

# Effect of Fuel Injection Pulse and Dwell Period in Split Injections on the Performance and Emissions of a Diesel Engine

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## Abstract

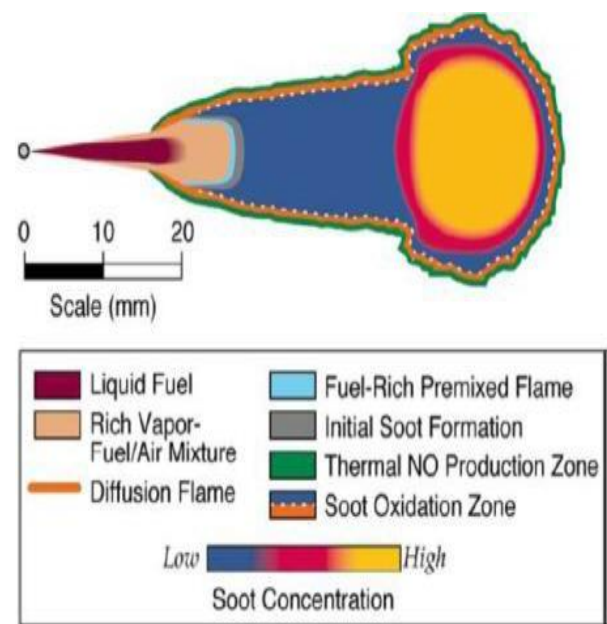
The diesel-fuelled engines are fuel efficient but are prone to high levels of NO<sub>x</sub> and soot emission. It has become easier to have precise control over fuel injection parameters with electronics-embedded diesel engine components mostly available on the common rail injection system of multi-cylinder engines. In the present competitive market, to utilize and continue the usage of diesel-fuelled engines efficiently, an attempt has been made with an in-house developed computational code to employ a split or multiple injection strategies for a direct injection single-cylinder diesel engine to obtain lower harmful emissions with appreciable engine performance. A quasi-dimensional model involving the engine parameters; equivalence ratio, injection timing, exhaust gas recirculation, pilot injection timing and its quantity. In addition, injection pulses with two different dwell periods are considered. The capabilities of the quasi-dimensional model have been verified and the trends are observed to be in line with the literature. The results of numerical experiments considering split injections with a variation of fuel quantities in the first pulse [10% -75%] with two dwell periods [8°CA and 3°CA] in conjunction with moderate boost pressures [10% to 30%] resulted in effective NO<sub>x</sub>-soot-piston work trade-off. Injection strategies with lower pilot quantity and larger dwell have resulted in lower NO<sub>x</sub> emissions and larger pilot quantity with retarded main injection timing has effectively addressed perennial NO<sub>x</sub>-soot trade-offs and would effectively address stringent emission norms.

**Keywords:** Direct Injection Diesel engine, Fuel injection parameters, Split injections, Boost pressure, NO<sub>x</sub>, Soot

## 1. INTRODUCTION

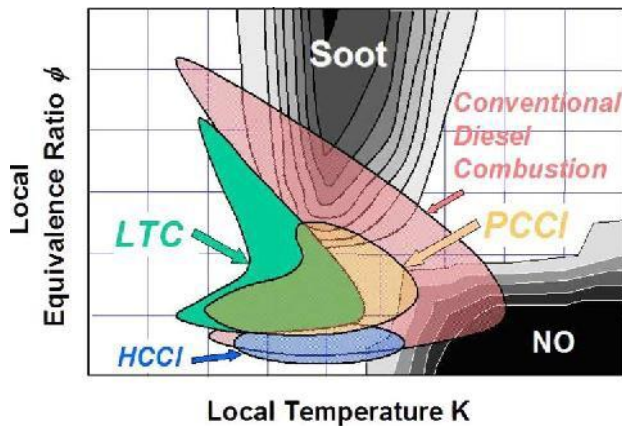
The diesel-fuelled compression ignition engines are fuel efficient relative to its counterpart-gasoline engines due to high compression ratio adaptability and hence preferred in all transportation applications. Fundamentally, the nature of combustion process is three dimensional, unsteady, diffusive and heterogeneous with typical timing ranges between 4 to 48 milliseconds depending on engine speed and thus is prone for release of high levels of tail-pipe obnoxious emissions such as NO<sub>x</sub> and PM or soot (Borman and Ragland 1998). The emission formation mechanism of NO<sub>x</sub> and PM are opposite to each

other and thus makes a tough task to control or simultaneous reduction of these two pollutant emissions and achieving NO<sub>x</sub>-PM trade off (Heywood 2018). The NO<sub>x</sub> emissions are both kinetically controlled thermally prompt, as depicted Fig.1 (Dec 1997). The NO<sub>x</sub> emissions are cyclically formed in regions of high temperatures and lean mixtures and the dominant during non-equilibrium conditions prevailing inside the combustion chamber.



**Figure.1.** Regimes in a diesel fuel jet under quasi-steady state.

From the days of its inception, diesel engines have undergone a multitude of developments. Many efforts have been made to mitigate engine out emission adopting alternative fuels; CNG, alcohols, hydrogen, biodiesel etc. and also with the alternative combustion processes that falls broadly under the category of low temperature combustion concept [LTC] viz; HCCI, PCCI and RCCI, resulting simultaneous reduction of NO<sub>x</sub> and PM emissions with each of these concepts has its own advantages and limitations (Ravichettu, Rao, and K 2017; A.K. Agarwal 2007; Rao and Sharma 2020).



**Figure. 2.** Regimes of combustion zones and emissions (Wang 2008).

Diesel engine performance and emissions are greatly affected by its geometrical and operating parameters. LTC conditions are also obtained through partial exhaust gas recirculation employed to lessen NO<sub>x</sub> emissions. EGR use though reduces NO<sub>x</sub> emissions, but enhances the soot formation thereby deteriorating the fuel economy, engine oil quality and hence reduces overall performance (Zheng, Reader, and Hawley 2004; D. Agarwal, Singh, and Agarwal 2011). The adoption of CRDI in multi-cylinder passenger vehicle segment, facilitates a precise control over fuel injection timing, pressure, quantity with reduced engine noise, emissions and improved performance. However, use of high compression ratio, advancing fuel injection timing, supercharging and increased charge temperature etc. are being predominantly used in diesel engines to for improved performance but results in a sharp increase in NO<sub>x</sub> emissions (Kouremenos et al. 2001; Zeng et al. 2006).

### 1.1. INJECTION MODES

In a conventional diesel engine, the load/speed control is done by varying the amount of fuel to be injected as per requirement and hence fuel injection parameters are pertinent for the regulation of performance as well as emissions. The development of CRDI system has provided the flexibility to alter timing of fuel delivery, duration of fuel injection in each pulse, etc and yields improved engine performance, lower emissions and associated low combustion noise levels of the engine. Additionally, dividing a single injection into number of injections over a single combustion event is proved to effectively address the perennial issue of NO<sub>x</sub>-soot trade-off (d'Ambrosio and Ferrari 2015).

Pilot injections, split injections and/or multiple injections are a part of multiple injection system. In pilot injection [sub-injection], fuel less than approximately 10%-15% of the total amount is injected initially at early combustion stages. Split injections refer to injections wherein fuel is supplied in varying proportions in different pulses. Duration in crank angle between two or more varying fuel injection pulses referred as dwell. In multiple injections double or more injections with different proportions of fuel quantities injected in each pulse with varying dwell periods between them. Potential benefits of split injections lie in the strong air-fuel mixing because of the definite dwell period between the injection pulses.

### 1.2. SPLIT INJECTIONS

An injection strategy with unequal fuel quantities injected over a single combustion event is termed as split injection. Splitting the fuelling process into two stages within a single cycle can significantly reduce combustion noise, peak cylinder pressure and engine-out emissions.

Experiments on a single cylinder diesel engine are done that incorporated electronically controlled with common rail direct injection system to render the flexibility to modify fuel delivery timings, pilot injection rate shaping and split injections. With an implementation of split injections, peak in-cylinder pressures reduced by around 45% when compared to conventional injection. The reduced fuel quantity significantly lessen NO<sub>x</sub> emissions in the first pulse of split injections without an increase soot (Nehmer and Reitz 1994). A phenomenological combustion model is proposed to predict NO<sub>x</sub> and soot emissions of a DI diesel engine operating at 1500rpm and at two engine loads; 50% and 100%. The optimum dwell was found to be 24°CA for reducing NO<sub>x</sub> and soot emissions at 100% load conditions, whereas at 50% load, 24°CA dwell has shown same amount of NO<sub>x</sub> and soot emissions as single injection when retarded. It was also found that when an amount of 10%-30% of the total fuel is injected in the second injection pulse with second pulse initiated at around 15°aTDC has significantly reduced soot emissions with no great increase in NO<sub>x</sub> emissions (Li et al. 1996). Split proportions of 40-60, 35-65, 30-70, 25-75 and 20-80 are tried with different crank angle intervals of 1, 2, 3, 4 and 5 between the main injection and sub-injection in predicting emissions particularly of NO emissions. The optimized split proportion was found to be 30-70 with a gap of 5°CA with lesser emissions (A.K. Babu and Devaradjane 2001). Experimental work with pilot injection implemented on a high-speed DI diesel engine revealed that engine-out emissions, combustion characteristics and performance of the engine are improved. Advancing the pilot amount fuel delivery timing was observed to reduce combustion noise at full load and low speed conditions. Further splitting the pilot injections into many smaller injections resulted in greater reduction of combustion noise whilst lowering BSFC rate and HC emissions (Hotta et al. 2005). Strategies involving optimization of PM, BSFC and NO<sub>x</sub> emissions and use of EGR reduced the NO<sub>x</sub> levels further but increased the PM emissions. Implementation of four injections followed by an after-injection resulted in lower emissions of PM emissions but increased the NO<sub>x</sub> emissions (Hardy and Reitz 2006).

