

Fixed point theorems for $(\psi - \phi)$ - contractions in G-fuzzy metric spaces

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Abstract

In this paper, we introduce generalized $(\psi - \phi)$ - contractions and prove a fixed point theorems in G-fuzzy metric spaces. Our results generalizes and extends many recent fixed point theorems in the literature. We justify our result by a suitable example.

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1. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [17] in 1965. Kramosil and Michalek [9] introduced the concept of fuzzy metric space in 1975, which can be regarded as a generalization of the statistical metric space. Clearly this work plays an essential role for the construction of fixed point theory in fuzzy metric spaces. Mustafa and Sims [11] introduced a new notion of a generalized metric space called G-metric space. Rao et al. [12] proved two unique common coupled fixed point theorems for three mappings in symmetric G- fuzzy metric spaces. Sun and Yang [14] introduced

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the concept of G- fuzzy metric spaces and proved two common fixed point theorems for four mappings Subsequently, in 1988. M. Grabiec [2] defined G-complete fuzzy metric space and extended the complete fuzzy metric spaces. Following Grabiec's work, many authors introduced and generalized the different types of fuzzy contractive mappings and investigate some fixed point theorems in fuzzy metric spaces. In 1994, George and Veeramani [1] modified the notion of M -complete fuzzy metric space with the help of continuous t-norms.

A number of fixed point theorem have been obtained by various authors in fuzzy metric spaces by using the concept of implicit relations, compatible maps, weakly compatible maps, R-weakly compatible maps. In 2019 Vishal Gupta et al.[4] proved fixed point theorem in V-fuzzy metric space employing the effectiveness of E.A. property and CLRg property. M.Jeyaraman et al.[6] validated unique common fixed point theorems for six weakly compatible mappings in intuitionistic generalized fuzzy metric spaces in 2020. Before giving our main result, we recall some of the basic concepts and results in G-metric spaces and G-fuzzy metric spaces.

2. PRELIMINARIES

Now, we begin with some basic concepts.

Definition 2.1. [13]. A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a continuous triangular norm (in short, continuous t-norm) if it satisfies the following conditions:

(TN-1) $*$ is commutative and associative.

(TN-2) $*$ is continuous.

(TN-3) $a * 1 = a$ for every $a \in [0, 1]$.

(TN-4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition 2.2. [1]. An ordered triple $(X, M, *)$ is called fuzzy metric space such that X is a nonempty set, $*$ defined a continuous t-norm and M is a fuzzy set on $X \times X \times (0, \infty)$, satisfying the following conditions, for all $x, y, z \in X, s, t > 0$:

(FM-1) $M(x, y, t) > 0$.

(FM-2) $M(x, y, t) = 1$ iff $x = y$.

(FM-3) $M(x, y, t) = M(y, x, t)$.

(FM-4) $(M(x, y, t) * M(y, z, s)) \leq M(x, z, t + s)$.

(FM-5) $M(x, y, *) : (0, \infty) \rightarrow (0, 1]$ is left continuous.

Definition 2.3. [11]. Let X be a nonempty set, and let $G : X \times X \times X \rightarrow [0, \infty)$ be a function satisfying the following properties:

(G-1) $G(x, y, z) = 0$ if $x = y = z$,

(G-2) $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$,

(G-3) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $y \neq z$,

(G-4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \dots$, symmetry in all three variables,

(G-5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$.

The function G is called a generalized metric or a G -metric on X and the pair (X, G) is called a G -metric space.

Definition 2.4. [11]. The G -metric space (X, G) is called symmetric if $G(x, x, y) = G(x, y, y)$ for all $x, y \in X$.

Definition 2.5. [14]. A 3-tuple $(X, G, *)$ is said to be a G -fuzzy metric space (denoted by GF space) if X is an arbitrary nonempty set, $*$ is a continuous t -norm and G is a fuzzy set on $G : X \times X \times X \rightarrow (0, +\infty)$ satisfying the following conditions for each $t, s > 0$:

(GF-1) $G(x, x, y, t) > 0$ for all $x, y \in X$ with $x \neq y$,

(GF-2) $G(x, x, y, t) \geq G(x, y, z, t)$ for all $x, y, z \in X$ with $y \neq z$,

(GF-3) $G(x, y, z, t) = 1$ if and only if $x = y = z$,

(GF-4) $G(x, y, z, t) = G(p(x, y, z), t)$, where p is a permutation function,

(GF-5) $G(x, a, a, t) * G(a, y, z, s) \leq G(x, y, z, t + s)$ (the triangle inequality),

(GF-6) $G(x, y, z, \cdot) : (0, \infty) \rightarrow [0, 1]$ is continuous.

