{\partial z^2}\omega-\gamma_{35})\varphi^* + (\gamma_{27}\frac{\partial^2}{\partial z^2}-\gamma_{34})T* = 0, \quad (32)$$

$$(\gamma_{38}\frac{\partial^2}{\partial z^2}-\gamma_{39})\varphi_2^* - \gamma_{24}T* = 0, \quad (33)$$

$$(\frac{\partial^2}{\partial z^2}-\lambda_6^2)\psi_1 * = 0, \quad (34)$$

$$(\frac{\partial^2}{\partial z^2}-\lambda_5^2)\psi_2 * = 0, \quad (35)$$

Where

$$\gamma_{25} = m^2(1+\gamma_1\omega\ ) + \gamma_{71}\omega^2 + \gamma_{71}\gamma_{72}\omega^2 m^2),$$

$$\gamma_{26} = (1+\gamma_8\omega \ + \gamma_{16}\omega^2),$$

$$\gamma_{27} = -m^2(1+\gamma_8)$$
$$-(\gamma_{11}+\gamma_{12}-\gamma_{14}) - \gamma_{15}\omega^2 - m^2\gamma_{16}\omega^2),$$

$$\gamma_{28} = m^2(\gamma_9+\omega\gamma_{10}),$$

$$\gamma_{29} = (\gamma_9+\omega\gamma_{10}),$$

$$\gamma_{30} = -m^2 \in_1 \omega^2(1+\tau_q\omega \ + \frac{{\tau_q}^2}{2}\omega^2),$$

$$\gamma_{31} = \frac{\gamma_{30}}{-m^2},$$

$$\gamma_{32} = -m^2\gamma_{33},$$

$$\gamma_{33} = [(\tau_T\,\omega^2+\omega) + \gamma_{17}(1+\tau_v\,\omega)]$$

Eliminating $\varphi_1 *, \varphi_2 *, \varphi * \, and \ T*$ from the resulting expressions, we obtain

$$\left(\frac{\partial^8}{\partial z^8} + M\frac{\partial^6}{\partial z^6} + N\frac{\partial^4}{\partial z^4} + O\frac{\partial^2}{\partial z^2} + P\right)(\varphi_1 *, \varphi_2 *, \varphi *, T*) = 0, \quad (36)$$

where

$$M = \{-\gamma_{39}(\gamma_{25}\gamma_{29}\gamma_{27}+\gamma_{25}\gamma_{13}\gamma_{16}) - \gamma_{38}(\gamma_{25}\gamma_{29}\gamma_{34}+\gamma_{25}\gamma_{30}\gamma_{27} + \gamma_{25}\gamma_{13}\gamma_{35} - \omega\gamma_{25}\gamma_{16}\gamma_{32} + \gamma_{26}\gamma_{29}\gamma_{27} + \omega\gamma_{26}\gamma_{13}\gamma_{16}) + \gamma_{18}\gamma_{24}\gamma_{25}\gamma_{29}\}/a_1,$$

$$N = [\gamma_{38}\{\gamma_{26}(\gamma_{34}\gamma_{29}+\gamma_{27}\gamma_{30}+\gamma_{13}\gamma_{35} - \omega\gamma_{16}\gamma_{26})\} - \gamma_{39}(\gamma_{25}\gamma_{29}\gamma_{34} + \gamma_{25}\gamma_{30}\gamma_{27}+\gamma_{25}\gamma_{13}\gamma_{35} - \omega\gamma_{25}\gamma_{16}\gamma_{32} + \gamma_{26}\gamma_{29}\gamma_{27}+\omega\gamma_{26}\gamma_{13}\gamma_{16}) + \gamma_{18}\gamma_{36}(\gamma_{19}\gamma_{30}+\gamma_{20}\gamma_{29}+\gamma_{27}\gamma_{31}) + m^2\gamma_{18}\gamma_{24}+\gamma_{25}\gamma_{29}]/a_1,$$

$$O = \{\gamma_{38}(\gamma_{26}\gamma_{32}\gamma_{35}-\gamma_{26}\gamma_{30}\gamma_{34}) - \gamma_{39}\{\gamma_{26}(\gamma_{34}\gamma_{29}+\gamma_{27}\gamma_{30} + \gamma_{13}\gamma_{35}-\omega\gamma_{16}\gamma_{26})\} - \gamma_{18}\gamma_{24}(m^2\gamma_{27}\gamma_{31}+\gamma_{26}\gamma_{30}) + m^2\gamma_{18}\gamma_{24}(\gamma_{25}\gamma_{30}+\gamma_{26}\gamma_{29}+\gamma_{27}\gamma_{31})\}/a_1$$

$$P = \{-\gamma_{39}(-\gamma_{26}\gamma_{30}\gamma_{34}+\gamma_{26}\gamma_{32}\gamma_{35}) - m^2\gamma_{18}\gamma_{24}(m^2\gamma_{27}\gamma_{31}+\gamma_{26}\gamma_{30})\}/a_1$$

$$a_1 = (\gamma_{25}\gamma_{29}\gamma_{27}+\gamma_{25}\gamma_{13}\gamma_{16})\gamma_{38}$$

The roots of equations (30) - (35) are $\pm\lambda_\ell(\ell = 1,2,3,4)$. we use the regularity condition at

z= ∞, the solutions of equations (30) - (35) may be written as

$$\varphi_1^* = A_1\bar{e}^{\lambda_1 z} + A_2\bar{e}^{\lambda_2 z} + A_3\bar{e}^{\lambda_3 z} + A_4\bar{e}^{\lambda_4 z}, \quad (37)$$

$$\varphi_2^* = b_1A_1\bar{e}^{\lambda_1 z} + b_2A_2\bar{e}^{\lambda_2 z} + b_3A_3\bar{e}^{\lambda_3 z} + b_4A_4\bar{e}^{\lambda_4 z}, \quad (38)$$

$$\varphi^* = g_1A_1\bar{e}^{\lambda_1 z} + g_2A_2\bar{e}^{\lambda_2 z} + g_3A_3\bar{e}^{\lambda_3 z} + g_4A_4\bar{e}^{\lambda_4 z}, \quad (39)$$

$$T^* = h_1A_1\bar{e}^{\lambda_1 z} + h_2A_2\bar{e}^{\lambda_2 z} + h_3A_3\bar{e}^{\lambda_3 z} + h_4A_4\bar{e}^{\lambda_4 z}, \quad (40)$$

$$\psi_1^* = A_6e^{-\lambda_6 z}. \quad (41)$$

$$\varphi_5^* = A_5e^{-\lambda_5} \quad (42)$$

Where

$$b_\ell = \frac{\lambda_\ell^4\gamma_{54}+\lambda_\ell^2\gamma_{55}+\gamma_{56}}{\lambda_\ell^4\gamma_{51}+\lambda_\ell^2\gamma_{52}+\gamma_{53}},$$

$$g_\ell = -\frac{\lambda_\ell^4\gamma_{41}+\lambda_\ell^2\gamma_{42}+\gamma_{43}}{\lambda_\ell^2\gamma_{44}+\gamma_{45}},$$

$$\square_\ell = \frac{\lambda_\ell^4\gamma_{48}+\lambda_\ell^2\gamma_{49}+\gamma_{50}}{\lambda_\ell^2\gamma_{46}+\gamma_{47}}, (\ell = 1,2,3,4),$$

$$\gamma_{41} = \gamma_{13}\gamma_{25}, \gamma_{42} = \gamma_{13}\gamma_{26} + \gamma_{25}\gamma_{32} - \gamma_{28}\gamma_{31},$$

$$\gamma_{43} = -\frac{\gamma_{27}}{\gamma_{24}}(\gamma_{27}\gamma_{39} + \gamma_{34}\gamma_{38} - \gamma_{24}\gamma_{18})$$

$$\gamma_{44} = \gamma_{27}\gamma_{13} - \gamma_{28}\gamma_{29}, \gamma_{45} = \gamma_{27}\gamma_{32} + \gamma_{28}\gamma_{30},$$

$$\gamma_{46} = -(\gamma_{28}\gamma_{29} + \gamma_{27}\gamma_{13}),$$

$$\gamma_{47} = (\gamma_{28}\gamma_{30} - \gamma_{27}\gamma_{32}), \gamma_{48} = \gamma_{25}\gamma_{29},$$

$$\gamma_{49} = -(\gamma_{25}\gamma_{30} + \gamma_{26}\gamma_{29} + \gamma_{27}\gamma_{31}),$$

$$\gamma_{50} = m^2\gamma_{27}\gamma_{31} + \gamma_{30}\gamma_{26},$$

$$\gamma_{51} = \frac{\gamma_{27}}{\gamma_{24}}(\gamma_{27}\gamma_{38}) + \frac{\gamma_{28}}{\gamma_{24}}(\omega\gamma_{16}\gamma_{38}),$$

$$\gamma_{52} = -\frac{\gamma_{27}}{\gamma_{24}}(\gamma_{27}\gamma_{39} + \gamma_{34}\gamma_{38} - \gamma_{24}\gamma_{18}) - \frac{\gamma_{28}}{\gamma_{24}}(\gamma_{38}\gamma_{35} + \omega\gamma_{39}\gamma_{16}),$$

$$\gamma_{53} = -\frac{\gamma_{27}}{\gamma_{24}}(\gamma_{34}\gamma_{39} - m^2\gamma_{18}\gamma_{24}) + \frac{\gamma_{28}}{\gamma_{24}}(\gamma_{35}\gamma_{39}),$$

$$\gamma_{54} = -\omega\gamma_{16}\gamma_{19},$$

$$\gamma_{55} = -\omega\gamma_{16}\gamma_{26} + \gamma_{19}\gamma_{35} + \gamma_{27}\gamma_{33},$$

$$\gamma_{56} = \gamma_{35}\gamma_{26} + \gamma_{27}\gamma_{29},$$

with $A_\ell(\ell = 1,2,3,4)$ being arbitrary constants.

5. Applications

Thermal source

In this case the boundary conditions are

$t_{33}(x,z,t) = 0,$

$t_{31}(x,z,t) = 0,$

$\frac{\partial\varphi}{\partial z} = 0\,,$

$q_{33} = 0,$

$q_{31} = 0$ at $z = 0,$

$$\frac{\partial T}{\partial z}(\text{x,z} = 0) = Pe^{\omega t+imx} \text{ at z} = 0, \text{ for the temperature gradient boundary,}$$

or

$$T(x, z = 0) = Pe^{\omega t+imx} \text{ at z} = 0, \text{ for the temperature input boundary,}. \quad (43)$$

The magnitude of constant temperature applied on the boundary is denoted by P.