The multi-dimensional KIVA code is utilized to optimize operating parameters of the engine. At medium load and high-speed operation of the engine, two stage combustion [TSC] concept was investigated. First stage of this was homogeneous charge combustion ignition [HCCI] and second stage was mixed-controlled burning of fuel at low oxygen levels of charge and high temperatures. The combustion stages were achieved using multiple injections. It is noted that with a combination of medium EGR rate, late inlet valve closing, retarded injection timing of second injection resulted in reduced emissions (Sun and Reitz 2006). Experimental studies involving pilot injection quantity and timing concluded that increase in pilot quantity and advancement of pilot injection will be helpful in reducing the harmful levels of emissions. Also, further dividing pilot

quantity into number of injections is useful in circumventing wall wetting of the cylinder (Okude et al. 2007).

It is reported the impact of splitting a single injection into two, three and four injections on engine performance and emissions. Tests were conducted on a research high speed direct injection diesel engine with single cylinder (Ehleskog, Ochoterena, and Andersson 2007). The effect of multiple injections is investigated on a low compression ratio diesel engine with high EGR rate. It is inferred that multiple injections can be better used to control spatial fuel distribution which improves air mixing aiding in the reduction of PM, potentially allowing higher rates of EGR (Mendez and Thirouard 2008).

Split injections on a heavy-duty diesel engine are implemented and at high EGR rates, and medium load operation, effects of variation in fuel mass injected, timing of injection was studied on engine performance, emissions and combustion noise. It was concluded that optimized triple injection strategy has resulted in lower soot and NO<sub>x</sub> emissions at medium load operation of the engine (Helmantel and Golovitchev 2009). Low temperature combustion [LTC] strategies are implemented on engines and it is observed that rate of pressure rise and NO<sub>x</sub> emissions showed similar trends with changing injection timings and dilution rates and reverse to that of HC and CO emissions. It is concluded that double injections resulted in lowered rise of pressure in combustion chamber while maintaining lower emissions (Horibe et al. 2009). The effects of split injections in combination is experimentally investigated with high EGR rates on a single cylinder engine's performance, combustion and emission characteristics. To implement split injections, engine was equipped with high pressure common rail direct injection system. It is noted that as the dwell angle increased, soot and UHC emissions decreased by about 70% and 50% respectively because of the improved fuel-air mixing and hence improving combustion process with almost similar IMEP values for advanced injection timings (Diez and Zhao 2010).

The effect of split injections is investigated in combination with variable injection pressures on NO<sub>x</sub> and soot emissions in a DI diesel engine using a 3D CFD code KIVA-3V model. The split injections with higher proportion of fuel [75%, 90% etc.] in the first injection maintained the engine power close to that of single injection. With the above injection schemes and a dwell period of 35°CA between the injection pulses, soot and NO<sub>x</sub> are found to be minimal. Soot and NO<sub>x</sub> were found to increase with an increase in the dwell period above 35°CA. Also, using an injection pressure of 1650 bar in the second injection pulse decreased soot emission by 65% with an ignorable decrease of 2% in NO<sub>x</sub> (Poorghasemi et al. 2012). Experimental studies are done with cooled EGR and high injection pressure split injections and single injections. Lower NO<sub>x</sub> levels with 10% EGR at maximum FIP, whereas with split injections, lower FIPs are required to obtain lower NO<sub>x</sub> at the same EGR rates (Edara et al. 2018).

Experiments with a blend of 20% lemon peel oil biodiesel and diesel are conducted under high injection pressures. It is noted that other than NO<sub>x</sub> emissions are greatly reduced with higher proportion of initial pulse. NO<sub>x</sub> levels have been reduced at lower pilot injection rate [10%] and low fuel injection pressures

(Ashok et al. 2018). Comparative studies are carried out with single and split injections and its effects on emissions. They varied injection pressure and injection timing independently. With split injection scheme, improved BSFC, emission levels with a short injection interval and injection timing close to TDC, with moderately high injection pressures (Park et al. 2018). Optical studies are performed to understand the characteristic flow and emission formation behaviour with split injection schemes both in non-reacting and reacting cases. It is observed that lower ignition delays under second injection with different behaviour of soot formation and higher amounts soot during shorter dwell period and longer pilot injection (Desantes et al. 2018).

Noise-cancelling peaks are observed between initial and other pulses when split injections are employed in a diesel engine (Fuyuto and Taki 2019). Experimental studies conducted on a single cylinder engine incorporated with CRDI systems with electronics operated injection kit. Compared to single injection, along with split injections, shorter combustion period, shorter ignition delay with reduced EGTs are obtained at increased EGR rates and smaller combustion duration, ignition delay relative to conventional techniques of fuel injection (Edara, Y.Murthy, and Nair 2019).

An investigation on engine for performance and stability with biodiesel-CNG/ethanol fuelled engines has been carried out. The authors have adopted response surface methodology [RSM] and estimated the importance of parameters influencing with split injection strategy. It is inferred that the strategy is superior in obtaining better performance with lower emissions with biodiesel fuelled engines also (Biswas et al. 2022). With split injection system, the influence of initial fuel timing and quantity on the performance of a CRDI diesel engine fuelled with micro-emulsions of diesel-palm oil-ethanol is studied. The study revealed a significant effect on NO<sub>x</sub> and modest effect on particulate number with varying PIT and PIQ. They observed no effect that micro-emulsions along with injection techniques would help in simultaneous reduction of both NO<sub>x</sub> and particulate matter [PM] in a diesel engine (Qi et al. 2022).

The influence of split injection, injection timing and engine speed, and HRRs at a given test condition is experimentally investigated. They observed that with increase in engine speed, high NO<sub>x</sub> levels, reduced ignition delay and increased peak ROHR values. Also, they studied the effect of injection fuel mass on the soot concentration and noted that with advanced injection timing, soot increased and the ultrafine particles increased with increased first injection (Yoon and Park 2022).

Both experiments and numerical work on a dual-fuel engine with natural gas as fuel were carried out. With advanced injection, lower HC emissions were noted. Also, with split injection better oxidation of methane was obtained (Altinkurt et al. 2023). The response surface methodology [RSM] and Bayesian neural network [BNN] are used for estimation and optimization of parameters affecting diesel engine performance and emissions fuelled with waste cooking oil biodiesel and diesel fuel blend. It was inferred that biocatalyst would be good in the synthesis of high yields of biodiesel and noted that ANN and RSM schemes are useful in accurate prediction of diesel engine performance (D. Babu et al. 2023).

A machine learning approach has been used for predicting the performance with prognostic model using Artificial Neural Network [ANN] linked with RSM approach is developed to estimate and optimize engine performance and emission parameters of a dual fuel engine fuelled with biodiesel prepared from biogas produced from waste and algae. They also validated the predicted results with experimental data. It is concluded that ANN-linked RSM is a better hybrid scheme for such studies (Sharma et al. 2023).

With the penetration of electronics into automotive engines in the form of sensors and actuators, the regulation of fuel has become more user friendly. In majority of the cases, the use of split/multiple injections has become more popular in multi-cylinder engines. Modern automobiles are now-a-days being provided with CRDI systems that facilitate multiple [split] injection strategies. Such systems enable to precisely control the injection duration, rate, etc. that in-turn affect NO<sub>x</sub> and PM emissions. Therefore, an attempt has been made to implement a split injection strategy for single cylinder diesel engines. To study its efficacy, numerical experiments have been performed by developing the in-house computer code. In the present work, split injections with variation of fuel quantities in first pulse [10% -75%] with two dwell periods [8°CA and 3°CA] in conjunction with moderate boost pressures [10% to 30%] has resulted in effectively achieving NO<sub>x</sub>-piston work trade-off. Injection strategies with lower pilot quantity and larger dwell have resulted in lower NO<sub>x</sub> emissions and larger pilot quantity with retarded main injection timing has effectively addressed perennial NO<sub>x</sub>-soot trade-off.

## 2. METHODOLOGY

The work is aimed at to identify the strategies minimizing the trade-offs between piston work [i.e. gross indicated work], NO<sub>x</sub> and soot emissions and to achieve the target, the following objectives are set-up.

1. To explore the possibility of split injection strategy implementation on a single cylinder engine for predicting the behaviour of heat release rate, in-cylinder pressure and temperature and emissions.
2. To study the combined effect of fuel injection schemes [variable fuel and variable dwell] and other operating parameters such as injection timing, cooled exhaust gas recirculation, boost pressure, etc. on engine emissions and performance.
3. To obtain NO<sub>x</sub>-soot trade-off using combined strategies of fuel injection, EGR and boost pressures.
4. To establish superiority of split injection strategy over conventional techniques for mitigation of emissions.

The chosen DI diesel engine is of quiescent combustion chamber with hemispherical piston bowl and study is carried out assuming a closed cycle, considering both the valves closed. While developing a model, it is accounted for different sub-models that facilitate estimation of main processes from IVC to EVO. Three different control volumes are used in the model to represent intake manifold, exhaust manifold and combustion chamber as shown in Fig. 3. The present section develops and details the fundamentals and mathematical equations governing

the processes in a CI engine. In this regard, control volumes are considered as shown in Fig.3 which illustrate the energy and mass interactions with one another.