Remark 2.6. [14]. Let $x = w, y = u, z = u, a = v$ in (GF - 5), we have $G(w, u, u, t + s) \geq G(w, v, v, t) * G(v, u, u, s)$, which implies that $G(u, u, w, s + t) \geq G(u, u, v, s) * G(v, v, w, t)$, for all $u, v, w \in X$ and $s, t > 0$. A GF space is said to be symmetric if $G(x, x, y, t) = G(x, y, y, t)$ for all $x, y \in X$ and for each $t > 0$.

Definition 2.7. [14]. Let $(X, G, *)$ be a GF space, then

(1) a sequence $\{x_n\}$ in X is said to be convergent to x (denoted by $\lim_{n \rightarrow \infty} x_n = x$) if $\lim_{n \rightarrow \infty} G(x_n, x_n, x, t) = 1$ for all $t > 0$.

(2) a sequence $\{x_n\}$ in X is said to be a Cauchy sequence if $\lim_{m \rightarrow \infty} G(x_n, x_n, x_m, t) = 1$, as $n, m \rightarrow \infty$ that is, for any $\epsilon > 0$ and for each $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $G(x_n, x_n, x_m, t) > 1 - \epsilon$, for $n, m \geq n_0$.

(3) A GF space $(X, G, *)$ is said to be complete if every Cauchy sequence in X is convergent.

Lemma 1. [14]. Let $(X, G, *)$ be a GF space. Then $G(x, y, z, t)$ is non-decreasing with respect to t for all $x, y, z \in X$.

Lemma 2. [5]. Φ denote the set of all continuous non decreasing function $\phi, \psi : [0, \infty) \rightarrow [0, \infty)$ such that $\phi^n(t) \rightarrow 0$ as $n \rightarrow \infty$ and $\psi^n(t) \rightarrow 1$ as $n \rightarrow \infty$ for all $t > 0$. It is clear that $\phi(t) > t, \psi(t) > t$ for all $t > 0$ and $\phi(0) = 0$ and $\psi(1) = 1$.

The objective of this work is to introduced generalized $(\psi - \phi)$ - contractions and prove fixed point theorems in G-fuzzy metric spaces. Our results generalize or improve many recent fixed point theorems in the literature. We furnish an example to validate our result.

3. MAIN RESULT

In this section, we establish fixed point theorem in G-fuzzy metric space based on the function $\psi, \phi \in \Phi$, we introduce the following definition.

Definition 3.1. Let $(X, G, *)$ be a G-fuzzy metric space. A mapping $f : X \rightarrow X$ is said to be a generalized $(\psi - \phi)$ - contractions if there exist $\phi, \psi \in \Phi$ such that, for any $x, y, z \in X$,

$$G(fx, fy, fz) \neq 1 \Rightarrow \phi(G(fx, fy, fz)) \geq \psi[\phi(H(x, y, z))], \quad (3.1)$$

where

$$H(x, y, z, t) = \min \left\{ \begin{array}{l} G(x, y, z, t), G(x, fx, fx, t), G(y, fy, fy, t), \\ G(z, fz, fz, t), \frac{1}{2}[G(x, fy, fy, t) + G(y, fz, fz, t)], \\ \frac{1}{3}[G(x, fy, fy, t) + G(y, fz, fz, t) + G(z, fx, fx, t)] \end{array} \right\}. \quad (3.2)$$

Theorem 3.2. Let $(X, G, *)$ be a complete G-fuzzy metric space and $f : X \rightarrow X$ be a generalized $(\psi - \phi)$ - contractions. Then f has a unique fixed point $x^* \in X$.

Proof: Let $x_0 \in X$ be any arbitrary point in X . There exist sequence $\{x_n\}$ in X such that $fx_n = x_{n+1}$ for all $n \in \mathbb{N}$. If $x_{n+1} = x_n$ for some $n \in \mathbb{N}$, then $x^* = x_n$ is a fixed point for f . Next we assume that $x_{n+1} \neq x_n$ for all $n \in \mathbb{N}$. Then $G(x_n, x_{n+1}, x_{n+1}, t) > 0$ for all $n \in \mathbb{N}$. Applying inequality (3.1) with $x = x_n, y = x_{n+1}, z = x_{n+1}$, we obtain

$$\phi(G(fx_n, fx_{n+1}, fx_{n+1}, t)) \geq \psi[\phi(H(x_n, x_{n+1}, x_{n+1}, t))] \quad (3.3)$$