We obtain the expressions for displacement components, components of the microtemperatures vector, stresses, temperature distribution, change in volume fraction field, and components of the first heat flux moment vector by using equations (1), (3), (13)-(15), (22) and substituting the values of $\varphi_1^*, \varphi_2^*, \psi_1^*, \psi_2^*, \mathrm{T}^*, \varphi^*$ from equations (37)–(42) in the boundary conditions (43)

$$u_1^* = \frac{P}{\Delta}\{im(\Delta_1'\bar{e}^{\lambda_1 z} + \Delta_2'\bar{e}^{\lambda_2 z} + \Delta_3'\bar{e}^{\lambda_3 z} + \Delta_4'\bar{e}^{\lambda_4 z}) + \Delta_6'\lambda_6\bar{e}^{\lambda_6 z}\}e^{\omega t+imx},$$

$$u_3^* = -\frac{P}{\Delta}\{(\lambda_1\Delta_1'\bar{e}^{\lambda_1 z} + \lambda_2\Delta_2'\bar{e}^{\lambda_2 z} + \lambda_3\Delta_3'\bar{e}^{\lambda_3 z} + \lambda_4\Delta_4'\bar{e}^{\lambda_4 z}) + im\Delta_6'\bar{e}^{\lambda_6 z}\}e^{\omega t+imx},$$

$$w_1^* = \frac{P}{\Delta}\{im(b_1\Delta_1'\bar{e}^{\lambda_1 z} + b_2\Delta_2'\bar{e}^{\lambda_2 z} + b_3\Delta_3'\bar{e}^{\lambda_3 z} + b_4\Delta_4'\bar{e}^{\lambda_4 z}) + \lambda_5\Delta_5'\bar{e}^{\lambda_5 z}\}e^{\omega t+imx},$$

$$w_3^* = -\frac{P}{\Delta}\{(\lambda_1 b_1\Delta_1'\bar{e}^{\lambda_1 z} + \lambda_2 b_2\Delta_2'\bar{e}^{\lambda_2 z} + \lambda_3 b_3\Delta_3'\bar{e}^{\lambda_3 z} + \lambda_4 b_4\Delta_4'\bar{e}^{\lambda_4 z}) - im\Delta_5'\bar{e}^{\lambda_5 z}\}e^{\omega t+imx},$$

$$\sigma_{33}^* = \frac{P}{\Delta}\{(n_1\Delta_1'\bar{e}^{\lambda_1 z} + n_2\Delta_2'\bar{e}^{\lambda_2 z} + n_3\Delta_3'\bar{e}^{\lambda_3 z} + n_4\Delta_4'\bar{e}^{\lambda_4 z}) + n_5\Delta_6'\bar{e}^{\lambda_6 z}\}e^{\omega t+imx},$$

$$\sigma_{31}^* = \frac{P}{\Delta}\{(n_6\Delta_1'\bar{e}^{\lambda_1 z} + n_7\Delta_2'\bar{e}^{\lambda_2 z} + n_8\Delta_3'\bar{e}^{\lambda_3 z} + n_9\Delta_4'\bar{e}^{\lambda_4 z}) + n_{10}\Delta_6'\bar{e}^{\lambda_6 z}\}e^{\omega t+imx},$$

$$\varphi^* = \frac{P}{\Delta}(g_1\Delta_1'\bar{e}^{\lambda_1 z} + g_2\Delta_2'\bar{e}^{\lambda_2 z} + g_3\Delta_3'\bar{e}^{\lambda_3 z} + g_4\Delta_4'\bar{e}^{\lambda_4 z})e^{\omega t+imx},$$

$$T^* = \frac{P}{\Delta}(h_1\Delta_1'\bar{e}^{\lambda_1 z} + h_2\Delta_2'\bar{e}^{\lambda_2 z} + h_3\Delta_3'\bar{e}^{\lambda_3 z} + h_4\Delta_4'\bar{e}^{\lambda_4 z})e^{\omega t+imx},$$

$$q_{33}^{*}=\frac{P}{\Delta}\{(n_{11}b_1\Delta_1^{'}\bar{e}^{\lambda_1 z}+n_{12}b_2\Delta_2^{'}\bar{e}^{\lambda_2 z}+n_{13}b_3\Delta_3^{'}\bar{e}^{\lambda_3 z}+n_{14}b_4\Delta_4^{'}\bar{e}^{\lambda_4 z})+n_{15}\Delta_5^{'}\bar{e}^{\lambda_5 z}\}e^{\omega t+imx},$$

$$q_{31}^{*}=\frac{P}{\Delta}\{2\big(n_{16}\Delta_1^{'}\bar{e}^{\lambda_1 z}+n_{17}\Delta_2^{'}\bar{e}^{\lambda_2 z}+n_{18}\Delta_3^{'}\bar{e}^{\lambda_3 z}+n_{19}\Delta_4^{'}\bar{e}^{\lambda_4 z}\big)+n_{20}\Delta_5^{'}\bar{e}^{\lambda_5 z}\big\}e^{\omega t+imx}. \tag{44}$$

where

$\Delta=\Delta_1*+\Delta_2*$

$$\Delta_1*=n_{15}(n_5n_9+n_{10}n_4)[\lambda_1g_1\{n_{17}(-\lambda_3+h)h_3-n_{18}(-\lambda_2+h)h_2\}-\lambda_2g_2\{n_{16}(-\lambda_3+h)h_3-n_{18}(-\lambda_1+h)h_1\}+\lambda_3g_3\{n_{16}(-\lambda_2+h)h_2-n_{17}(-\lambda_1+h)h_1\}]$$

$$\Delta_2*=n_{20}(n_5n_9+n_{10}n_4)[\lambda_1g_1\{n_{12}d_2(-\lambda_3+h)h_3-n_{13}d_3(-\lambda_2+h)h_2\}-\lambda_2g_2\{n_{11}d_1(-\lambda_3+h)h_3-n_{13}d_3(-\lambda_1+h)h_1\}+\lambda_3g_3\{n_{11}d_1(-\lambda_2+h)h_2-n_{12}d_2(-\lambda_1+h)h_1\}]$$

$$\Delta_1'=n_5\Delta''_1-\Delta_1'''n_{10}$$

$$\Delta''_1=-n_{15}[n_7\{\lambda_3g_3n_{19}-\lambda_4g_4n_{18}\}-n_8\{\lambda_2g_2n_{19}-\lambda_4g_4n_{17}\}+n_9\{\lambda_2g_2n_{18}-\lambda_3g_3n_{17}\}]+n_{20}[n_7\{\lambda_3g_3n_{14}b_4-\lambda_4g_4n_{13}b_3\}-n_8\{\lambda_2g_2n_{13}b_3-\lambda_3g_3n_{12}b_2\}+n_9\{\lambda_2g_2n_{13}b_3-\lambda_3g_3n_{12}b_2\}]$$

$$\Delta'''_1=-n_{15}[n_2\{\lambda_3g_3n_{19}-\lambda_4g_4n_{18}\}-n_3\{\lambda_2g_2n_{19}-\lambda_4g_4n_{17}\}+n_4\{\lambda_2g_2n_{18}-\lambda_3g_3n_{17}\}]+n_{20}[n_2\{\lambda_3g_3n_{14}b_4-\lambda_4g_4n_{13}b_3\}-n_3\{\lambda_2g_2n_{14}b_4-\lambda_4g_4n_{12}b_2\}+n_4\{\lambda_2g_2n_{13}b_3-\lambda_3g_3n_{12}b_2\}]=$$

$$\Delta_2'=-n_5\Delta''_2+\Delta_2'''n_{10}$$

$$\Delta''_2=-n_{15}[n_6\{\lambda_3g_3n_{19}-\lambda_4g_4n_{18}\}-n_8\{\lambda_1g_1n_{19}-\lambda_4g_4n_{16}\}+n_9\{\lambda_1g_1n_{18}-\lambda_3g_3n_{16}\}]+n_{20}[n_6\{\lambda_3g_3n_{14}b_4-\lambda_4g_4n_{13}b_3\}-n_8\{\lambda_1g_1n_{14}b_3-\lambda_4g_4n_{11}b_1\}+n_9\{\lambda_1g_1n_{13}b_3-\lambda_3g_3n_{11}b_1\}]$$

$$\Delta^{\square}{}_2=-n_{15}[n_1\{\lambda_3g_3n_{19}-\lambda_4g_4n_{18}\}-n_3\{\lambda_1g_1n_{19}-\lambda_4g_4n_{16}\}+n_4\{\lambda_1g_1n_{18}-\lambda_3g_3n_{16}\}]+n_{20}[n_1\{\lambda_3g_3n_{14}b_4-\lambda_4g_4n_{13}b_3\}-n_3\{\lambda_1g_1n_{14}b_4-\lambda_4g_4n_{11}b_1\}+n_4\{\lambda_1g_1n_{13}b_3-\lambda_3g_3n_{11}b_1\}]$$

$$\Delta_3'=-n_5\Delta_3''+\Delta_3'''n_{10}$$

$$\Delta''_3=-n_{15}[n_1\{\lambda_2g_2n_{19}-\lambda_4g_4n_{17}\}-n_7\{\lambda_1g_1n_{19}-\lambda_4g_4n_{16}\}+n_9\{\lambda_1g_1n_{17}-\lambda_2g_2n_{16}\}]+n_{20}[n_6\{\lambda_2g_2n_{1}-\lambda_4g_4n_{17}\}-n_7\{\lambda_1g_1n_{14}-\lambda_4g_4n_{16}\}+n_9\{\lambda_1g_1n_{17}-\lambda_2g_2n_{16}\}]$$

$$\Delta^{\square}{}_3=n_{15}[n_1\{\lambda_2g_2n_{19}-\lambda_4g_4n_{17}\}-n_2\{\lambda_1g_1n_{19}-\lambda_4g_4n_{16}\}+n_4\{\lambda_1g_1n_{18}-\lambda_2g_2n_{16}\}]+n_{20}[n_1\{\lambda_2g_2n_{14}b_4-\lambda_4g_4n_{12}b_2\}-n_2\{\lambda_1g_1n_{14}b_4-\lambda_4g_4n_{11}b_1\}+n_4\{\lambda_1g_1n_{12}b_2-\lambda_2g_2n_{11}b_1\}]$$

$$\Delta_4'=n_{15}\Delta''_4+\Delta_4'''n_{20}$$

$$\Delta''_4=-n_5[n_6\{\lambda_2g_2n_{18}-\lambda_3g_3n_{17}\}-n_7\{\lambda_1g_1n_{18}-\lambda_3g_3n_{16}\}+n_8\{\lambda_1g_1n_{17}-\lambda_2g_2n_{16}\}]+n_{10}[n_1\{\lambda_2g_2n_{18}-\lambda_3g_3n_{17}\}-n_2\{\lambda_1g_1n_{18}-\lambda_3g_3n_{16}\}+n_3\{\lambda_1g_1n_{17}-\lambda_2g_2n_{16}\}]$$

$$\begin{aligned}\Delta^{\square}{}_4 = &-n_5[n_6\{\lambda_2 g_2 n_{13} b_3 - \lambda_3 g_3 n_{12} b_2\} \\ &- n_7\{\lambda_1 g_1 n_{13} b_3 \\ &- \lambda_3 g_3 n_{11} b_1\} \\ &+ n_8\{\lambda_1 g_1 n_{12} b_2 \\ &- \lambda_2 g_2 n_{11} b_1\}] \\ &+n_{10}[n_1\{\lambda_2 g_2 n_{13} b_4 - \lambda_3 g_3 n_{12} b_2\} \\ &- n_2\{\lambda_1 g_1 n_{13} b_3 \\ &- \lambda_3 g_3 n_{11} b_1\} \\ &+ n_3\{\lambda_1 g_1 n_{12} b_2 \\ &- \lambda_2 g_2 n_{11} b_1\}]\end{aligned}$$

$$\Delta'_5 = -n_5 \Delta''{}_5 + \Delta'''_5 n_{10}$$

$$\begin{aligned}\Delta''{}_5 = &\, n_6[\lambda_2 g_2\{n_{13} b_3 n_{19} - n_{14} b_4 n_{18}\} \\ &- \lambda_3 g_3\{n_{12} b_2 n_{19} \\ &- n_{14} b_4 n_{17}\} \\ &+ \lambda_4 g_4\{n_{12} b_2 n_{18} \\ &- n_{13} b_3 n_{17}\}] \\ &-n_7[\lambda_1 g_1\{n_{13} b_3 n_{19} - n_{14} b_4 n_{18}\} \\ &- \lambda_3 g_3\{n_{11} b_1 n_{19} \\ &- n_{14} b_4 n_{16}\} \\ &+ \lambda_4 g_4\{n_{11} b_1 n_{18} \\ &- n_{13} b_3 n_{16}\}] \\ &+n_8[\lambda_1 g_1\{n_{12} b_2 n_{19} - n_{14} b_4 n_{17}\} \\ &- \lambda_2 g_2\{n_{11} b_1 n_{19} \\ &- n_{14} b_4 n_{16}\} \\ &+ \lambda_4 g_4\{n_{11} b_1 n_{17} \\ &- n_{12} b_2 n_{16}\}] \\ &-n_9[\lambda_1 g_1\{n_{12} b_2 n_{18} - n_{13} b_3 n_{17}\} \\ &- \lambda_2 g_2\{n_{11} b_1 n_{18} \\ &- n_{13} b_3 n_{16}\} \\ &+ \lambda_3 g_3\{n_{11} b_1 n_{17} \\ &- n_{12} b_2 n_{16}\}]\end{aligned}$$