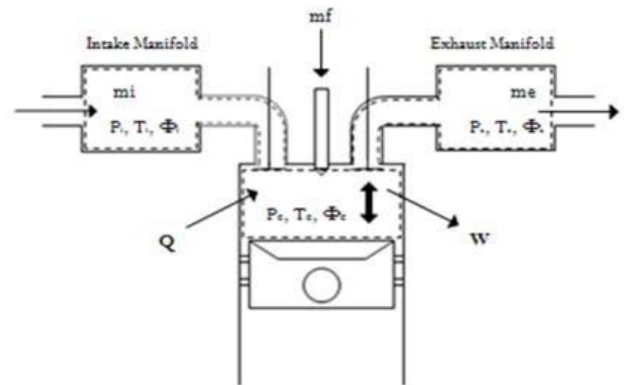


Figure. 3. Schematic of engine control volumes

### 2.1 Model Assumptions

The cylinder with gas mixture of clean air, vaporized fuel, burnt gases, constant temperatures of cylinder head, cylinder walls, piston crown and the requisite its thermodynamic properties are estimated assuming perfect gas law and temperature dependent specific heats and with uniform cylinder pressure and temperature but vary crank angle.

### 2.2 Formulation of Computational Procedure

The numerical computations are developed for various sub-models of engine processes. The numerical simulations have done for the following sub-models to obtain data of in-cylinder pressure, temperature and emissions with respect to crank angle.

#### 2.2.1 Sub-model for Engine Dynamics

Considering simple piston-cylinder arrangement, as depicted in Fig. 4, equations available for calculation of time dependent stroke, change of volume with crankangle, relations of slider crank mechanism are used and time dependent volume and derived the time dependent volume expressed in radians derived from crankangle values (Heywood 2018).

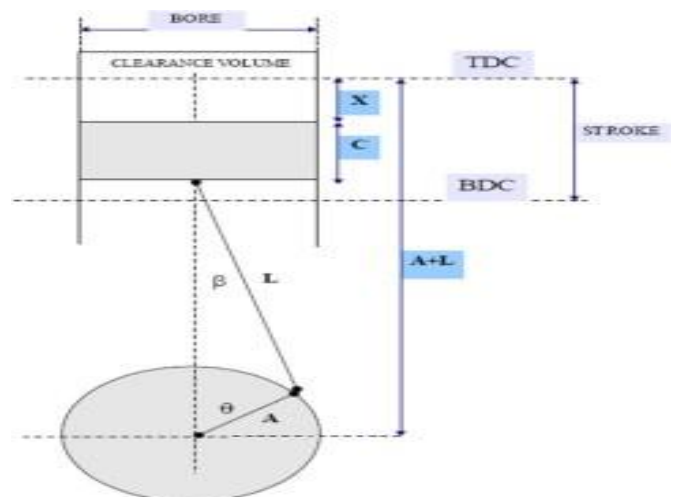


Figure. 4. Engine piston-cylinder geometry

The cylinder geometry is used for calculating instantaneous stroke, area, cylinder volume with crank angle.

$$\frac{dV}{dt} = 3V_d N \left[ \sin \frac{\pi N}{30} t + \frac{\sin \frac{\pi N}{30} t \cos \frac{\pi N}{30} t}{\left(r^2 - \sin^2 \frac{\pi N}{30} t\right)^{\frac{1}{2}}} \right] \quad [1]$$

### 2.2.2. Sub-model for Gas Exchange: Flow Rate through Valves

The respective equations for gas exchange phenomena are used as detailed below, using mass flow rate through a restriction, as depicted in Fig. 5.

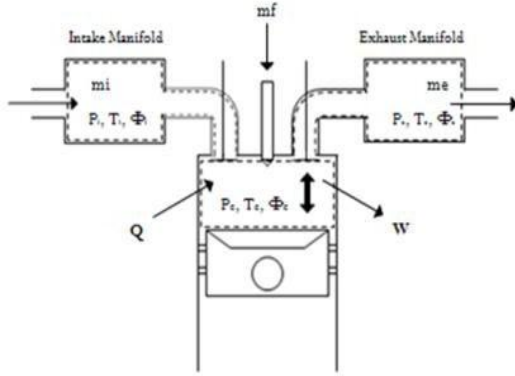


Figure 5. Flow phenomena of an engine

Making use of energy equations from first law of thermodynamic, the mass flow rate is calculated.

$$\frac{dM}{dt} = AP_{im} \sqrt{\left(\frac{2\gamma}{(\gamma-1)RT_{im}}\right) \left(\frac{P}{P_{im}}\right)^{\frac{2}{\gamma}} \left(1 - \left(\frac{P}{P_{im}}\right)^{\frac{\gamma-1}{\gamma}}\right)} \quad [2]$$

### 2.2.3. Sub-model for EGR Calculations

To allow a provision for the use of variations intake pressure and temperatures, the following equations are used with varying proportions of EGR.

$$T[0] = \frac{m_a T_a + m_{egr} T_{egr}}{(m_a + m_{egr})} \quad [3]$$

$$P[0]V[0] = (m_a + m_{egr})R_g T[0] \quad [4]$$

Mass of fuel injected per cycle is given by

$$\dot{m}_i = \rho_f A_{noz} u_{inj} \quad [5]$$

Where  $\rho_f$  (density of liquid fuel),  $A_{noz}$  (nozzle orifice area) and  $u_{inj}$  (injection velocity) given by

$$u_{inj} = C_D \sqrt{2\Delta P / \rho_f} \quad [6a]$$

Where  $\Delta P$  (pressure drop across nozzle) and  $C_D$  (discharge coefficient). Therefore, fuel injection rate is calculated as

Where

$$\dot{m}_i = C_D A_{noz} \sqrt{2\rho_f \Delta P} \quad [6b]$$

$$\left(\frac{dot; m_f}{m_{ft}}\right) = \left(\frac{\pi}{2\Delta\theta_{injection}}\right) \sin\left(\frac{\pi(\theta - \theta_0)}{\Delta\theta_{injection}}\right) dot; \theta \quad [6c]$$

### 2.2.4. Sub-model for Ignition Delay

Ignition delay [IGD] of a fuel is a characterise feature of CI engine fuel has pronouncing effect on engine combustion noise and emissions. It is estimated, at the start of fuel injection, model proposed by Hardenberg and Hase has been utilized along with fuel properties, mixture pressure, temperature and equivalence ratio (Hardenberg and Hase 1979).

$$IGD(deg) = (0.36 + 0.22\bar{S}_p) \exp \left[ E_a \left( \frac{1}{RT} - \frac{1}{17190} \right) + \left( \frac{21.2}{P - 12.4} \right)^{0.63} \right] \quad [7]$$

$$E_a = \frac{618840}{CN + 25} \quad [7a]$$

$$\int_{t_{inj}}^{t_{ign}} \frac{1}{\tau} dt = 1 \quad [7b]$$

### 2.2.5. Sub-model for Heat Transfer Analysis

Empirical expressions are adopted for modelling heat losses between hot gases and cylinder walls considering forced convective heat transfer (Woschni 1967).

$$Q_{loss} = \bar{h}_c A(\theta)_{cyl} (\bar{T}_{gas} - \bar{T}_{wall}) \quad [8]$$

$$A(\theta)_{cyl} = A_{ch} + A_{pc} + A_{lat}(\theta) \quad [8a]$$

$$\bar{h}_c = a D^{\bar{m}-1} P^{\bar{m}} T_g^{0.75-1.62\bar{m}} w^{\bar{m}} \quad [8b]$$

Where

$$w = \left[ C_{1w} \bar{S}_p + C_{2w} \frac{V_d T_{ref}}{P_{ref} V_{ref}} (P - P_m) \right] \quad [8c]$$

Process	C <sub>1</sub>	C <sub>2</sub>
Gas Exchange	6.18	0.0
Compression	2.28	0.0
Combustion and Expansion	2.28	0.000324

### 2.2.6. Sub-model for Heat Release Rates

The sub-model is a crucial one as it helps to arrive at the pressures and emissions from the combustion. The diesel fuel combustion process is associated with heterogeneities due to variations in fuel-air ratios and without external sources of ignition and for its prediction through the quasi-dimensional model four different Wiebe functions to predict heat release rates i.e., one for initial pulse, two for premixed and diffusion combustion phases and finally one after burning phase (Serrano et al. 2008). The different engine flow processes are illustrated in Fig. 6.

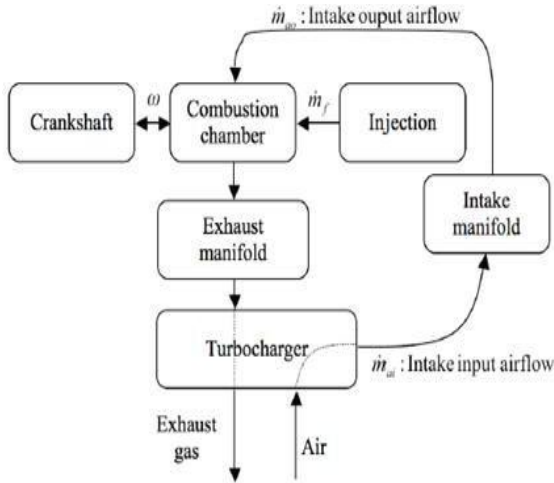
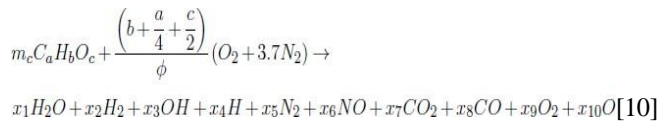


Figure 6. Depiction of engine flow processes

In addition to the above the equations related to heat transfer sub-models have been used further as provided (Ravichettu, Rao, and K 2017).

### 2.2.7 Chemistry of Combustion

The following expression is used for estimating burnt gas composition exited of a typical diesel fuel combustion with air.