where, $H(x_n, x_{n+1}, x_{n+1}, t)$

$$= \min \left\{ \begin{array}{l} G(x_n, x_{n+1}, x_{n+1}, t), G(x_n, fx_n, fx_n, t), G(x_{n+1}, fx_{n+1}, fx_{n+1}, t), G(x_{n+1}, \\ fx_{n+1}, fx_{n+1}, t), \frac{1}{2}[G(x_n, fx_{n+1}, fx_{n+1}, t) + G(x_{n+1}, fx_{n+1}, fx_{n+1}, t)], \\ \frac{1}{3}[G(x_n, fx_{n+1}, fx_{n+1}, t) + G(x_{n+1}, fx_{n+1}, fx_{n+1}, t) + G(x_{n+1}, fx_n, fx_n, t)] \end{array} \right\}$$

$$\begin{aligned}
&= \min \left\{ \begin{array}{l} G(x_n, x_{n+1}, x_{n+1}, t), G(x_n, x_{n+1}, x_{n+1}, t), G(x_{n+1}, x_{n+2}, x_{n+2}, t), \\ G(x_{n+1}, x_{n+2}, x_{n+2}, t), \frac{1}{2}[G(x_n, x_{n+2}, x_{n+2}, t) + G(x_{n+1}, x_{n+2}, x_{n+2}, t)] \\ \frac{1}{3}[G(x_n, x_{n+2}, x_{n+2}, t) + G(x_{n+1}, x_{n+2}, x_{n+2}, t) + G(x_{n+1}, x_{n+1}, x_{n+1}, t)] \end{array} \right\} \\
&= \min\{G(x_n, x_{n+1}, x_{n+1}, t), G(x_{n+1}, x_{n+2}, x_{n+2}, t)\}.
\end{aligned}$$

If $H(x_n, x_{n+1}, x_{n+1}, t) = G(x_{n+1}, x_{n+2}, x_{n+2}, t)$, then it follows from (3.1) that

$$\begin{aligned}
\phi(G(x_{n+1}, x_{n+2}, x_{n+2}, t)) &= \phi(G(fx_n, fx_{n+1}, fx_{n+1}, t)) \\
&\geq \psi[\phi(H(x_n, x_{n+1}, x_{n+1}, t))] \\
&= \psi[\phi(G(x_{n+1}, x_{n+2}, x_{n+2}, t))] \\
&> \phi(G(x_{n+1}, x_{n+2}, x_{n+2}, t)),
\end{aligned}$$

by Lemma 2, which is a contradiction. Hence for all $n \in \mathbb{N}$,

$$H(x_n, x_{n+1}, x_{n+1}, t) = G(x_n, x_{n+1}, x_{n+1}, t). \quad (3.4)$$

Thus, (3.1) becomes

$$\phi(G(fx_n, fx_{n+1}, fx_{n+1}, t)) \geq \psi[\phi(G(fx_n, fx_{n+1}, fx_{n+1}, t))]. \quad (3.5)$$

Repeating this process, we get

$$\begin{aligned}
\phi(G(x_n, x_{n+1}, x_{n+1}, t)) &= \phi(G(fx_{n-1}, fx_n, fx_n, t)) \\
&\geq \psi[\phi(G(x_{n-1}, x_n, x_n, t))] \\
&\geq \psi^2[\phi(G(x_{n-2}, x_{n-1}, x_{n-1}, t))] \\
&\geq \psi^3[\phi(G(x_{n-3}, x_{n-2}, x_{n-2}, t))] \\
&\geq \dots \geq \psi^n[\phi(G(x_0, x_1, x_1, t))].
\end{aligned}$$

We have $\phi(G(x_n, x_{n+1}, x_{n+1}, t)) \geq \psi[\phi(G(x_{n-1}, x_n, x_n, t))] \geq \dots \geq \psi^n[\phi(G(x_0, x_1, x_1, t))]$. By the definition of ψ and ϕ , we have

$$\lim_{n \rightarrow \infty} \psi^n[\phi(G(x_0, x_1, x_1, t))] = 1, \quad (3.6)$$

and

$$\lim_{n \rightarrow \infty} \phi(G(x_n, x_{n+1}, x_{n+1}, t)) = 0. \quad (3.7)$$