$$\begin{aligned}\Delta'''{}_5 = &\, n_1[\lambda_2 g_2\{n_{13} b_3 n_{19} - n_{14} b_4 n_{18}\} \\ &- \lambda_3 g_3\{n_{12} b_2 n_{19} \\ &- n_{14} b_4 n_{17}\} \\ &+ \lambda_4 g_4\{n_{12} b_2 n_{18} \\ &- n_{13} b_3 n_{17}\}] \\ &-n_2[\lambda_1 g_1\{n_{13} b_4 n_{19} - n_{14} b_4 n_{18}\} \\ &- \lambda_3 g_3\{n_{11} b_1 n_{19} \\ &- n_{14} b_4 n_{16}\} \\ &+ \lambda_4 g_4\{n_{11} b_1 n_{18} \\ &- n_{13} b_4 n_{16}\}] \\ &+n_3[\lambda_1 g_1\{n_{12} b_2 n_{19} - n_{14} b_4 n_{17}\} \\ &- \lambda_2 g_2\{n_{11} b_1 n_{19} \\ &- n_{14} b_4 n_{16}\} \\ &+ \lambda_4 g_4\{n_{11} b_1 n_{17} \\ &- n_{12} b_2 n_{16}\}] \\ &+n_4[\lambda_1 g_1\{n_{12} b_2 n_{18} - n_{13} b_3 n_{17}\} \\ &- \lambda_2 g_2\{n_{11} b_1 n_{18} \\ &- n_{13} b_3 n_{16}\} \\ &+ \lambda_3 g_3\{n_{11} b_1 n_{17} \\ &- n_{12} b_2 n_{16}\}]\end{aligned}$$

$$\Delta'_6 = -n_{15} \Delta''{}_6 + \Delta'''_6 n_{20}$$

$$\begin{aligned}\Delta''{}_6 = &\, n_1[n_7(\lambda_3 g_3 n_{19} - \lambda_4 g_4 n_{18}) \\ &- n_8(\lambda_2 g_2 n_{19} - \lambda_4 g_4 n_{17}) \\ &+ n_9(\lambda_2 g_2 n_{18} - \lambda_3 g_3 n_{17})] \\ &-n_2[n_6(\lambda_3 g_3 n_{19} - \lambda_4 g_4 n_{18}) \\ &- n_8(\lambda_1 g_1 n_{19} - \lambda_4 g_4 n_{16}) \\ &+ n_9(\lambda_1 g_1 n_{18} - \lambda_3 g_3 n_{16})] \\ &+n_3[n_6(\lambda_2 g_2 n_{19} - \lambda_4 g_4 n_{17}) \\ &- n_7(\lambda_1 g_1 n_{19} - \lambda_4 g_4 n_{16}) \\ &+ n_9(\lambda_1 g_1 n_{17} - \lambda_2 g_2 n_{16})] \\ &+n_4[n_6(\lambda_2 g_2 n_{18} - \lambda_3 g_3 n_{17}) \\ &- n_7(\lambda_1 g_1 n_{18} - \lambda_3 g_3 n_{16}) \\ &+ n_8(\lambda_1 g_1 n_{17} - \lambda_2 g_2 n_{16})]\end{aligned}$$

$$\begin{aligned}\Delta'''_6 = &\, n_1[n_7(\lambda_3 g_3 n_{14} b_4 - \lambda_4 g_4 n_{13} b_3) \\ &- n_8(\lambda_2 g_2 n_{14} b_4 \\ &- \lambda_4 g_4 n_{12} b_2) \\ &+ n_9(\lambda_2 g_2 n_{13} b_3 \\ &- \lambda_3 g_3 n_{12} b_2)] \\ &-n_2[n_6(\lambda_3 g_3 n_{14} b_4 - \lambda_4 g_4 n_{13} b_3) \\ &- n_8(\lambda_1 g_1 n_{14} b_4 \\ &- \lambda_4 g_4 n_{11} b_1) \\ &+ n_9(\lambda_1 g_1 n_{13} b_3 \\ &- \lambda_3 g_3 n_{11} b_1)]\end{aligned}$$

$$+n_3[n_6(\lambda_2 g_2 n_{14} b_4 - \lambda_4 g_4 n_{12} b_2) - n_7(\lambda_1 g_1 n_{14} b_4 - \lambda_4 g_4 n_{11} b_1) + n_9(\lambda_1 g_1 n_{12} b_2 - \lambda_2 g_2 n_{11} b_1)]$$

$$+n_4[n_6(\lambda_2 g_2 n_{13} b_3 - \lambda_3 g_3 n_{12} b_2) - n_7(\lambda_1 g_1 n_{13} b_3 - \lambda_3 g_3 n_{11} b_1) + n_8(\lambda_1 g_1 n_{12} b_2 - \lambda_2 g_2 n_{11} b_1)]$$

$$n_1 = [-(\lambda+\omega\lambda^*)m^2 + \{(\lambda+\omega\lambda^*) + 2(\mu+\omega\mu^*)\}\lambda_1^2 + (b + \omega b^*)\frac{c_2^2 g_1}{\omega_1^{*2} K} - \beta T_0(1+\tau_1\delta_{2k}\omega)h_1],$$

$$n_2 = [-(\lambda+\omega\lambda^*)m^2 + \{(\lambda+\omega\lambda^*) + 2(\mu+\omega\mu^*)\}\lambda_2^2 + (b + \omega b^*)\frac{c_2^2 g_1}{\omega_1^{*2} K} - \beta T_0(1+\tau_1\delta_{2k}\omega)h_1],$$

$$n_3 = [-(\lambda+\omega\lambda^*)m^2 + \{(\lambda+\omega\lambda^*) + 2(\mu+\omega\mu^*)\}\lambda_3^2 + (b + \omega b^*)\frac{c_2^2 g_1}{\omega_1^{*2} K} - \beta T_0(1+\tau_1\delta_{2k}\omega)\square_1],$$

$$n_4 = [-(\lambda+\omega\lambda^*)m^2 + \{(\lambda+\omega\lambda^*) + 2(\mu+\omega\mu^*)\}\lambda_4^2 + (b + \omega b^*)\frac{c_2^2 g_1}{\omega_1^{*2} K} - \beta T_0(1+\tau_1\delta_{2k}\omega)h_1],$$

$$n_5 = -2mi\lambda_6(\mu+\omega\mu^*),$$

$$n_6 = -2mi\lambda_1, n_7 = -2mi\lambda_2, n_8 = -2mi\lambda_3,$$
$$n_9 = -2mi\lambda_4, n_{10} = -(mi\lambda_6^2 + m^2),$$

$$n_{11} = -(s_1\lambda_4 m^2 + s_2 + \lambda_1^2),$$
$$n_{12} = -(s_1\lambda_4 m^2 + s_2 + \lambda_2^2),$$

$$n_{13} = -(s_1\lambda_4 m^2 + s_2 + \lambda_3^2),$$

$$n_{14} = -(s_1\lambda_4 m^2 + s_2 + \lambda_4^2),$$
$$n_{15} = ims_1\lambda_4\lambda_5 - ims_2, n_{16} = ims_1\lambda_1, n_{17} = s_1 b_2\lambda_2,$$

$$n_{18} = s_1 b_2 \lambda_3, n_{19} = s_1 b_2 \lambda_4,$$
$$n_{20} = s_1 im + s_1\lambda_5^2, s_1 = \frac{\omega_1^{*3}\rho^2}{\beta T_0 \mu^2}, s_2 = s_1 + (K_4 + K_5 + K_6),$$

The expressions for temperature gradient boundary and temperature input boundary are obtained by replacing $\Delta\ by$ Δ_1^* and Δ_2^*, respectively.

PATICULAR CASE: On Neglecting the microtemperature, voids and viscosity effect i.e.,($q_{ij} = \alpha = b = \xi_1 = m^* = \chi = 0$, $\mu^* = \lambda^* = b^* = \alpha^* = \gamma^* = \xi^* =$ K*=0) in equation (44), the corresponding expressions of stresses, displacement and temperature distribution for thermoelastic half-space are obtained.

Special case 1: For L–S theory, we obtain the corresponding expressions of thermoelastic half-space with voids and microtemperature by taking k = 1, $\tau_1 = \mu^* = \lambda^* = b^* = \alpha^* = \gamma^* = \xi^* =$ K*=0 in equation (44), respectively, .

Special case 2: The expressions of thermoelastic half-space with voids and microtemperature, are obtained by taking k=2, $\mu^* = \lambda^* = b^* = \alpha^* = \gamma^* = \xi^* =$ K*=0 in equation (44) for G–L theory

Special case 3: In case of coupled theory of thermoelasticity, the expressions of thermoelastic half-space with voids and microtemperature are obtained by taking $\tau_0 = \tau_1 = 0$,$\mu^* = \lambda^* = b^* = \alpha^* = \gamma^* = \xi^* =$ K*=0 in equation (44).

Special case 4: For uncoupled theory of thermoelasticity, we take $\epsilon_1 = 0, \tau_0 = \tau_1 = 0$, $\mu^* = \lambda^* = b^* = \alpha^* = \gamma^* = \xi^* =$ K*=0 in equation (44), and obtain the expressions of thermoelastic half-space with voids and microtemperature, respectively,.

6. Numerical results and discussion

Following Tomar et. al. (2013) the hypothetical values of the relevant parameters are

λ = 1.5 x 10^3 Nm^{-2}, μ = 2.5 x 10^4 Nm^{-2}, a=2 x 10^{-8} N/m^{2}K^2 , b=2.1 x 10^4 N/m^2,

α = 4 x 10^{-3} N, β =4 x 10^{-3} N/m^{2}K , $\xi = 40 N/m^2, m^* = 5 \times 10^{-3} N/m^2 K,$

K=0.016 x 10^{-3} Nsec^{-1}K, ρ = 2.6 x 10^6 kgm^{-3} , λ^* = 2.6 x 10^5 Nsec/m^2, μ^*= 1.0 x 10^2 Nsec/m^2,

b^*=1.2 x 10^4 Nsec/m^2, $\xi^* = 4 \times 10^4$ Nsec/m^2, α^* =1.6 Nsec, $\gamma^* = 8.0 \times 10^3$ Nsec/m^2, $\tau^* = 6.0 \times 10^{-6}$NK^{-1}, χ =0.2 x 10^{-6} m^2, ς =0.2 x 10^{-8} N, T_0 = 300 ^{0} K,

and c_e = 1.04 x 10^3 J kg^{-1}degree^{-1} , P=1, $K^* = c_e(\frac{\lambda_1 + 2\mu_1}{4})$

and other physical constants are (Steeb et. al.(2013))

$$K_1 = 2\times 10^{10} Wm^{-1}, K_2 = 0.1\times 10^{10} Wm^{-1},$$
$$K_3 = 0.4\times 10^{10} Wm^{-1}, K_4 = 0.3\times 10^{10} Wm^{-1},$$
$$K_5 = 0.5\times 10^{10} Wm^{-1}, K_6 = 0.7\times 10^{10} Wm^{-1},$$
$$b^\circ = 1.3849 \times 10^{10} N,$$

Figures 1-14 graphically compare the values of components of the microtemperatures vectors ($w_1^*$and w_3^*), tangential stress σ_{31}^*, normal stress σ_{33}^*, boundary temperature field T^* and change in volume fraction field φ^* , the components of the first heat flux moment vector q_{33}^* with distance x , for G-N, L-S and G-L theories, for non-dimensional relaxation times$\tau_0 = 0.02, \tau_1 = 0.05$

and $\omega = \omega_0 + i\eta, \omega_0 = 2.0, \eta = 0.1, m = 0.9$. For time t=1.0 and t=2.0, in the range $0 \le x \le 10$ at z=1, the computation has been done. The variation for G-N theory with viscous effect is shown by black lines with and without centre symbols in figures 1-7, whereas the variation for G-N theory without viscous effect is represented by red lines with and without centre symbols in figures 1-7. The variation for L-S theory is represented by black lines with and without centre symbols in figures 8-14, whereas the variation for G-L theory is represented by red lines with and without centre symbols. The temperature gradient boundary is represented in the figures.