### 2.2.8 Modelling emissions formation

#### 2.2.8 (i). NOx formation mechanism

For the purpose of NO formation estimation, kinetics model proposed by Lavoie et al., an extended Zeldovich mechanism has been employed, with following temperature dependent relations (Altinkurt et al. 2023).

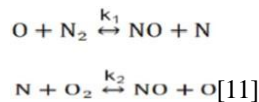


Table 1a and 1b-Engine specifications used for validation purpose.

Table 1a. Ricardo–Cussons Hydra engine (Rakopoulos et al. 2004)				Table 1b. Kirloskar AV1 model engine.			
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
Bore, mm X	80.226	Intake valve open, bTDC	8° CA	Bore, mm X	80 X 110	Intake valve open, bTDC	4.5° CA
Stroke, mm	X 88.90	Intake valve close, aBDC	42° CA	Stroke, mm		Intake valve close, aBDC	35.5° CA
Compression ratio	19.8:1	Exhaust valve open, bBDC	60° CA	Compression ratio	16.5:1	Exhaust valve open, bBDC	35.5° CA
Engine speed, rpm	1000-4500	Exhaust valve open, aTDC	12° CA	Engine speed	1500 rpm	Exhaust valve open, aTDC	4.5° CA
Static Fuel injection timing	0-40° CA bTDC			Static Fuel injection timing	23° CA bTDC		

### 3.1 Validation of the proposed model

The predicted values of cylinder pressure and emission values obtained from the model developed and practical two engines are compared in the following Figs.7a -7c.

$$\frac{d[NO]}{dt} = \frac{2R_1 \{1 - ([NO]/[NO]_e)^2\}}{1 + ([NO]/[NO]_e)(R_1/R_2)} [11a]$$

Where

$$R_1 = K_1^+ [O]_e [N_2]_e$$

$$R_2 = K_2^- [NO]_e [O]_e$$

$$K_1^+ = 7.6 \cdot 10^{13} \left( \frac{-38000}{T} \right) [11b]$$

$$K_2^- = 1.5 \cdot 10^9 \left( \frac{-19500}{T} \right) [11c]$$

#### 2.2.8 (ii).Soot formation mechanism [Two-rate Hiroyasu Model]

For the estimation of time variant net soot formation, models for, soot formation and soot oxidation, relations proposed by Lipkea and DeJoode[44] have been utilized;

$$\frac{dm_s}{dt} = \frac{dm_{sf}}{dt} - \frac{dm_{so}}{dt} [12]$$

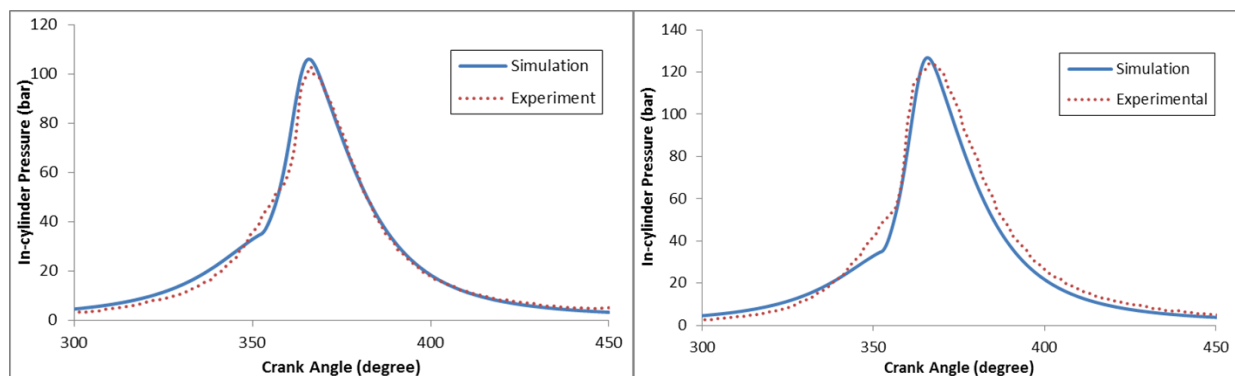
$$\frac{dm_{sf}}{dt} = 0.3m_{fb} \exp\left(\frac{-3000}{T_{cf}}\right) [12a]$$

$$\frac{dm_{so}}{dt} = \frac{0.4m_s \exp(-10000/T) \sqrt{P_{O_2}}}{6\rho_s d_s N} [12b]$$

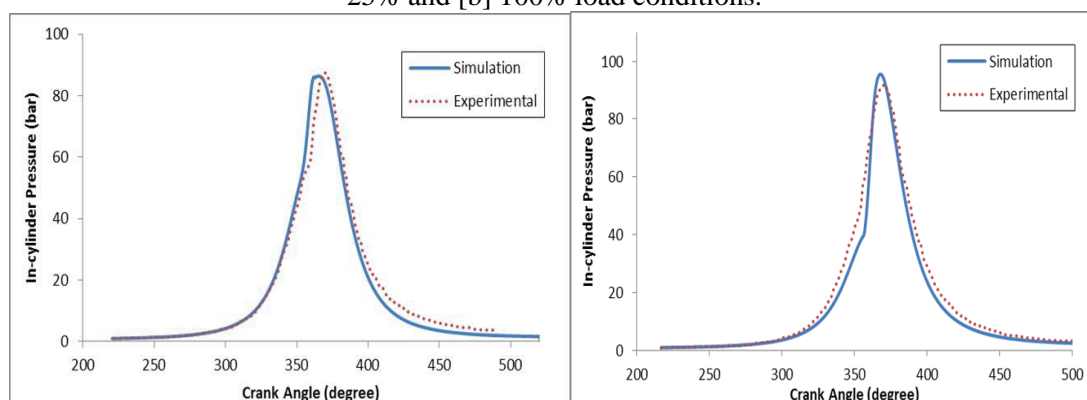
$$\frac{dm_{fb}}{dt} = \frac{m_{fb} \beta k_1 k_2 (1 - \hat{\theta}^{k_1})^{\beta} \hat{\theta}^{(k_1-1)} + k_3 k_4 (1 - \beta) \hat{\theta}^{(k_4-1)} \exp(-k_3 \hat{\theta}^{k_4})}{\Delta \theta_{comb}} [12c]$$

### 3. Computational Work

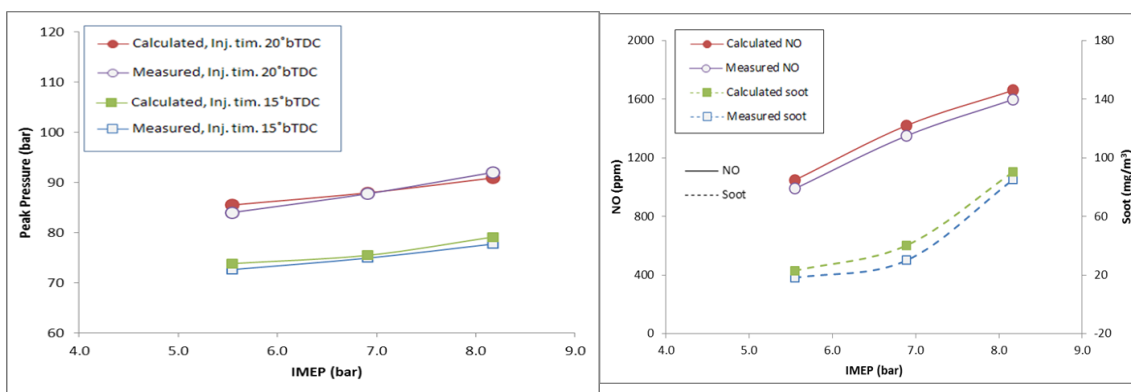
A phenomenological model (quasi-dimensional) is developed in C++ applying the all the above well-established empirical, semi-empirical and physical models for a typical DI diesel engine performance and emission analysis and implemented for predicting the behaviour of a single cylinder diesel engine and verifying the capability to predict the in-cylinder pressure and temperature histories over a series of working cycles and validated with two different engine configurations as given in the following Table 1a and 1b.



**Figure 7a.** Validation of predicted and experimental values for Kirloskar-AV1 model engine running at 1500rpm at [a] 25% and [b] 100% load conditions.



**Figure 7b.** Validation of predicted and experimental values for Ricardo-Hydra engine at [a] 2500rpm, 80% load and 20°bTDC and [b] 2000rpm, 75% load and 29°bTDC conditions (Rakopoulos et al. 2004).



**Figure 7c.** Validation between predicted and experimental results for [a] Peak combustion pressures and [b] NO and soot densities (Rakopoulos et al. 2004).

The computational work has been carried out with a fixed step, ode5 [Dormand-Prince] solver, and the computational time taken for single working cycle is observed to be less than a second.

