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A brief Review-Mathematical Modeling on Water Pollution and Its Effects on Aquatic Species, resembling Bagmati River

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Abstract

We have reviewed four mathematical models to control and decrease the pollution of river, then resemble them to Bagmati-river. We reviewed a one-dimensional method for the disposal of pollutants of Bagmati-river. Generally in such cases a model is formulated by using the system of non-linear ordinary differential equations to resolve pollution problem of any river. But here we co-relate it to Bagmati-river to control its water pollution.

Keywords: Water pollution, mathematical modeling, one dimensional method, non-linear ordinary differential equations, biological oxygen demand, equilibrium condition state, water quality.

1.Introduction:

The population density of Bihar (India) is very high. Due to high population density the natural environment of this state is negatively affected. All types of pollutions are seen here. We have also destroyed the aquatic ecosystem. Water is one of the very important gift of nature on this planet. In water a number of aquatic animals live. Water body is polluted every day due to release of untreated waste, sewage, agricultural discharge along with toxic metals. As water is their natural habitat. If we pollute any reservoir, it will impact negatively on whole reservoir ecosystem. By doing so, we decrease the oxygen level of water in that reservoir. Since the dissolve oxygen is only the basis of life air of the aquatic animals in any aquatic ecosystem. These pollutants put very harmful effect by changing the color, taste, as well as odor of water and whole water body, making it unfit to use.

2. Review of related as well as co-related literature:

Bagmati is one of the holy river in Nepal as well as in Bihar (India). We have reviewed the book Bagmati Ki Sadgati! By Dr. Dinesh Mishra. From holy river to flowing filth by Dinesh Kumar Mishra. Auther highlights the challenges faced by much abused Bagmati River in bihar because of solid waste, industrial effluents, encroachment and rampant construction along the river's banks. Dirty water of the Riga Sugar Millis let into the Bagmati and the result is that ground water in five-mile stretch of the river has become very polluted, that even the cattle refuse to drink it. It was Damoder Jha, member of Legislative Assembly, who pointed out such matter of pollution in Bihar Legislative Assembly on 1 March 1955. We have tried to tackle the increasing pollution of Bagmati river by reviewing different mathematical models as well as by resembling Ganga river's pollution. We have studied the mathematical modeling on water pollution and self-purification of river Gangaes by Rajat Kaushik, 2015, Pelagia Research Library, Mathematical modeling of pollutant in Rivers by R. V. Waghmare and S. B. Kiwne, 2017, International Journal of Computational and Applied Mathematics, 835-842, Mathematical modeling of the survival of abiological species in polluted water bodies by J. B. Shukala, A. K. Mishra and Peeyussh Chandra, 2007, a mathematical approach to study the effect of pollutants as well as Toxicants in aquatic environment by Anita Chaturvedi, Kokila Ramessh, Vatsala 2017 and made own review to decrease pollution of river water.

3. Material and methods:

(A) Model on Self-purification of River water on its own:
A Review of Mathematical modeling on water pollution and self-purification of river Bagmati:

As Bagmati-river enters India at Bairgania, then it flows across the districts Darbhanga, Sitamarhi, Sheohar, Muzaffarpur as well as Khagaria in Bihar. It meets River Kamla at Jagmohra in Samastipur. These districts of Bihar are heavily densely populated. So a large number of untreated waste, sewage and toxic chemical substances are released in this river. The river self-purifies itself as it moves forward on its own. The governing equations are following, $\Delta S_t = M + FC - F\frac{S_0 - S_t}{2V}$, $S_t = \frac{Mt + v_1 A_1 C - \frac{S_0}{2V} v_2 A_2}{\frac{1}{t} + \frac{v_2 A_2}{2V}}$. If the velocity flow rate at initial as well as end point remains same as $v_1=v_2=\ v$ then $S_t=\frac{M+v(A_1C-\frac{S_0}{2V}A_2)}{\frac{1}{t}+\frac{vA_2}{2V}}$. If S_0 be initial pollutants in Bagmati-river when t=0. S_t be the pollutants in Bagmati-river when $t=t$. Therefore, total increase in pollutants after t days $\Delta S_t=S_t-S_0$. Let M be the amount of pollutants released per day in considered area of river. F be the daily flow of river. C be the concentration of pollutants in water in considered area of river. FC be the total pollutants (sewage, untreated waste, industrial waste) in considered area of river. $\frac{S_0}{V}$ be the initial concentration of polluted water in the considered area of river if total volume of water in considered area is V. $\frac{S_t}{V}$ be the concentration of pollutants after t days. Thus in t days total pollutants concentration in river in considered area will be $\frac{S_0-S_t}{2V}$. If we correct the sewage treatment, and there should be a ban on daily release of pollutants then the pollution of Bagmati-river can be decreased.

Thus we say that if $M\rightarrow 0$ then we get as $(S_t-S_0)(1+\frac{Ft}{2V})=FCt-\frac{FS_0}{2V}t \Rightarrow (S_t-S_0)=\frac{F(C-C_0)t}{(1+\frac{Ft}{2V})}$.We have found that river Bagmati gets polluted heavily (mostly due to untreated sewage) in considered area Sitamarhi to Khagaria, so $C-C_0<0$ and $S_t-S_0<0$. Therefore, the river will recover after sufficient time and equilibrium state will be reached, at $S_t-S_0=0$ as well as $C-C_0=0$.