Thermal source (Temperature gradient boundary)

Figure 1. depicts the variation of microtemperatures vector $w_1^*$with distance *x.* Initially, the values w_1^* remain the same in the range 0 ≤ x ≤ 5 for viscous and without viscous effect. The values of w_1^* for the case without viscous effect are more than with viscous effect in the range 5 □ x ≤ 10 for time t=1.0 and t=2.0. Figure 2. displays the variation of microtemperatures vector $w_3^*$with distance *x.* For viscous effect and without viscous effect the values of w_3^* at time t=1 are more than that at time t=2 in the whole range 0≤ x ≤ 10. Figure 3. shows the variation of temperature distribution T^* with distance *x.* For viscous and without viscous effect the value of T^* shows the opposite oscillatory pattern in the range 0≤ x ≤ 10. The variation of tangential stress σ_{31}^* with distance x has been shown in Figure 4.

For viscous and without viscous effect the values of σ_{31}^* start from zero and decrease with an increase in distance x. Figure 5 depicts the variation of $\sigma_{33}^*$with distance x. For viscous effect the values of σ_{33}^* at time t=3 are more than at time t=1 and for without viscous effect the values of σ_{33}^* at time t=1 are more than at time t=3 in the whole range. Figure 6 shows the variation of change in volume fraction field φ^* with distance x. The values of φ^* are the same for viscous and without viscous effect in the range 0 ≤ x ≤ 3 and increase with an increase in distance x in the range 3.1 ≤ x ≤ 10 for viscous and without viscous effect. The variation of q_{33}^* has been shown in Figure7. The values of q_{33}^* increases as time decreases for viscous and without viscous effect in the whole range.

Figure 8. shows the variation of microtemperatures vector $w_1^*$with distance *x.* Near the point of application of source, in the range, 0 ≤ x ≤ 3.5, the values of w_1^* for L-S and G-L theories show very small differences and in the range 3.6< x ≤ 10 an appreciable difference is observed, i.e., the values of w_1^* for L-S theory are more than G-L theory for time t=1 and t=2. Figure 9 displays the variation of microtemperatures vector $w_3^*$with distance *x.* The values ofw_3^* for time t=2 lies between the time t=1 for L-S and G-L theories. Figure 10 depicts the variation of temperature distribution T^* with distance *x.* The value of T^* for L-S and G-L theories start from zero and decreases with an increase in distance x for both values of time in the whole range. The variation of tangential stress σ_{31}^* with distance x has been shown in Figure 11. Initially, the values of tangential stress for both the theories are the same and the values of L-S theory are less than G-L theory for both values of time in the range 2 ≤ x ≤ 10. Figure 12 depicts the variation of $\sigma_{33}^*$with distance x. The values of σ_{33}^* start from zero and decrease with an increase in distance x for both the theories and for both values of the time. Figure13 shows the variation of φ^* with distance x. The values of φ^* for L-S theory are more than those of G-L theory for both values of time in the whole range. The variation of the first heat flux moment vector q_{33}^* has been shown in Figure14. The values of q_{33}^* decrease with an increase in distance x in the range 0 ≤ x ≤ 10 for both L-S and G-L theories and for time t=1 and t=2.

CONCLUSIONS

1. The thermo-viscoelastic materials with voids has application in the distribution of field quantities.

2. For finding the solution of the problem the normal mode analysis technique has been used.

3. The comparison G-N theory, L-S, G-L theories with viscous and without viscous effect has been depicted graphically for temperature gradient boundary.

4. It is noticed that the viscous effect plays an important role in all considered physical quantities.

5. It is observed that the deformation of a body depends on viscous effect, the nature of the applied force as well as the type of boundary conditions. The problem Investigated here is applicable in the field of earthquake engineering, seismology and geophysics.

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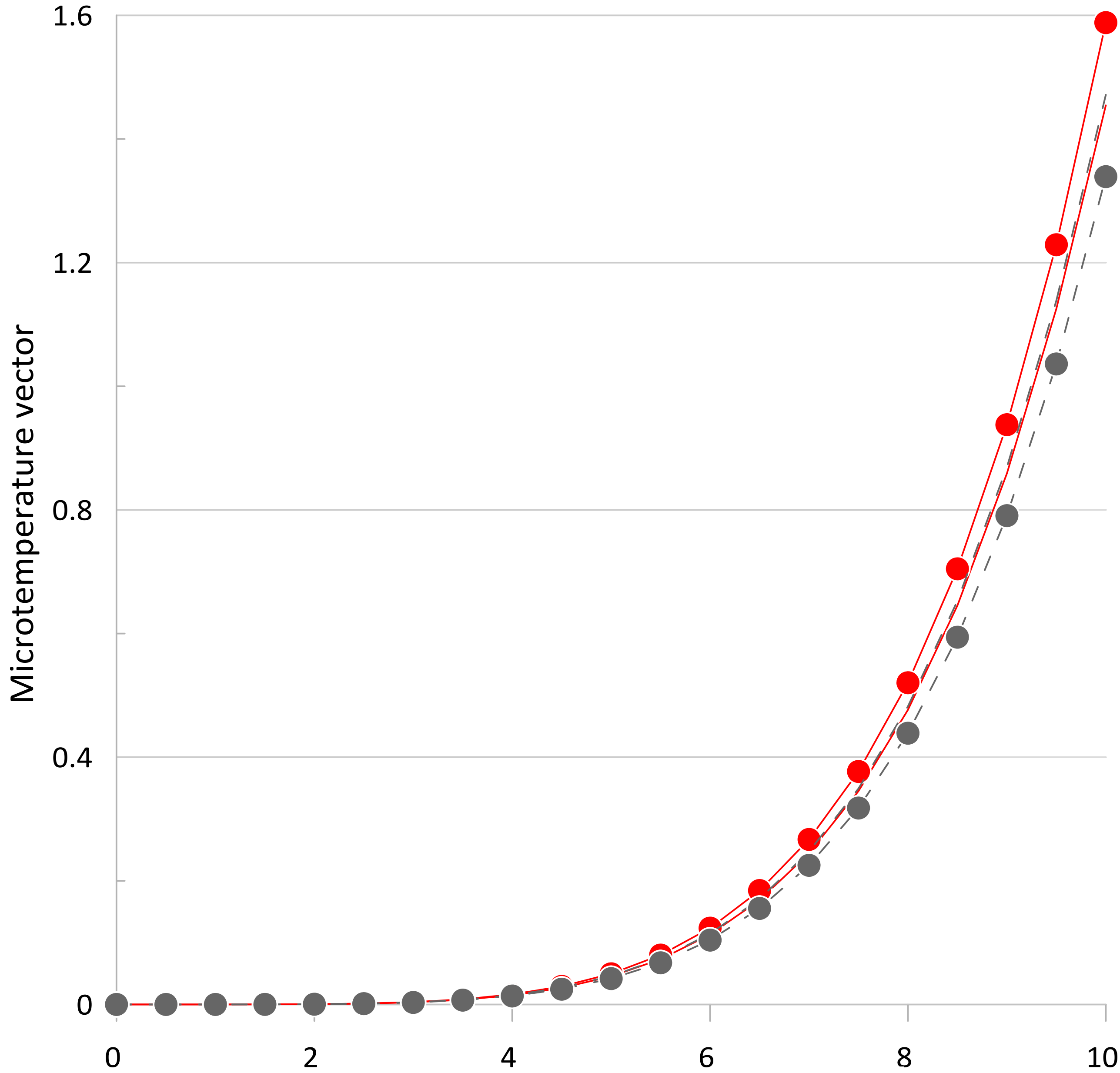
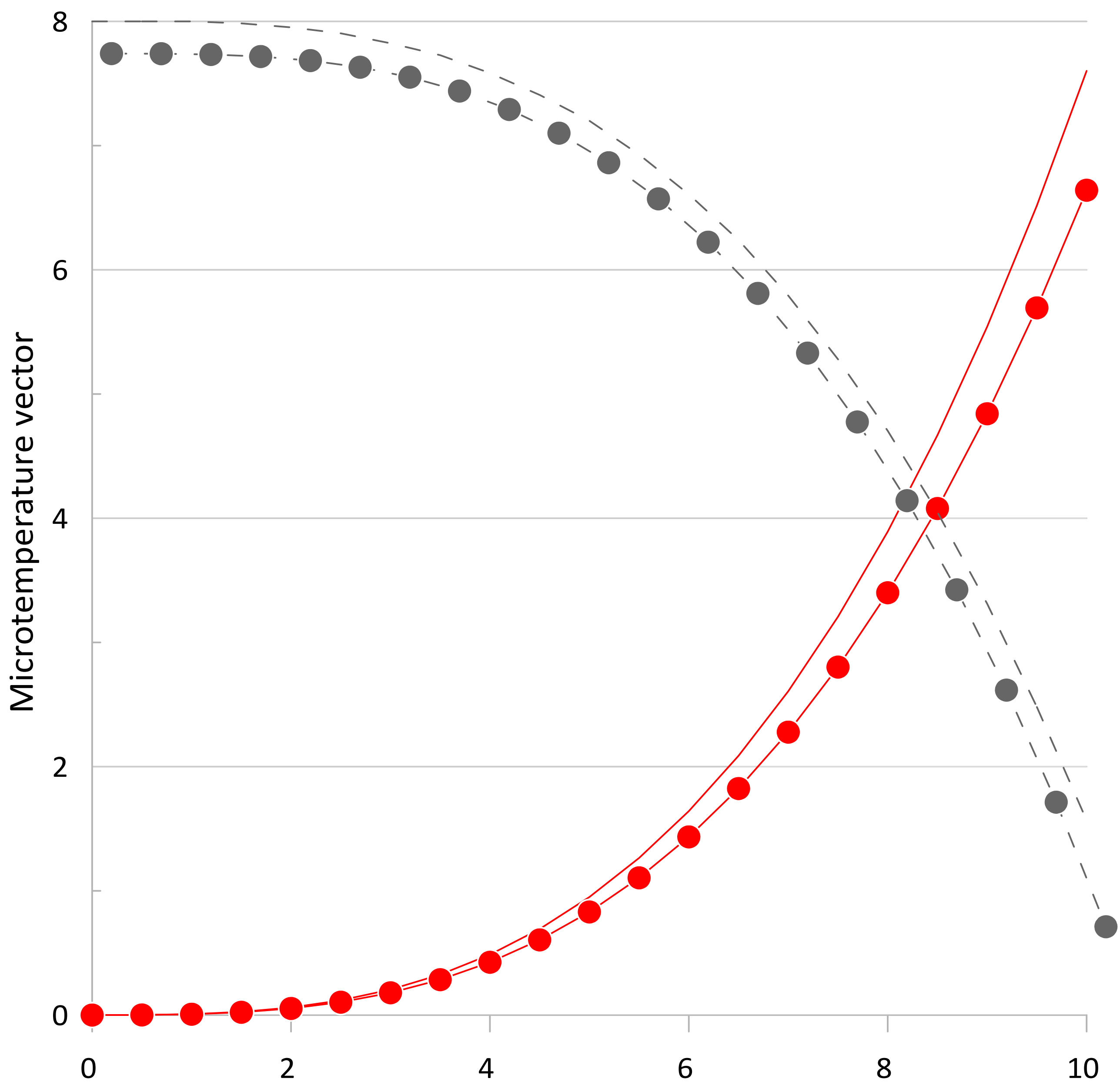