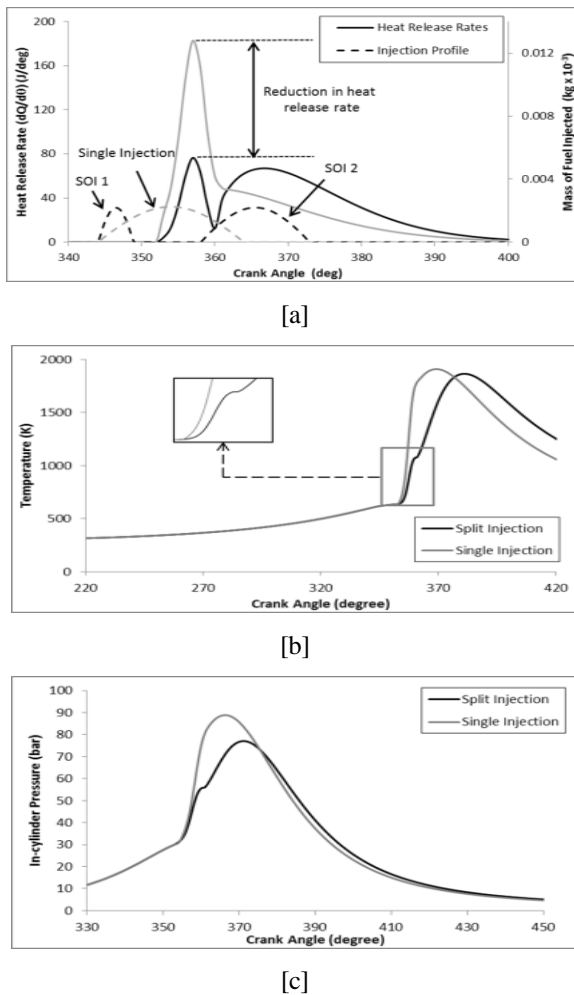
#### 4 Results and Discussion

Based on the numerical work as described in the methodology, the results are presented and discussed. Extensive numerical experiments are performed on the chosen engine configuration considering single fuel injection and split injection schemes [4]. The work has been carried out maintaining total fuel injection duration as 16°CA. Both single injection [conventional] and split injection have been incorporated. The HRRs, in-cylinder

pressures, NO and soot emissions are predicted with the help of in-house code developed based on phenomenological combustion model. While implementing the split injections the whole injection is divided into two pulses with different percentages of fuel in the first and second pulses, maintaining a constant dwell period between two pulses.

##### 4.1 Engine performance and emissions

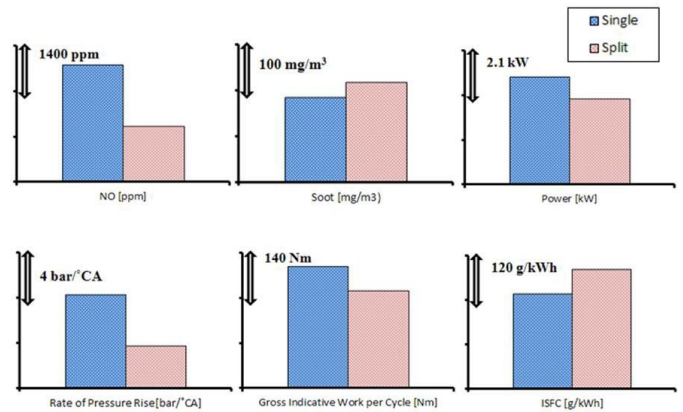
The predicted patterns of HRRs, in-cylinder temperatures and pressures are depicted in Fig. 8, for single and split injections for the same injection timing [rated timing 16°CA] with and for the fixed quantity of fuel injected.



**Figure. 8.** Patterns of HRR, in-cylinder temperature and pressures for single and split injections

SOI1 and SOI2 represent the two injection pulses respectively. As far as split injection scheme is concerned, the overall premixed phase of combustion is significantly reduced and the diffusion phase has been increased in comparison with conventional single injection. In split injection, considering a case of 10%-25% of the total fuel injected as first pulse, relatively lesser of fuel is injected. Since the fuel is divided in two pulses, a lesser amount is injection allowed during SOI1, the rise of heat is small. The smaller amount initially injected would help in combustion of next part of fuel by creating a favourable condition for the fuel to be fuel later, by appropriate reduction of ignition delay for the later part of fuel and thus leading to mixed-controlled combustion phase. With the splitting of fuel in small and larger pulses respectively reduced the peak heat release rate modestly and shifted the occurrence of the peak values a little away from TDC as seen in Fig. 8[a]. The in-cylinder temperature profiles are compared in Fig. 8[b] for split and single injections. In split scheme, as the first pulse quantity relatively small resulting in a lower rise pre-mixed phase of combustion with consequent lower as well as delayed peak in-cylinder temperatures in comparison with a single injection scheme. Conventionally, a large amount of gets accumulated during the ignition delay period and a little after

the fuel spontaneously burns with sudden rise in HRR and in-cylinder temperatures with a larger heat release (Heywood 2018). The smaller amounts of fuel being taken part in first stage of combustion with associated lower HRR, the in-cylinder pressures are lowered as can be observed from Fig. 8[c] probably resulting lower thermal and mechanical forces and hence lower combustion noise as well. The cumulative values of NO, soot, rate of pressure rise and piston work are compared as shown in Fig. 9, for both the single and split injection cases.



**Figure.9.** Comparison of parameter for single and split injections.

The theory of NO<sub>x</sub> formation suggests that greater the amount of fuel taking part in combustion in the pre-mixed phase, leading to higher temperatures and thereby produces higher levels NO<sub>x</sub> emissions and this has been evidenced in single injection as depicted in Fig. 9. However, the NO levels have been more than halved in case of split scheme.

As described, the split strategy with a constant dwell has given rise to lower first phase whereas the second phase has significant led to diffusion phase resulting greater soot emissions as obviously be seen in Fig. 9. A marginal reduction in piston work is also observed with split injection when compared to single injection due to delayed/retarded combustion in addition to effect of only a small amount initially and larger portion second pulse of fuel. A significant reduction in rate of pressure rise is observed with split injection thus reducing the associated combustion noise levels (Carlucci 2003), can be compromised with the substantial reduction in harmful NO levels than soot emission. It is thus inferred that the reduced amount of fuel injected initially reduces the NO<sub>x</sub> emissions and later injections decreases the soot emissions influencing both engine performance and emissions. Therefore, the amount of fuel injected varied in different proportions. Therefore, different injection schemes with varying dwell periods are designed and implemented, maintaining constant the total injection duration in crank angle, as illustrated in Fig.10. Hence meeting these requirements, a total of eight schemes designed. The percentage of fuel injected in the first pulse has been increased from 10% to 75% and two different dwell periods were considered [8°CA and 3°CA] between the injection pulses.



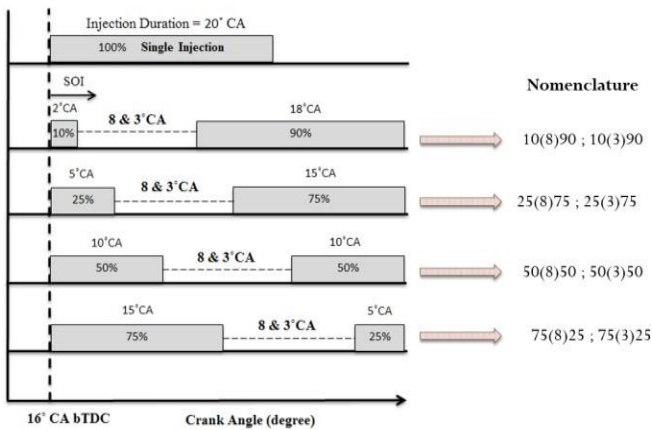


Figure 10. Different injection schemes

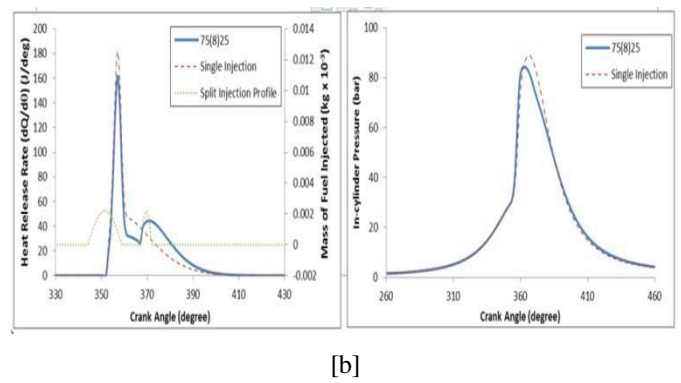


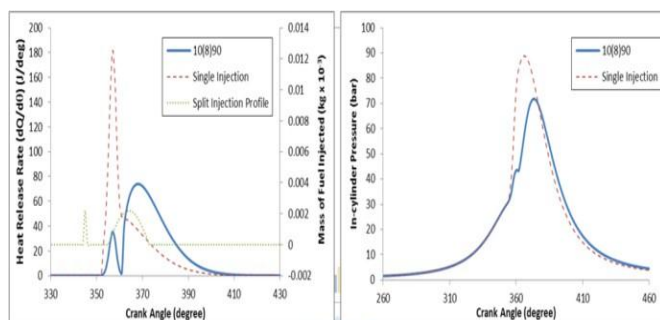
Figure 11. Predicted HRRs and in-cylinder pressures for 10[8]90 and 75[8]25 injection pulses respectively for a dwell period of 8°CA.

The percentage of fuel injected in the first pulse has been increased from 10% to 75% and two different dwell periods were considered [8°CA and 3°CA] between the injection pulses. The designated schemes are, for example, the 25 [8]75 injection scheme represents 25% of fuel injected in the first pulse and 75% of fuel injected in the second pulse. The number in the bracket represents the injection pause [or dwell period] between the injection pulses which is 8 °CA in this case. Therefore, a total of nine injection schemes are designed consisting of a single injection [16°bTDC] and 8 split injection schemes [10[8]90, 10[3]90, 25[8]75, 25[3]75, 50[8]50, 50[3]50, 75[8]25 and 75[3]25]. In all the cases, the SOI is considered as 16°bTDC, accounting to the full load equivalent of fuel as unchanged [36.84 mg/cycle] and 20°CA as the duration of fuel injection for all the schemes, however duration of each pulse varied as depicted in Fig.10.

#### 4.1.1 Engine performance a constant dwell period of 8 °CA

The Figs. 11[a and b] represent the predicted HRRs and in-cylinder pressures for 10[8]90 and 75[8]25 injection pulses respectively for a dwell period of 8°CA. The comparison is made in such a way that to observe the variation in first pulse and second pulse with constant dwell period and also corresponding plots obtained with single injection are superimposed.