(B) A Review of Model on the disposal of Pollutants in river Bagmati:

As it is known to us that the polluted waste are discharged in Bagmati-river from Sitamarhi to Khagaria. This waste disposal contains dissolved oxygen, bacteria, nutrients, **PH**, toxic which worsen the physical, chemical as well as biological properties of river water [9]. Water quality is determined naturally by external agents such as rainfall, wind, solar radiation etc. But on the other hand water quality is also affected by pollutants wastes released by human. So the concentration of these substances is calculated by the dispersion and advection characteristics of the river water. Here we use one dimensional method, because Bagmati-river is longer less wider and more deeper. Therefore, longitudinal variation of constituents concentration are solved in the form of cross-section average value. The mass conservation equation over the cross-section of river can be presented as first order decay-process as,

$$\frac{\delta C}{\delta t}=-u\frac{\partial C}{\partial x}+\frac{\partial}{\partial x}[(D_x+D_L)\frac{\partial C}{\partial x}]-KC+\sum I.$$

Here,

x = longitudinal distance along river, L and t is time with dimension T.

D_L =longitudinal dispersion coefficient, L^2/T. We will use the formula to determine dispersion coefficient $D_L=0.011\frac{u^2B^2}{Hu*}$

Here D_L is longitudinal dispersion coefficient, L^2/T

u is the cross section averaged velocity, L/T

B is the width, L

H is the stream depth, L

u_* is the shear velocity,

L/T=$\sqrt{gHs},$

We suppose g as gravitational acceleration, L/T^2 and s is the stream slope. Here we will find the analytical solution from [9] of distribution of concentration of pollutants through which it has been predicted the changes in river water quality and the calculation of continuous discharge in this river upon the concerned area. This mathematical model is based on the longitudinal vibrational to get in the form of cross-section-averaged values through one dimensional method. With careful and prompt release of pollutants in river, the maximum concentration of these materials in water decreases with time due to decay and dispersion.

(C) A Review of Model on the survival of Biological species in Polluted Water of Bagmati-river:

Here we will discuss upon a non-linear mathematical model for depletion of dissolved oxygen level in Bagmati-river due to discharge of pollutants. We will consider a food chain type of system on the river by taking pollutants, bacteria, protozoa, dissolve oxygen, and biological population. Amount of these things in river put a deep effect on the level of dissolve oxygen. Thus we have a system of non-linear differential equations as,

$$\frac{dT}{dt}=Q-\alpha_0 T-\frac{K_1TB}{K_{12}+K_{11}T},\ \frac{dB}{dt}=\frac{\lambda_1K_1TB}{K_{12}+K_{11}T}-\alpha_1B-\lambda_{10}B^2-\frac{K_2BP}{K_{21}+K_{22}B},\ \frac{dP}{dt}=\frac{\lambda_2K_2BP}{K_{21}+K_{22}B}-\alpha_2P-\lambda_{20}P^2,\ \frac{dC}{dt}=Q-\alpha_3C-\frac{\lambda_{12}K_1BP}{K_{12}+K_{11}T}-\frac{\lambda_{23}K_2BP}{K_{21}+K_{22}T}-\lambda_{11}\alpha_1B-\lambda_{22}\alpha_2P-K_3CF,\ \frac{dF}{dt}=\lambda_3K_3CF-\alpha_4F-\lambda_{30}F^2$$

Here, $T(0)>0, B(0)>0, P(0)>0, C(0)>0, F(0)>0$. Since α_i is depletion rate coefficients and K_i is proportionality constants, so theses should be positive. Also for the feasibility of this model, the growth rate of bacteria and protozoa must be positive. Hence we have, $\lambda_1K_1-K_{11}\alpha_1>0$ and $\lambda_2K_2-K_{22}\alpha_2>0$. Our assumptions are as follow,

T is the accumulative concentration of organic pollutants,
B is the density of bacteria,
P is the density of protozoa,
C is the concentration of dissolve oxygen,
F is the density of biological population and
Q is the accumulative rate of discharge of organic pollutants into the water body.

Thus we can say that, *T* becomes proportional to the concentration of organic pollutants. Hence, it is clear that as the rate of discharge of pollutants increases the equilibrium, the concentration of dissolve oxygen decreases. When the amount of discharge of pollutants is very high then the equilibrium concentration of dissolved oxygen tends to zero. This makes the water unfit for survival of biological living beings.

(D) A Review of mathematical approach to study the effect of pollutants and toxicants in aquatic environments:

Since the acidic material is released by pollutants, which decrease the **PH** level of river water. Therefore we will show a non-linear differential equation of four variables. These dependent variables are (a) amount of acid and metal in water (b) density of favourable resources (c) density of fish population (d) concentration of nutrients in water. Here we will assume that the quantity of metal is less than the acid present in river water. Now, we will present it in non-linear form of Mathematical model as follow,

$\frac{dS}{dt}=S_0-aS-gSP-\alpha(T_1+qC_m)+kcP+kbF+k_1F^2$, $\frac{dP}{dt}=gSP-cP-fFP$, $\frac{dF}{dt}=fPF-bF-kF^2$,

$\frac{d(T_1+qC_m)}{dt}=Q_0-\alpha(T_1+qC_m)-\alpha_1(T_1+qC_m)S$.

Thus we have the initial condition as $S(0)=S_{10}>0, P(0)=P_{10}>0, F(0)=F_{10}>0$, and $T(0)=T_{10}>0$. It is seen that all dependent variables should be positive and in equilibrium state aquatic environment will be stable .These all four dependent variables must be equal at two points in equilibrium state.

4. Conclusions:

On the basis of reviews of four mathematical models, we have found that one dimensional method is used to dispose pollutants from the Bagmati-river. The model is formulated using the system of non-linear ordinary differential equations. We have found that a Mathematical model can make a complex situation simple. It has been observed that the mathematical model helps us to make predictions. We have also found from these reviews that the Mathematical modelling helps us to find a better understanding of real world problem. Thus we have reviewed four mathematical models by different authors and resembling them to decrease Bagmati-river water pollutions and to make its water suitable for living beings.

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