Figure 1. Variation of microtemperatures vector w_1^* with distance x



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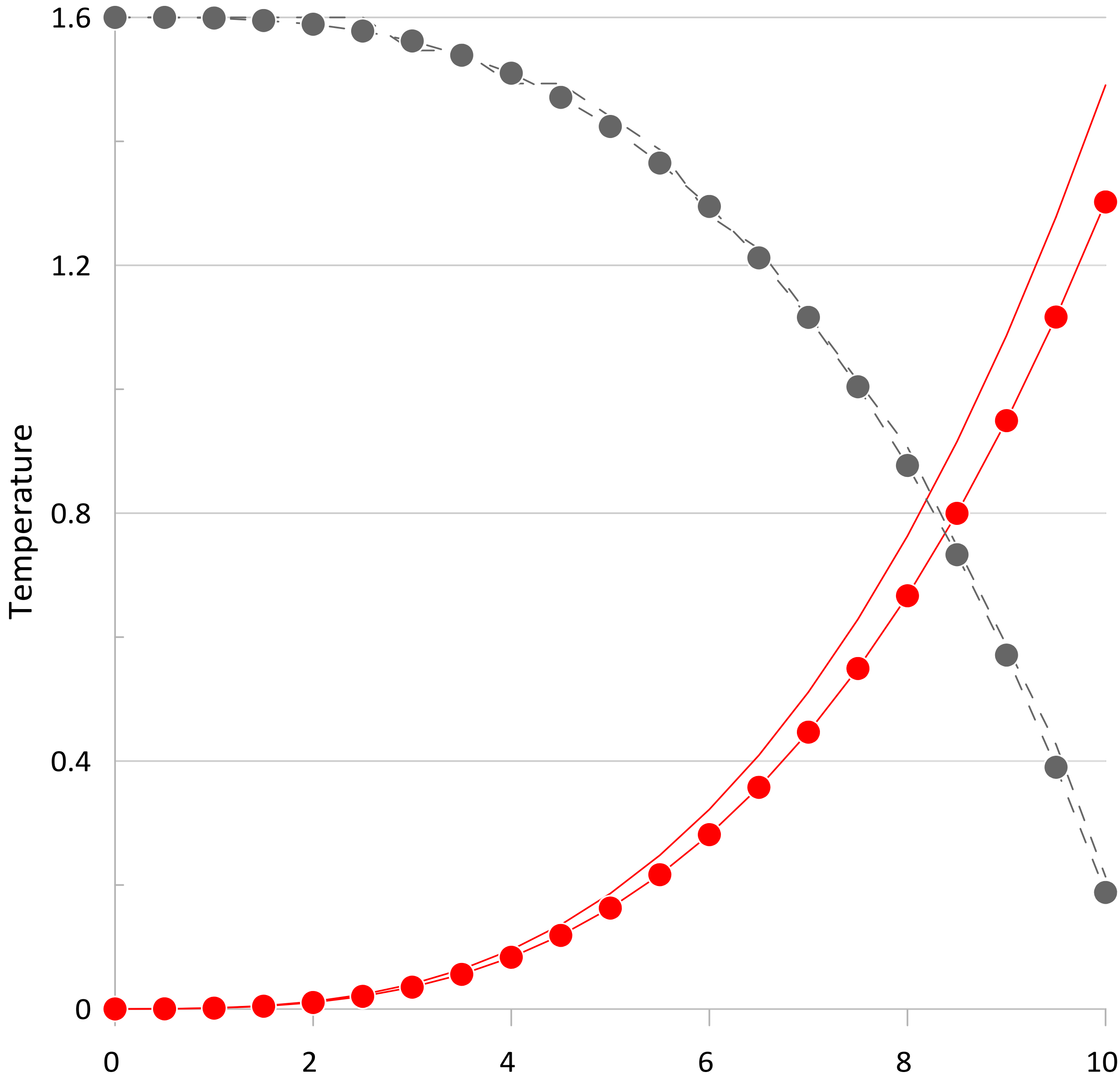

Figure 3. Variation of temperature distribution T^* with distance x.

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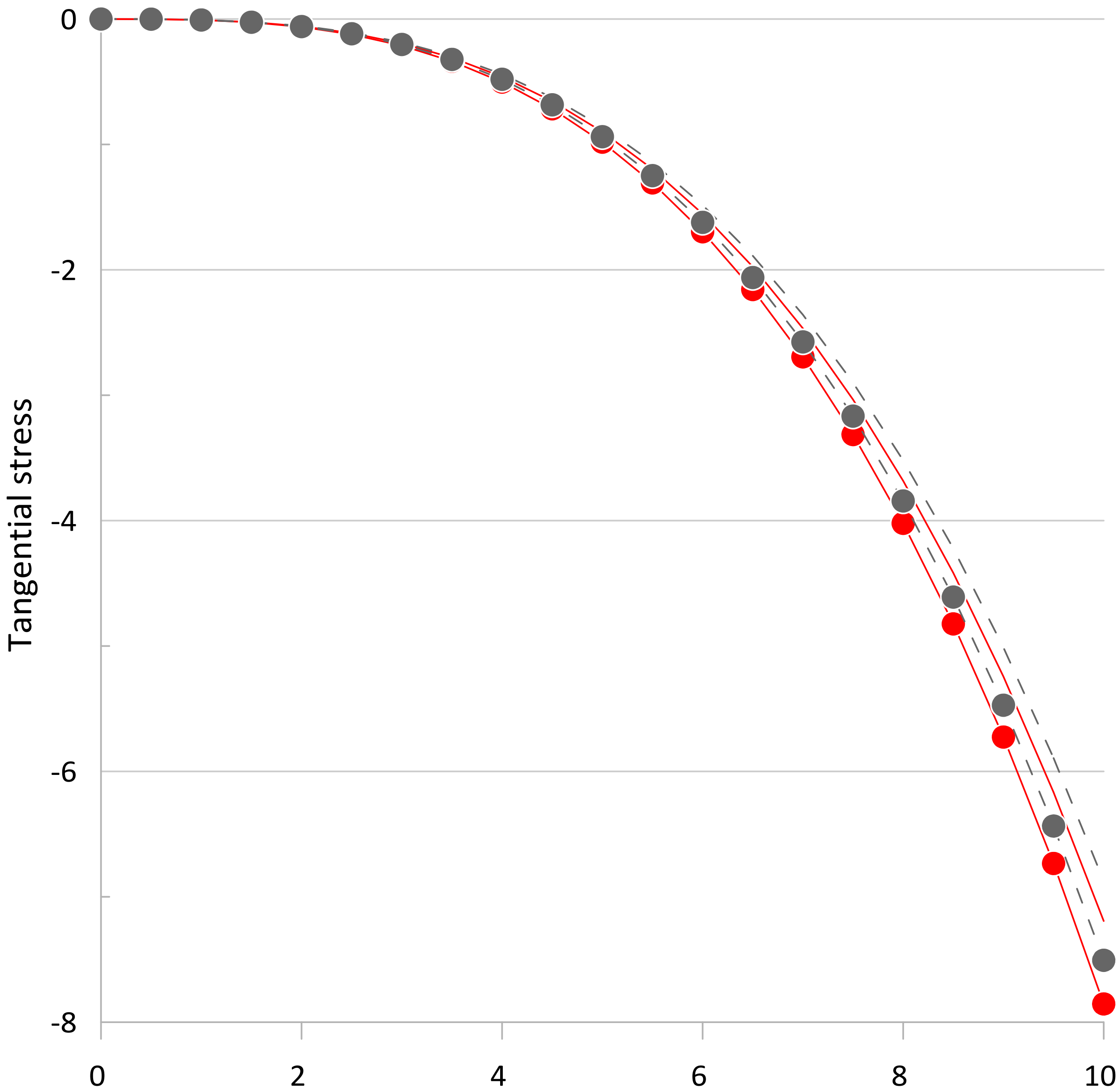

Figure 4. The variation of tangential stress σ_{31}^{*} with distance x.

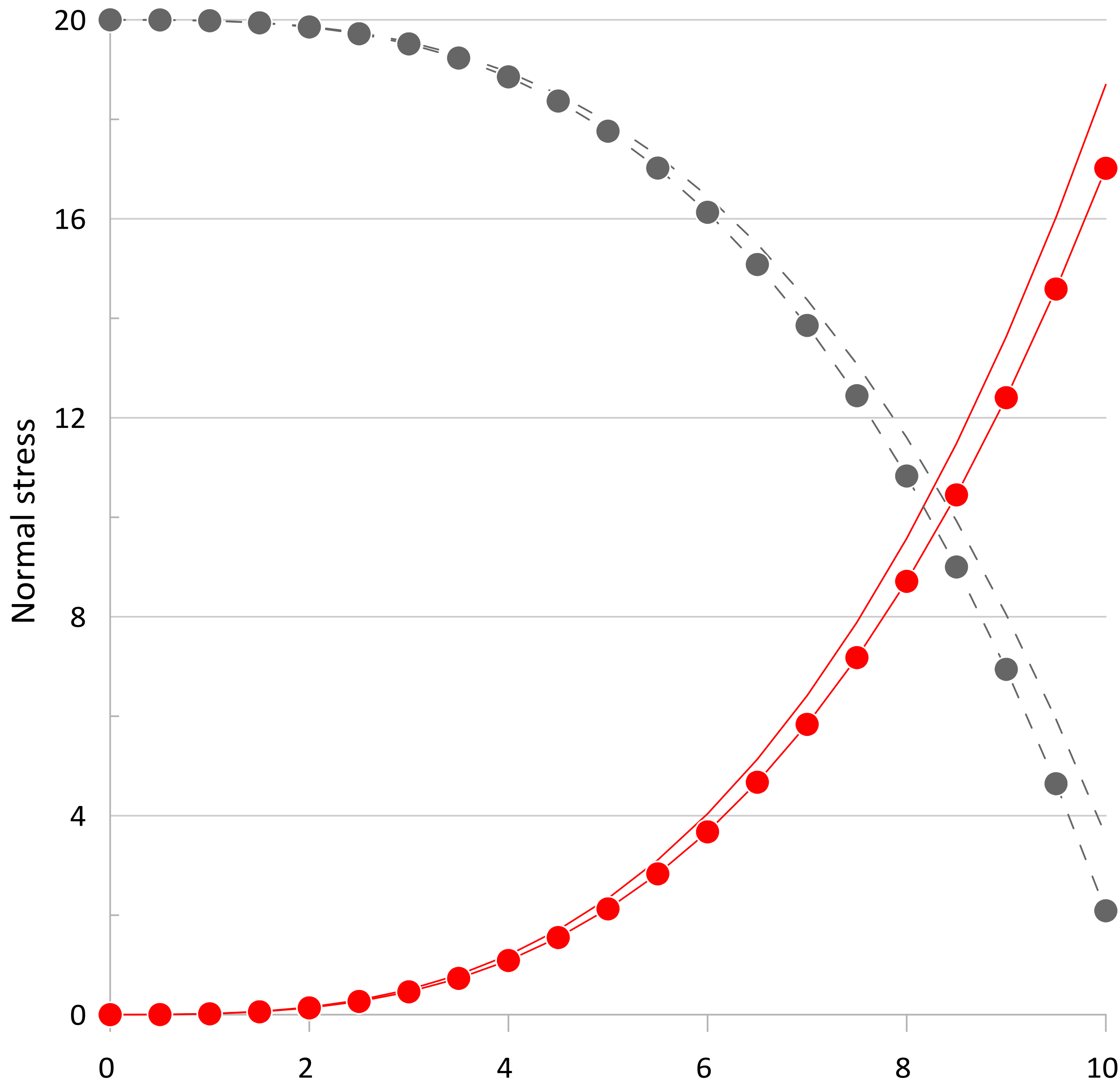

Figure 5. The variation of normal stress σ_{33}^{*} with distance x.