The first pulse is varied from 10% to 75% and the amount of fuel injected in the second pulse thus is reduced proportionally from 90% to 25% respectively. Fig. 11 represents comparison of the HRR profiles injection and in-cylinder pressures for selected injection strategies.



[a]

The Figs. 11[a and b] depicts the traces of comparison of HRRs and in-cylinder pressures for split injection schemes of 10[8]90 and 75[8]25 respectively with corresponding plots of single injection compared. It is clear that the with increase in amount of fuel pulse in first phase, the heat release as well as peak pressures increased significantly for split injection scheme. By maintaining dwell of 8°CA, the peak values have moved away from TDC with values showing near to single injection. It can be noted that for a 10[8]90 scheme, the initial phase of peak in-cylinder pressure reduces indicating larger portion of heat release into the expansion stroke relative to single injection. Moreover, the area under P - θ diagrams increased yielding enhanced engine power. Also, the rate of pressure rise also increased with increase in premixed phase of combustion. The increase in fuel injected in the first pulse would lead to an increase in combustion noise. Therefore, a trade-off has been established between associated combustion noise and piston work of the engine with increased fuel amount injected in the first pulse. Fig.12 reflects such a trend of increasing rate of pressure rise and probably increased combustion noise with corresponding increase in engine power [29].

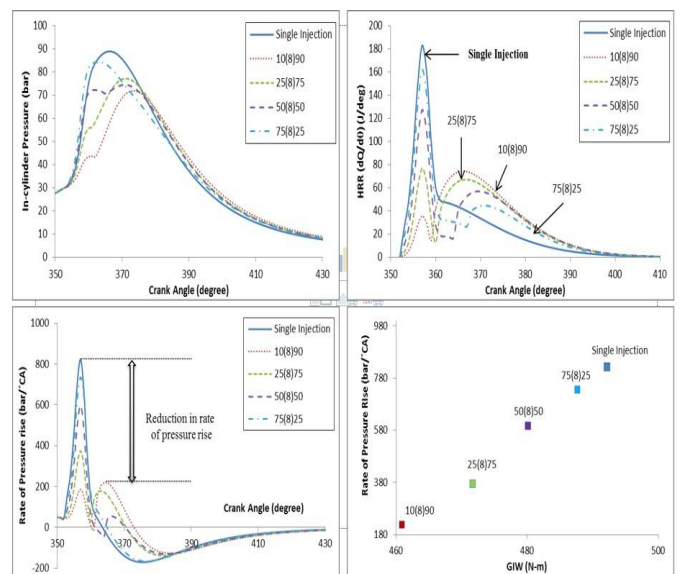


Figure 12. The traces of in-cylinder pressures, HRRs, rate of pressure rise and its trade-offs.

## 4.2 Engine performance and emissions with varying dwell period on the

### 4.2.1. Comparison of emissions with 8°CA as a dwell period

The NO and soot emissions formed during the combustion and expansion stages for different injection pulses have been compared in Fig. 13 as a function of crank angle. The split injections engine-out emissions have been significantly affected with split injections with lower values of NO for small portion of first pulse of fuel injection relative to single injection. On the contrary with increase in first pulse, the soot emission formation has been greatly reduced, with a shift in peak soot values towards far right of TDC position. The situation is associated with a moderate reduction in soot emission from baseline [single injection]. Therefore, it can be concluded that there is a simultaneous reduction in NO and soot emissions with split injection schemes.

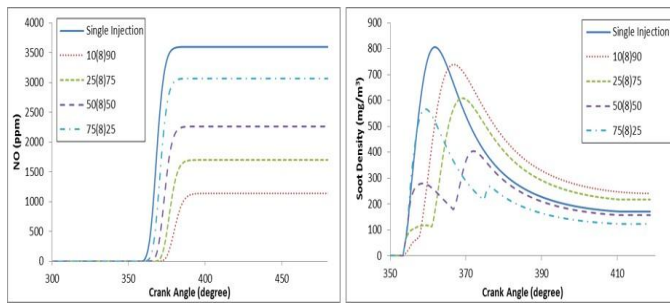


Figure.13. Predicted NO and soot formation histories for different split injection schemes.

The effect of dwell period under split injection is observed considering heat release rate profiles. Fig. 14 shows HRR profiles and NO formation rates of single injection, 75[8]25, 50[8]50, 25[8]75 and 10[8]90 compared with single injection case. It is noticed that as the fuel injected in the first pulse decreases from 100% to 75%, 50%, 25% and 10%, respectively, the peak HRRs, and NO emissions are reduced.

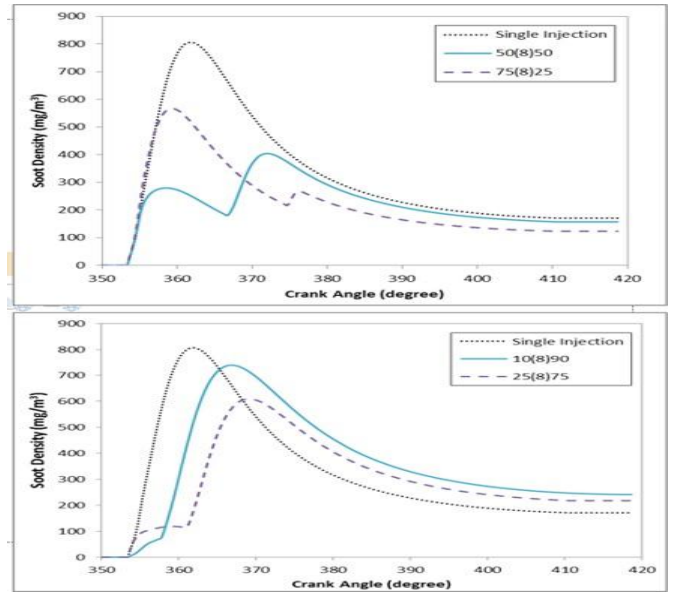
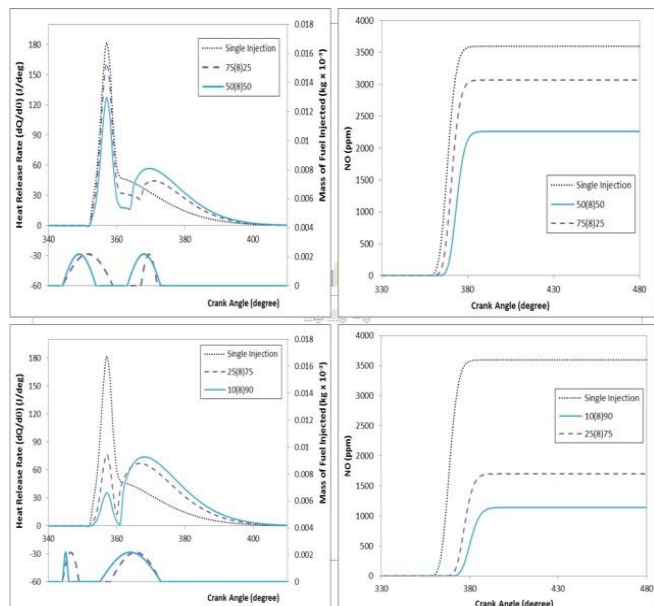


Figure. 14. HRRs, NO and soot formation histories for different injection schemes.

From Fig.14, it can be obviously noted that premixed combustion phase of combustion increased respectively from 10[8]90, 25[8]75, 50[8]50 to 75[8]25 sequentially. The effect of dwell period can be compared for two first pulses of 10[8]90 or 25[8]75 and it can be noticed that there is a delayed/retarded equivalent to a dwell angle resembling retarded combustion leading to lower NO emission formation. As far as higher first pulses of fuel such as in 75[8]25 case, the NO emissions are comparable with conventional single injection. Fig. 15 depicts soot emission profiles for the considered injection schemes. The values obtained in different injection schemes for NO and soot formation are compared for a dwell of 8°CA. As described for small quantities of first pulses, the NO levels have been seen to lower with corresponding higher values for soot emission. By reducing second pulse of fuel, the soot values have been reduced substantially and with increased second pulse, the combustion has been moved into diffusion phase, even maintaining lower relative to single injection, resulting enhanced diffusion combustion phase with higher soot formation, due to combustion of rich mixture.

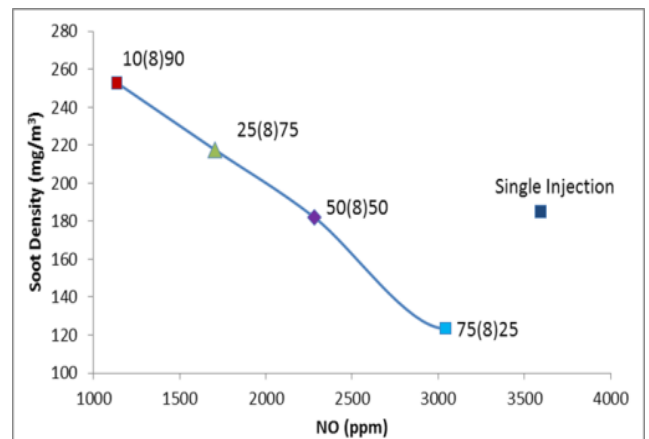
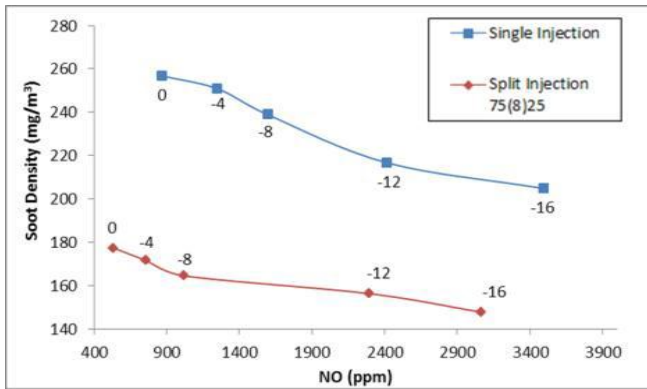


Figure. 15. NO-soot trends for the different injection schemes.