We shall prove that $\{x_n\}$ is G-Cauchy sequence in X , now for $m > n$, we have $H(x_n, x_n, x_m, t)$

$$\begin{aligned}
&= \min \left\{ \begin{array}{l} G(x_n, x_n, x_m, t), G(x_n, fx_n, fx_n, t), G(x_n, fx_n, fx_n, t), \\ G(x_m, fx_m, fx_m, t), \frac{1}{2}[G(x_n, fx_n, fx_n, t) + G(x_n, fx_m, fx_m, t)], \\ \frac{1}{3}[G(x_n, fx_n, fx_n, t) + G(x_n, fx_m, fx_m, t) + G(x_m, fx_n, fx_n, t)] \end{array} \right\} \\
&= \min \left\{ \begin{array}{l} G(x_n, x_n, x_m, t), G(x_n, x_{n+1}, x_{n+1}, t), G(x_n, x_{n+1}, x_{n+1}, t), \\ G(x_m, x_{m+1}, x_{m+1}, t), \frac{1}{2}[G(x_n, x_{n+1}, x_{n+1}, t) + G(x_n, x_{m+1}, x_{m+1}, t)], \\ \frac{1}{3}[(G(x_n, x_{n+1}, x_{n+1}, t) + G(x_n, x_{m+1}, x_{m+1}, t) + G(x_m, x_{n+1}, x_{n+1}, t))] \end{array} \right\} \\
&\geq \min\{G(x_n, x_n, x_m, t), G(x_{m+1}, x_{m+1}, x_n, t)\} \\
&> 1 - \epsilon \text{ for } n, m \geq n_0.
\end{aligned}$$

$$\begin{aligned}
\text{Now } G(x_n, x_n, x_m, t) &\geq \psi[G(x_n, x_n, x_{n+1}, t)] + \psi^2[\phi(G(x_{n+1}, x_{n+1}, x_{n+2}, t))] + \dots \dots \\
&\quad + \psi^n[\phi(G(x_{m-1}, x_{m-1}, x_m, t))] \\
&\geq \psi^n[\phi(G(y_0, y_1, y_1, t))] + \psi^{n+1}[\phi(G(y_0, y_1, y_1, t))] + \dots \dots \\
&\quad + \psi^{m-1}[\phi(G(y_0, y_1, y_1, t))] \\
&\rightarrow 1 \text{ as } n, m \rightarrow \infty.
\end{aligned}$$

Since $\phi^n(t) \rightarrow 1$ as $n \rightarrow \infty$ for all $t > 0$. Using the condition (3.1)

$$\begin{aligned}
\phi(G(x_{n+1}, x_{m+1}, x_{m+1}, t)) &= \phi(G(fx_n, fx_m, fx_m, t)) \\
&\geq \psi[\phi(H(x_n, x_m, x_m, t))].
\end{aligned}$$

Passing to limit $n, m \rightarrow \infty$, then we get $\phi(1 - \epsilon) \geq \psi[\phi(1 - \epsilon)]$, by Lemma 2, $\psi[\phi(1 - \epsilon)] > \phi(1 - \epsilon)$, then $\phi(1 - \epsilon) \geq \psi[\phi(1 - \epsilon)] > \phi(1 - \epsilon)$, which is a contradiction. Hence $\{x_n\}$ is G-Cauchy. Since $f(X)$ is G-complete. Then there exist $x^* \in X$ such that $\{x_n\}$ convergence to x^* . In particular,

$$\lim_{n \rightarrow \infty} G(x_n, x^*, x^*) = 1. \quad (3.8)$$

Using the fact that G is continuous on each variable,

$$G(x^*, fx^*, fx^*) = \lim_{n \rightarrow \infty} G(x_{n+1}, fx^*, fx^*). \quad (3.9)$$

We claim that x^* is a fixed point of f . Suppose, on the contrary, if $x^* \neq fx^*$, then by (3.8) and (3.9)

$$H(x_n, x^*, x^*, t) = \min \left\{ \begin{array}{l} G(x_n, x^*, x^*, t), G(x_n, fx_n, fx_n, t), G(x^*, fx^*, fx^*, t), \\ G(x^*, fx^*, fx^*, t), \frac{1}{2}[G(x_n, fx^*, fx^*, t) + G(x^*, fx^*, fx^*, t)], \\ \frac{1}{3}[G(x_n, fx^*, fx^*, t) + G(x^*, fx^*, fx^*, t) + G(x^*, fx_n, fx_n, t)] \end{array} \right\} \quad (3.10)$$

$$\rightarrow G(x^*, fx^*, fx^*, t),$$

as $n \rightarrow \infty$, using the condition (3.1),

$$\begin{aligned} \phi(G(x_{n+1}, fx^*, fx^*, t)) &= \phi(G(fx_n, fx^*, fx^*, t)) \\ &\geq \psi[\phi(H(x_n, x^*, x^*, t))]. \end{aligned} \quad (3.11)$$