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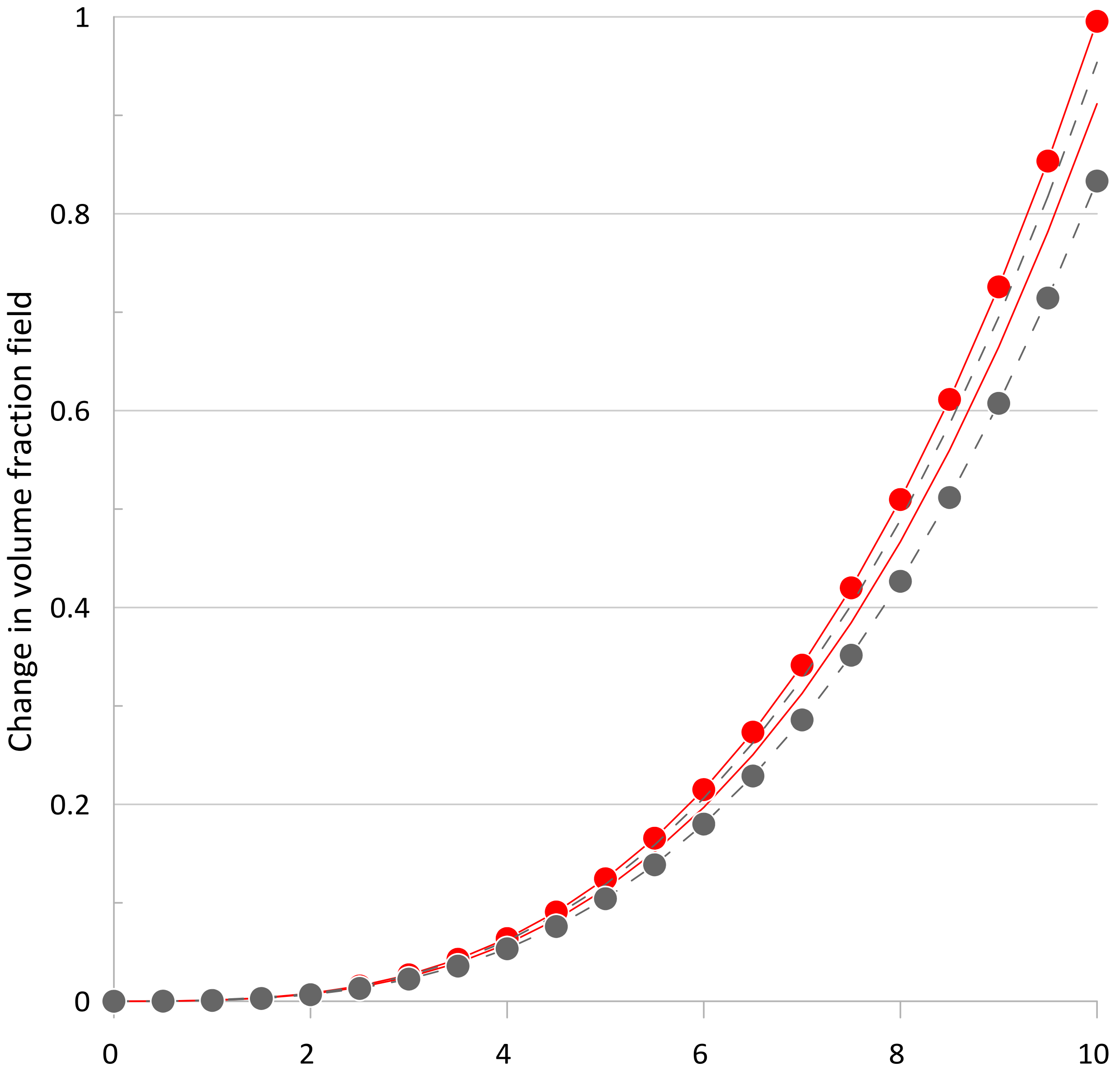

Figure6. Variation of change in volume fraction field φ^* with distance x.

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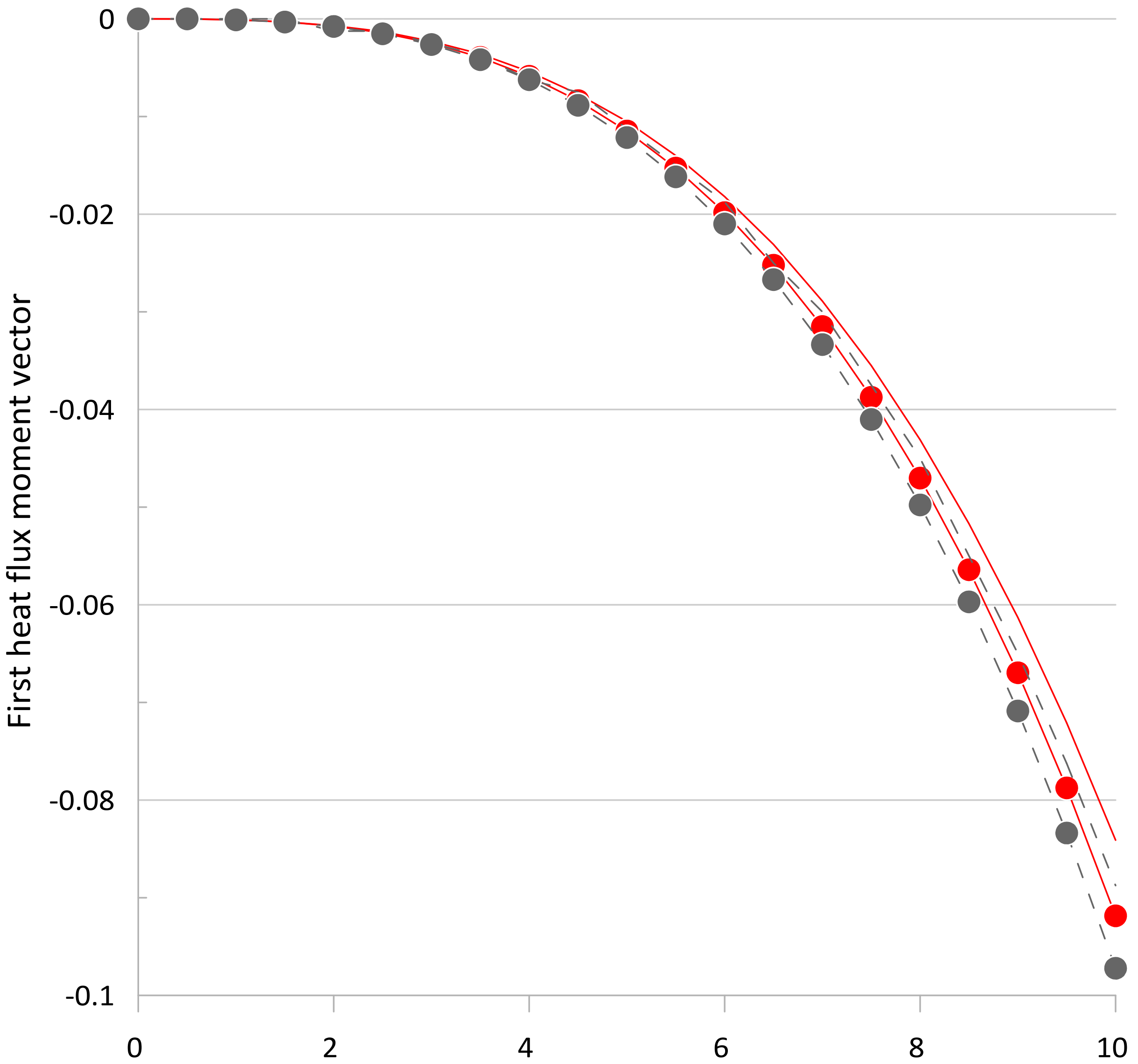

Figure7. Variation of the first heat flux moment vecto q_{33}^{*} with distance x.

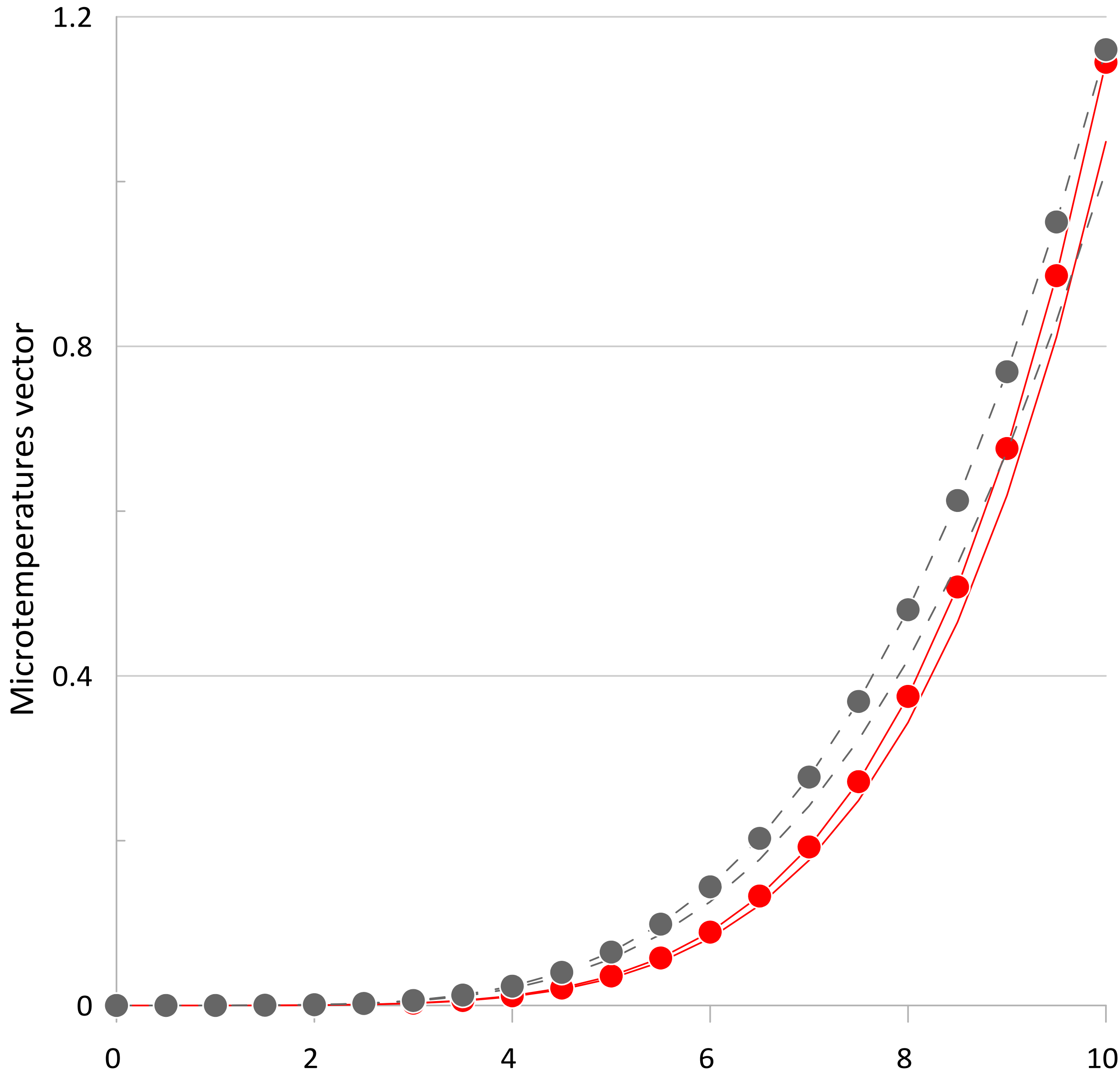

Figure 8. Variation of microtemperatures vector w_1^* with distance x.