From the Fig. 15, a NO-soot trade-off can be inferred and it is establishing a fact that, split injections will have a potential to regulate the emission release as per the requirement.

To explore the influence of split injection schemes, a case of 75[8]25 injection has been considered and the values of NO and soot are replotted in Fig. 16, while taking the values obtained with retarded fuel injection timing. The plot is aimed at to compare the superiority of split injection in comparison with single injection, as retarded injection scheme is more commonly adopted with a view to reduce NO emissions. It is inferred that the split scheme, along with retarded fuel injection timing, is favourable with better trade-off between NO-soot emissions, over single injection scheme.

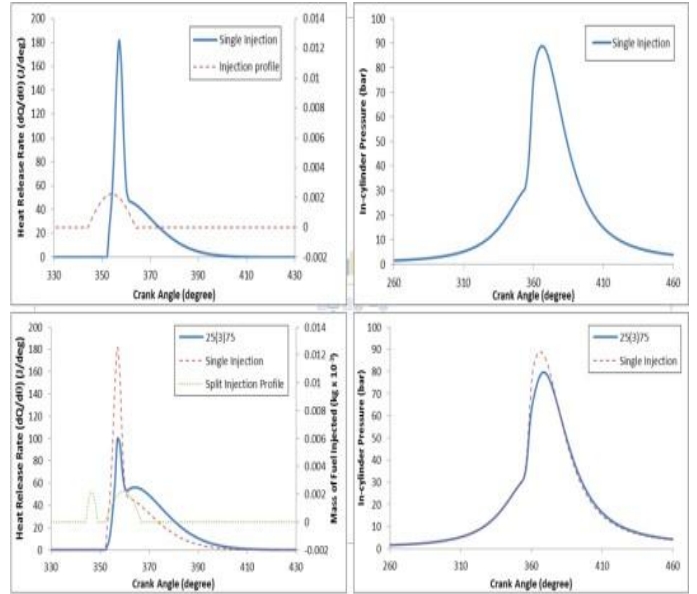
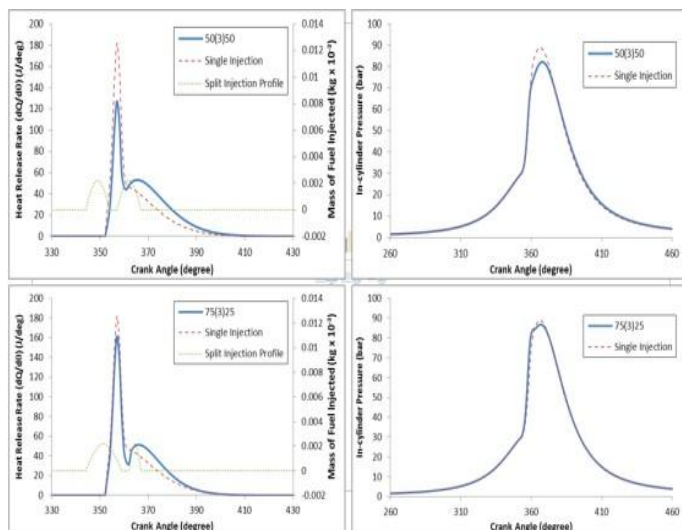


**Figure 16.** NO-soot trade-off trends with retarded injection timings for single and split injections.

Numbers in the graph indicate injection timing [aTDC].

#### 4.2.2. Comparison of emissions for a dwell period of 3 °CA

To investigate the effect of split injection, pause between injection pulses, dwell period has been fixed at 3°CA for all the injection schemes considered. The injection schemes considered are single injection [16°bTDC] and 75[3]25, 50[3]50 and 25[3]75 respectively. Fig. 20 shows HRRs and in-cylinder pressures chosen split injection schemes plotted and compared against single injection.

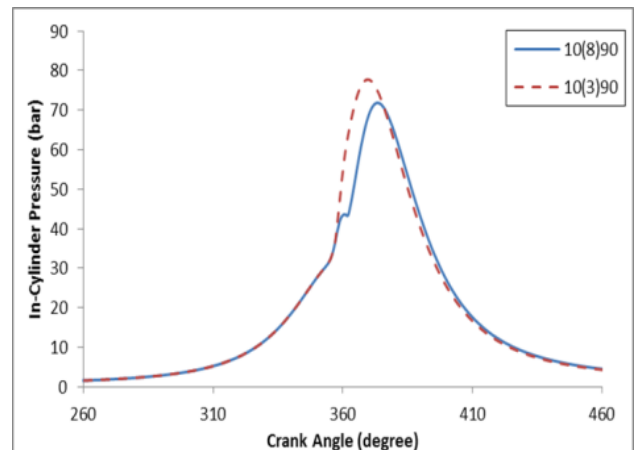


**Figure. 17.** HRRs and cylinder pressures prediction for split injections with 3°CA as dwell period.

From Fig. 20, it can be observed that irrespective of the dwell period between injection pulses, premixed combustion phase increases with increase in amount of fuel injected in the first pulse.

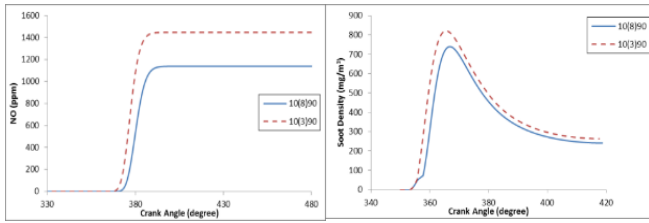
#### 4.3 Comparison between two dwell periods

Engine HRRs and in-cylinder pressures are predicted for two dwell periods to evaluate chosen engine performance and emissions. Fig. 18 shows in-cylinder pressure histories for two dwell periods with a split proportion of 10-90.



**Figure.18.** Comparison between in-cylinder pressure histories for 8°CA and 3°CA dwell periods.

From Fig. 19 it can be observed that with a lower dwell period [3°CA] area under P –θ diagram moderately increased due to early burning [or advanced] of fuel injected in the second injection pulse. The effect is further compounded with the ongoing compression stroke because of the rising piston. With a longer dwell period, power is released more into the expansion stroke resulting in reduced pressures.



**Figure.19.**NO and soot emissions for dwell periods of 8°C and 3°C.

Fig. 22 represents NO formation histories for two dwell periods of 8 °CA and 3 °CA, for a split proportion of 10-

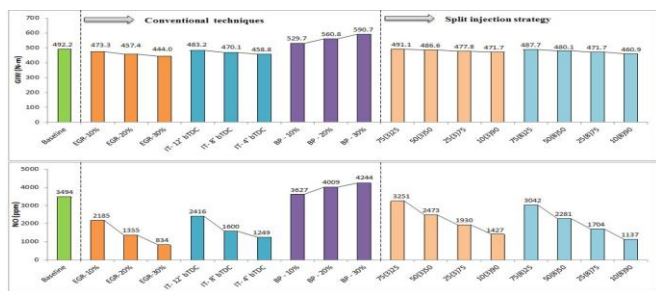
90. From the Fig., it can be observed that NO emissions increase with a decrease in the dwell period. Hence a dwell period of 3 °CA has resulted in a higher NO emission when compared to 8 °CA, due to the increased in-cylinder temperatures because of the relatively early injection of a second pulse of fuel. From the NO formation histories for both dwell periods it can also be observed that NO formation starts a little before in the case of 10[3]90 than 10[8]90. On Fig. 19, the soot formation histories are compared for two dwell periods [8 °CA and 3

CA], for a split proportion of 10-90. It is observed that with reduction in dwell period between injection pulses, soot formation increased due to availability of fuel sooner than higher dwell, and hence rich mixture in the combustion chamber. Moreover, with higher gas mixture temperatures, the NO emissions also increased for lower dwell period and soot formation tendencies increased with rich mixture at higher temperatures. Therefore, it is

Noted concluded that all the designed injection schemes have lowered NO emission either substantially or marginally but only few injection schemes have lowered soot emissions.

#### 4.4 Comparison between Piston Work and NO Emissions for Conventional and Split Injection Strategies

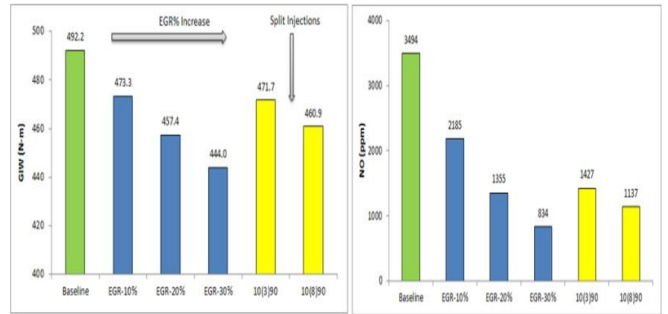
A comparison of cumulative values of piston work [PW] and NO emissions obtained by variation of conventional techniques such as EGR [0-30%], injection timing [12°bTDC, 8 °bTDC and 4 °bTDC] and boost pressure [10%, 20% and 30%] and designed split injection strategies has been made with baseline operation [EGR-0%, IT- 16°bTDC and BP-0%]. Fig. 20 shows the cumulative values of variation conventional techniques and proposed split injection schemes implemented in lowering NO emissions or improving piston work of the engine.