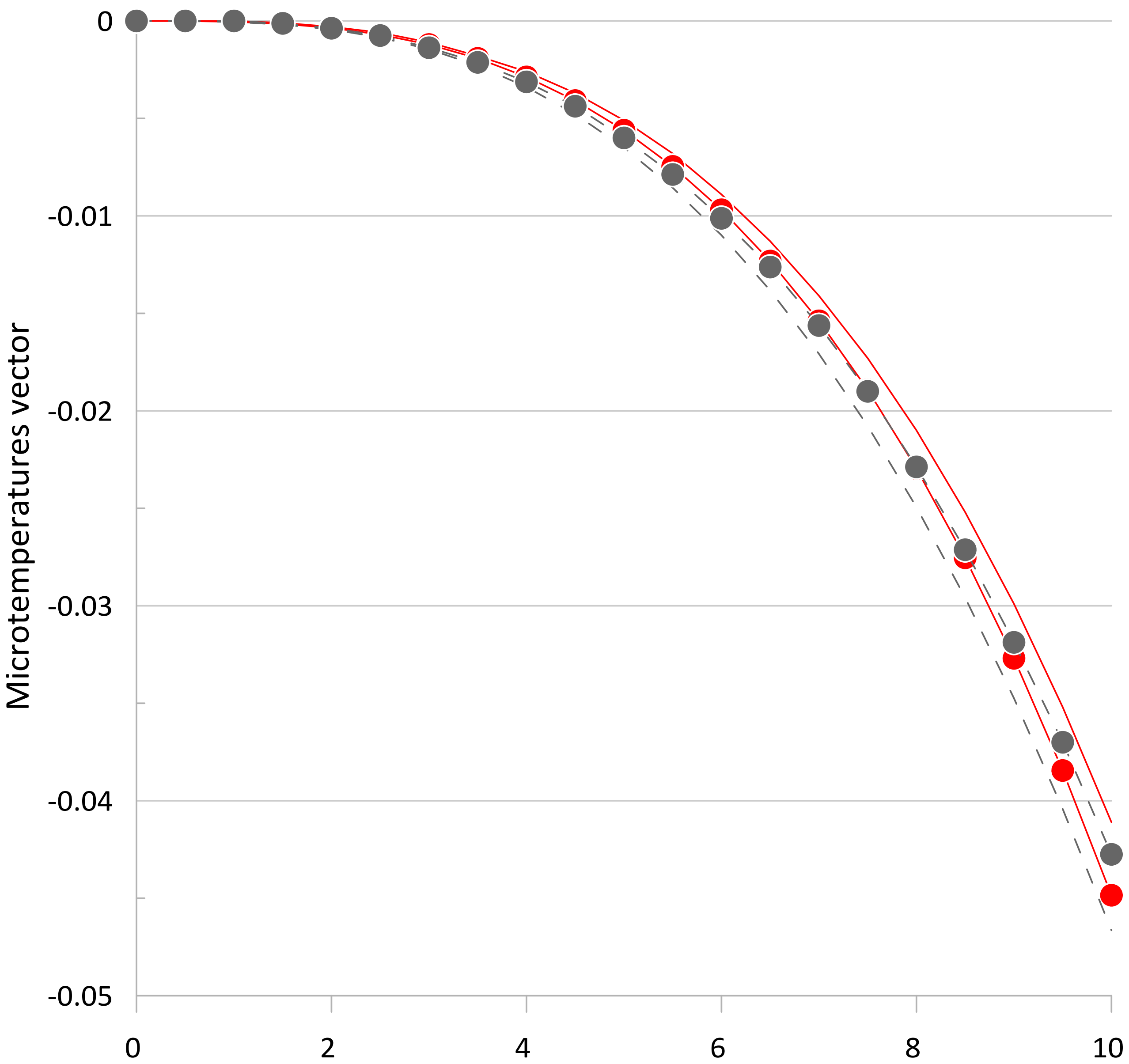

Figure 9. Variation of microtemperatures vector w_3^* with distance x.

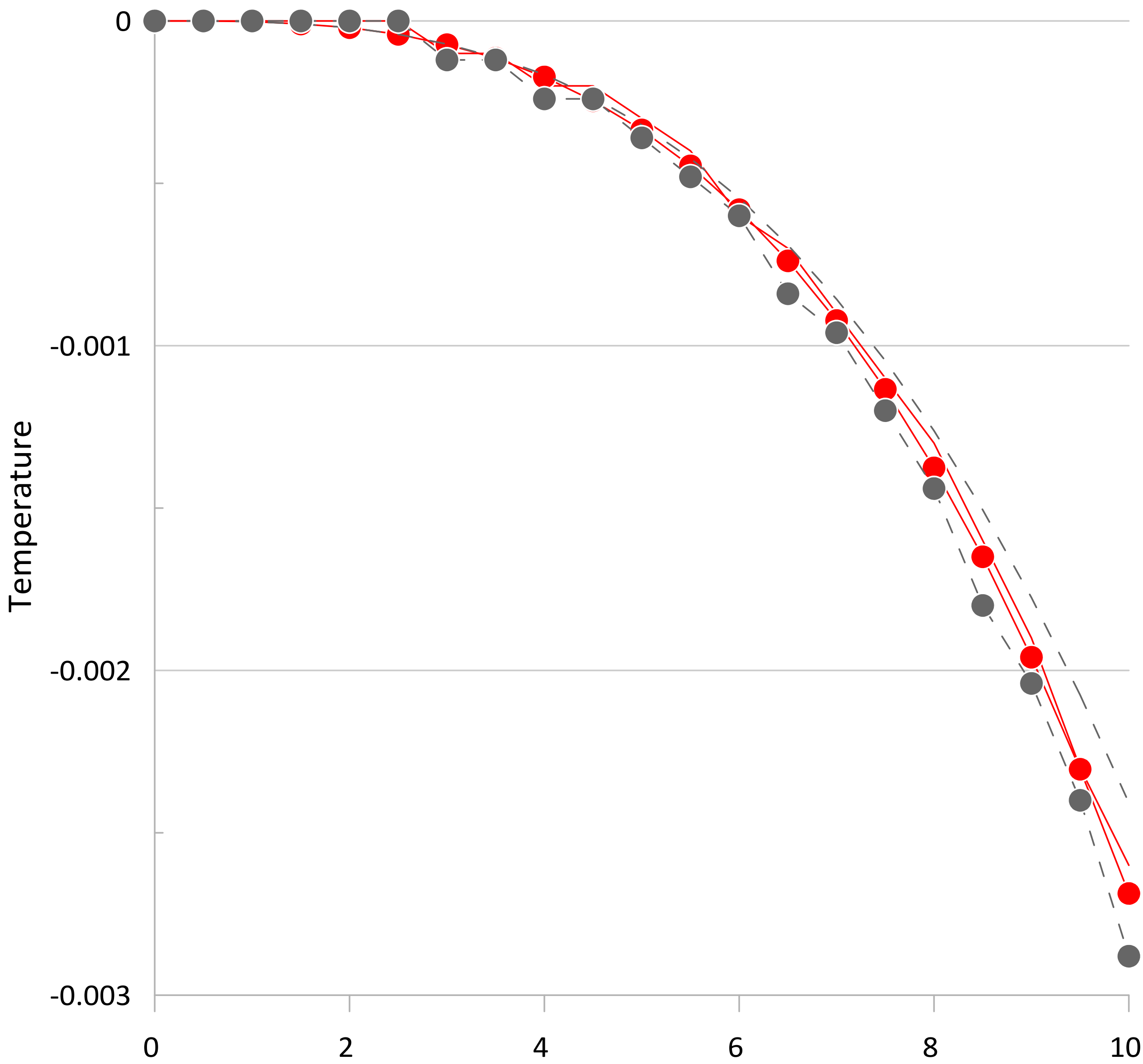

Figure 10. Variation of temperature distributior T^* with distance x.

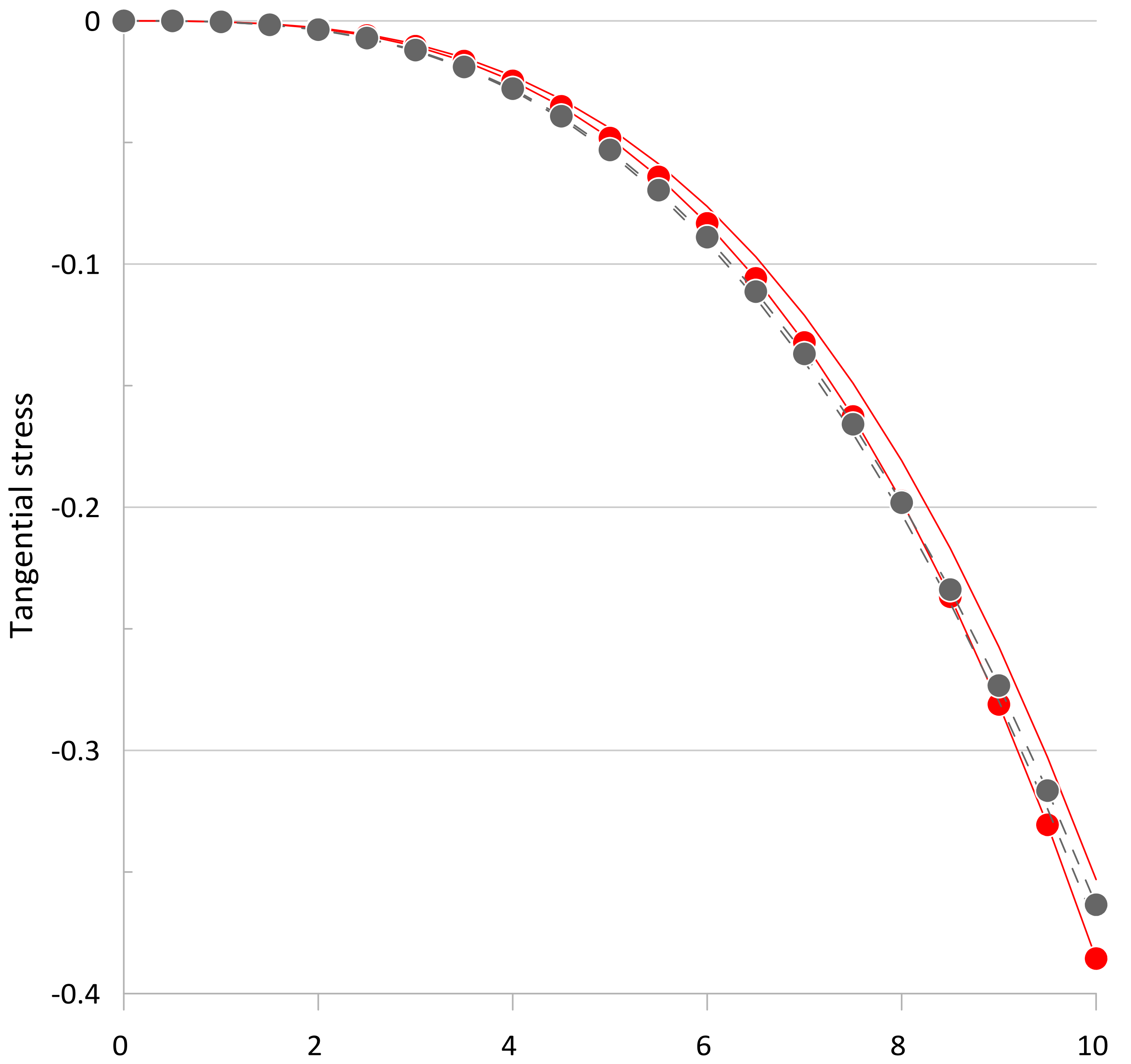

Figure 11. The variation of tangential stress σ_{31}^{*} with distance x .