**Figure.20.**Comparison of PW and NO emissions of conventional and split injection techniques.

#### 4.5 Comparison of Split Injection with EGR

It is well known that either retarding injection timing or use of EGR reduces the NO emissions. It is established that split injection scheme will reduce the NO emissions with varying first pulses. Hence a comparison is made in Fig. 21 between cumulative values of PW and NO emissions for increasing EGR levels and two split injection strategies of 10[8]90 and 10[3]90 with baseline operation.

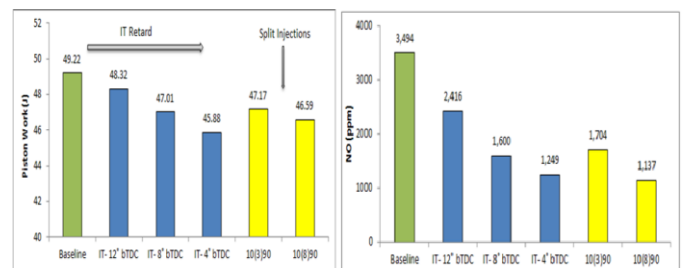


**Figure.21.** Comparison of EGR and split injection techniques with baseline.

It can be noticed that for a split injection strategy of 10[3]90, the NO values are compared with conventional technique of EGR at 20%. It can be observed that from the baseline operation, EGR-20% has decreased PW by 7.07% whereas split injection of 10[3]90 has decreased PW by 4.16%. A comparison of NO emissions obtained from EGR-20% and above-mentioned split injection schemes with baseline value has revealed that EGR-20% has reduced NO emissions by 61.2% and split injection of 10[3]90 has reduced it by 59% from baseline value.

#### 4.6 Comparison of Split Injection with Retarded Injection Timings

Fig. 22 compares cumulative values of PW and NO emissions obtained from two designed split injection strategies which generated lower NO emissions, viz; 10[3]90 and 10[8]90 with retarding injection timings of single injections with baseline values.



**Figure. 22.** Comparison of retarded injection timings and split injection techniques with baseline.

It can be seen that retarded injection timing of 8°bTDC has generated similar NO levels as that of split injection, 10[8]90. Retardation of injection timing to 8 °bTDC has shown to reduce NO emission levels by 54.2% and split injection of 10[8]90 has shown to reduce NO emission level by 67.4% from baseline

value. However, for the same cases, PW was observed to reduce by 4.44% and 5.36% for IT-8 °bTDC and 10[8]90 cases respectively.

#### 4.7 Comparison of PW and NO emissions for Different Strategies Involving Lowering of NO Emissions

A comparison is made between cumulative PW and NO emission values obtained by employing various conventional techniques and split injection techniques with an aim to lower NO emissions, have been made with baseline values. Fig. 23 depicts the cumulative values of PW and NO emissions for various conventional techniques [EGR and retarded injection timings] and split injection techniques compared with baseline values.

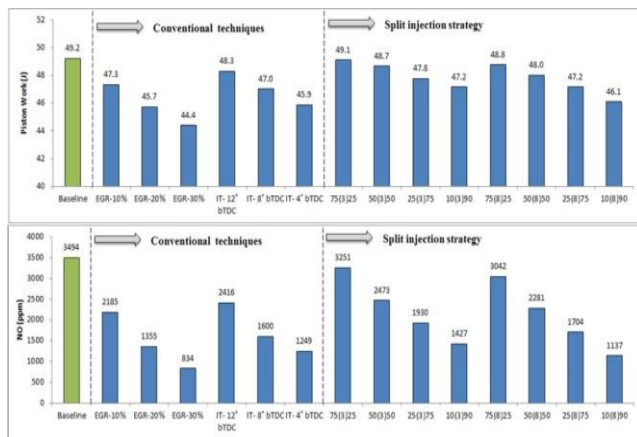


Figure. 23. Comparison of cumulative values of PW and NO emissions of various strategies with baseline values.

From the strategies [both conventional and proposed split injection technique], which yielded lower NO emissions have been taken into consideration as shown in Fig. 24. It can be observed that, two conventional techniques, viz; EGR-20% and IT-4 °bTDC and split injection of 10[8]90 have been observed to reduce NO emissions significantly in comparison with baseline value. It is to be noted that, EGR-30% case has not been considered as it significantly deteriorates the engine performance.

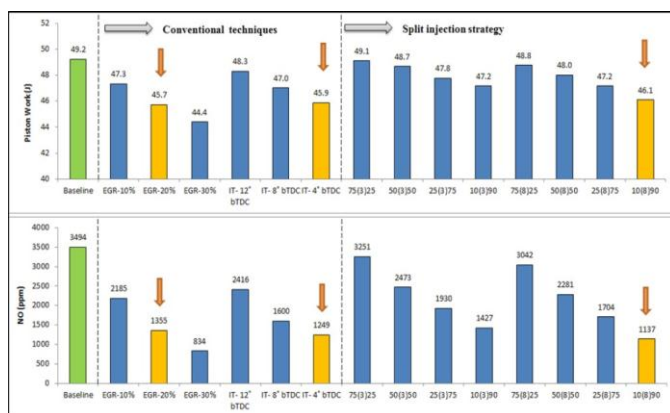


Figure. 24. Strategies yielding lower NO emissions from conventional techniques and split injection techniques are considered. Bars in orange colour indicate the chosen strategies.

#### 4.8 Effect of boost pressure and comparison among different performance and emission parameters

The strategies chosen to lower NO emissions [EGR-20%, IT-4 °bTDC and 10[8]90] have resulted in NO<sub>x</sub>-piston work trade-off. Hence an attempt has been made to increase the engine performance of chosen strategies near to baseline operation employing moderate use of conventional techniques such as boost pressures and advanced injection timings as shown in Fig. 25.

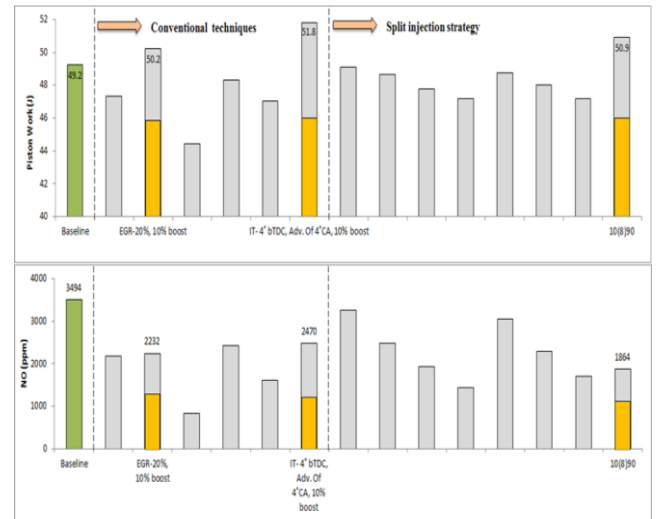


Figure.25. Comparison of moderate boost pressures and advanced injection timings.

Moderate levels [10%] of boost pressure has been added to strategies of EGR-20%, IT-4 °bTDC and 10[8]90. However, an advancement of injection timing by 4 °CA has also been made to strategy of IT-4 °bTDC to improve its engine performance near to baseline operation.

It can be thus inferred that, in conventional techniques, a combination of 2 or 3 techniques is required to raise their performance near to baseline with nearly 50% increase in NO emissions. With split injection, adding moderate boost pressures of 10% to 10[8]90 has enhanced engine performance with 39% increase in NO emissions, potentially allowing 20% EGR reduction.

#### CONCLUSIONS

The in-house code is developed using phenomenological modelling approach, [with a step size of 0.9° CA] has given insight into the strategies for improving the performance and lowering the engine exhaust emissions of single cylinder DI diesel engine. The engine model can be used as a real-time model for hardware-in-loop testing and also as a non-real-time model to predict engine performance and emissions under steady state and transient conditions. The present model is observed to be insensitive to the engine geometry. Prominent parameters affecting the engine emissions and performance are identified. Increase in boost pressure has been realized in higher power from the engine.

Split injection strategy is observed to have more control over NO as well as soot emissions. Given a choice between the two split injection schemes lowest NO emissions are realized with lowest amount of pilot quantity. Dwell period between injection pulses is observed to have a significant effect on NO emissions and piston work. Split injection strategy is observed to be superior in controlling NO than the EGR without sacrificing the performance of engine.

The trade-off between NO-soot emissions is effectively established with split injection strategy. With EGR -10% and EGR - 20%, GIW was reduced by 3.8%, 7.2% with a significant reduction of 37%, 61.2% in NO emissions

Respectively. With retarded ITs of 12° bTDC and 4°bTDC a reduction of 1.9%, 4.4% in GIW with 30% & 54% reduction in NO was achieved. Boost of 30% has increased GIW by 16% with an increase in NO by 17%. Split of 75[8]25 has resulted in marginal reduction of 0.68% in GIW with 12% reduction NO emissions and 22% reduction in soot emissions.

Split of 10[8]90 has reduced GIW by 6.1% and has led to a significant reduction in NO emissions by 66% and an increase in soot emissions by 35.4%. For the same split strategy [10[8]90] and dwell of 3° CA has resulted in 3% reduction in GIW and 50% reduction in NO emissions with almost 38% increase in soot emissions. To achieve baseline GIW using conventional techniques of EGR 20% and boost 10%, a reduction of 36% NO was observed from baseline conditions. To achieve baseline GIW using 3 conventional techniques of 4° bTDC and advancement of 4° CA with 10% boost has resulted in 29% reduction in NO emissions. With split strategy of 10[8]90 and boost of 10% has resulted in baseline GIW performance with 46% reduction NO emissions. Therefore, it is concluded that with split injection scheme, simultaneous reduction of NO and soot can be achieved without resorting to advanced combustion concepts.

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