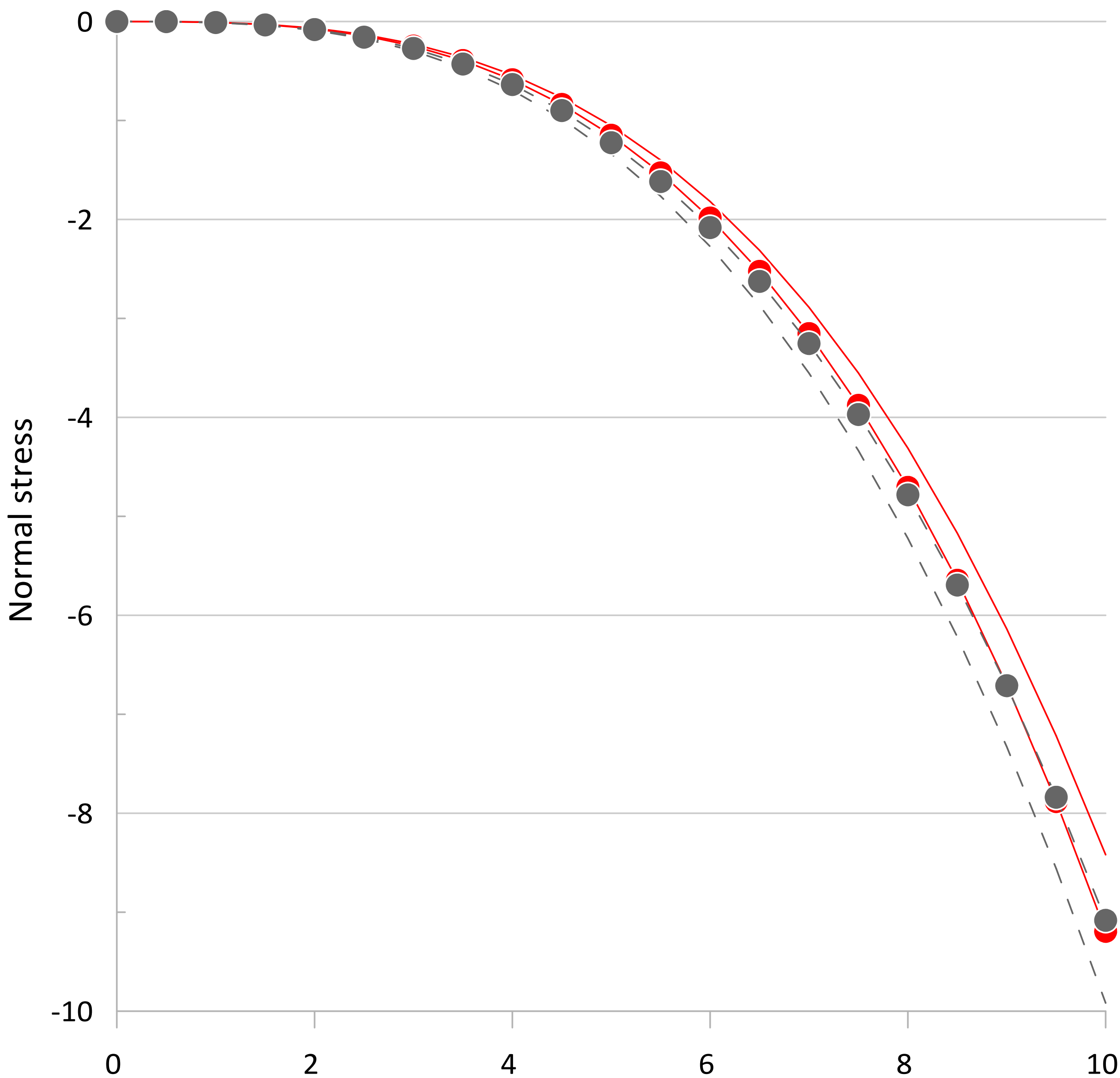

Figure 12. The variation of normal stress σ_{33}^{*} with distance x .

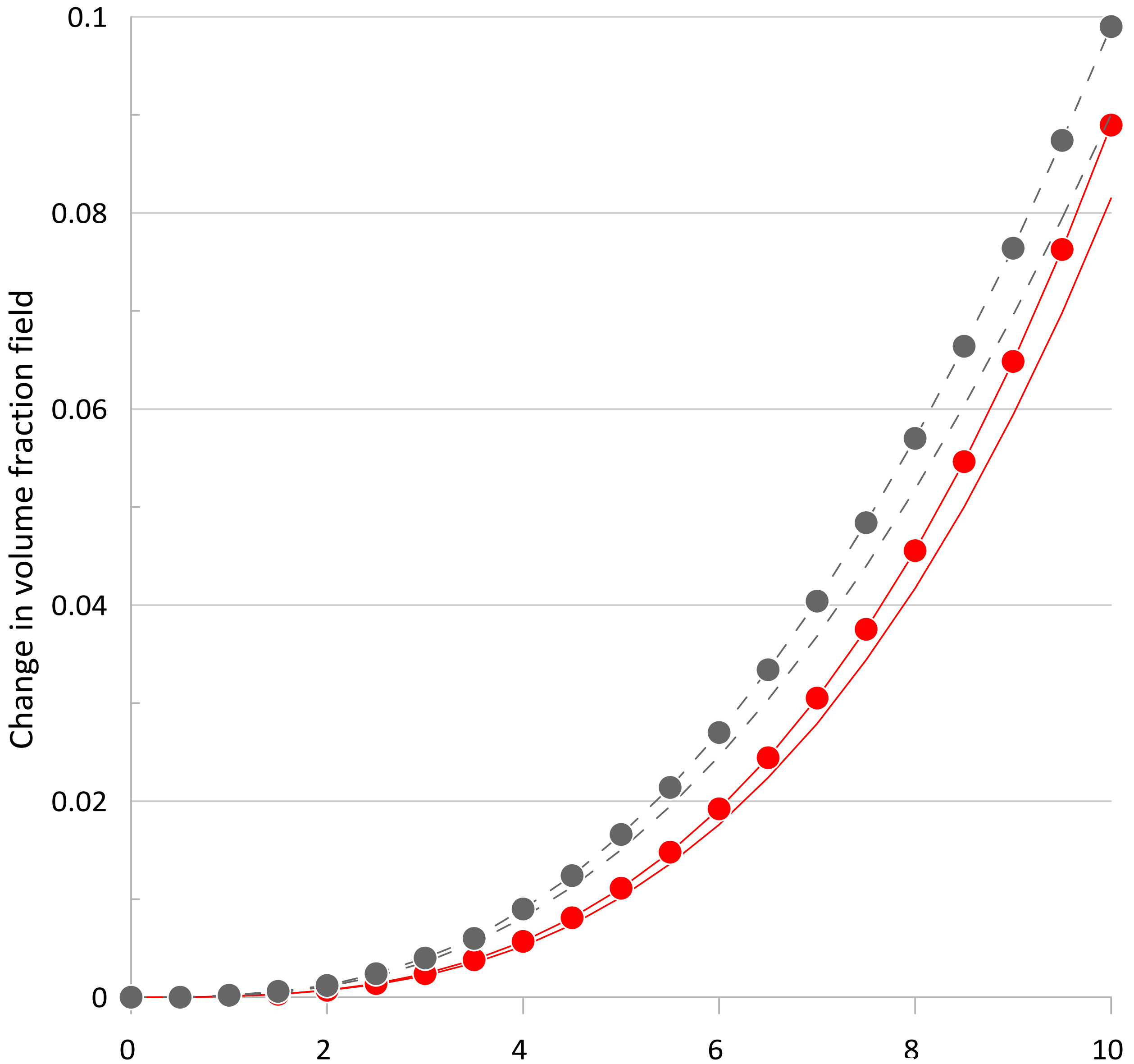

Figure13. Variation of change in volume fraction fielc φ^* with distance x.

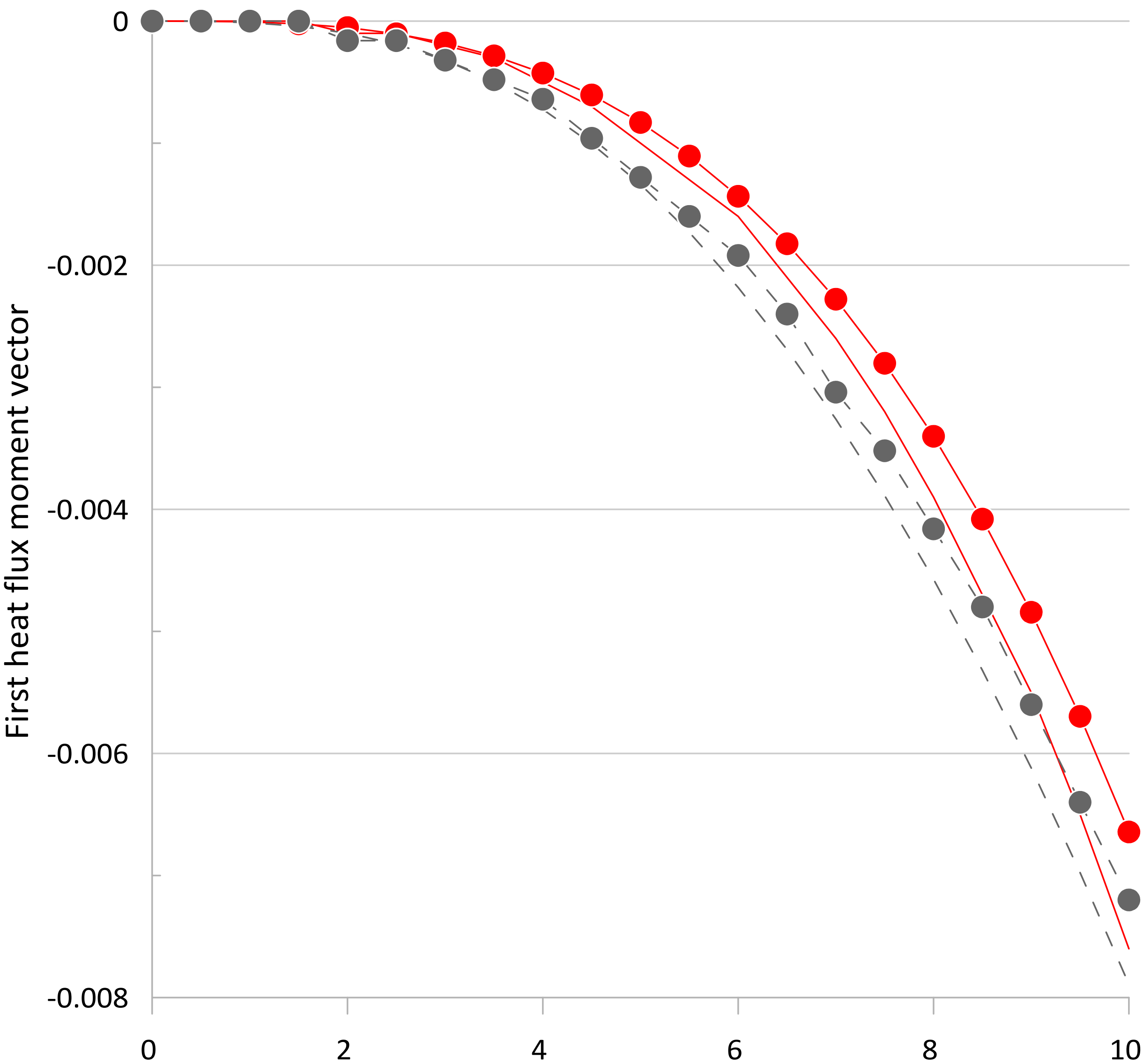

Figure14. Variation of the first heat flux moment vector q_{33}^{*} with distance x.