Passing to limit as $n \rightarrow \infty$, then we have

$$\phi(G(x^*, fx^*, fx^*, t)) \geq \psi[\phi(G(x^*, fx^*, fx^*, t))]. \quad (3.12)$$

By Lemma 2, $\psi[\phi(G(x^*, fx^*, fx^*, t))] > \phi(G(x^*, fx^*, fx^*, t))$. Then

$$\phi(G(x^*, fx^*, fx^*, t)) \geq \psi[\phi(G(x^*, fx^*, fx^*, t))] > \phi(G(x^*, fx^*, fx^*, t)), \quad (3.13)$$

which is a contradiction. As a consequence, we conclude that $fx^* = x^*$. Now, we will prove that f has at most one fixed point. Suppose, on the contrary, that there exists another distinct fixed point y^* of f such that $fx^* = x^* \neq fy^* = y^*$. Then $G(fx^*, fy^*, fy^*, t) = G(x^*, y^*, y^*, t) > 0$ and $H(x^*, y^*, y^*, t) = G(x^*, y^*, y^*, t)$, and then by (3.1)

$$\begin{aligned} \phi(G(fx, fy, fy)) &= \psi(G(fx^*, fy^*, fy^*, t)) \\ &\geq \psi[\phi(H(x^*, y^*, y^*, t))] \\ &= \psi[\phi(G(x^*, y^*, y^*, t))], \end{aligned}$$

and by Lemma 2, $\phi(G(x^*, y^*, y^*, t)) \geq \psi[\phi(G(x^*, y^*, y^*, t))] > \phi(G(x^*, y^*, x^*, t))$, which is a contradiction. Therefore, the fixed point of f is unique.

Theorem 3.3. Let $(X, G, *)$ be a complete G -fuzzy metric space and $f : X \rightarrow X$ be a self-mapping which satisfies the following condition, for all $x, y \in X$,

$$G(fx, fy, fy, t) \geq \min \left\{ \begin{array}{l} aG(x, y, y, t), b[G(x, fx, fx, t) + 2G(y, fy, fy, t)], \\ b[G(x, fy, fy, t) + G(y, fy, fy, t) + G(y, fx, fx, t)] \end{array} \right\}, \quad (3.14)$$

where $0 \leq a < 1$ and $0 \leq b < \frac{1}{3}$. Then f has a unique fixed point $x^* \in X$.

Proof: Let $\lambda = \min\{a, 3b\}$, then $0 \leq \lambda < 1$. And let $\phi(t) = t^\lambda, \psi(t) = t$, then $\phi, \psi \in \Phi$. Since

$$\begin{aligned}
& \min \left\{ aG(x, y, y, t), b[G(x, fx, fx, t) + 2G(y, fy, fy, t)], \right. \\
& \quad \left. b[G(x, fy, fy, t) + G(y, fy, fy, t) + G(y, fx, fx, t)] \right\} \\
& \geq \min \left\{ G(x, y, y, t), \frac{1}{3}[G(x, fx, fx, t) + 2G(y, fy, fy, t)], \right. \\
& \quad \left. \frac{1}{3}[G(x, fy, fy, t) + G(y, fy, fy, t) + G(y, fx, fx, t)] \right\} \quad (3.15) \\
& \geq \min \left\{ G(x, y, y, t), G(x, fx, fx, t), G(y, fy, fy, t), \right. \\
& \quad \left. \frac{1}{3}[G(x, fy, fy, t) + G(y, fy, fy, t) + G(y, fx, fx, t)] \right\} \\
& \geq \lambda H(x, y, y).
\end{aligned}$$

Therefore,

$$\begin{aligned}
\phi(G(fx, fy, fy)) &= (G(fx, fy, fy))^\lambda \geq (\lambda H(x, y, y))^\lambda \\
&= \phi(\lambda H(x, y, y)) \\
&= \psi(\phi(\lambda H(x, y, y))).
\end{aligned}$$

Therefore, f has a unique fixed point $x^* \in X$.

4. EXAMPLE

In this section, we give an example to validate our results.

Example. Let $X = \{\frac{1}{n} : n \in \mathbb{N}\} \cup \{0\}$ be endowed with the G-fuzzy metric space $G(x, y, z, t) = 1 + |x - y| + |y - z| + |z - x|$ for all $x, y, z \in X$. Then $(X, G, *)$ is a complete G-fuzzy metric space. Define the mapping $f : X \rightarrow X$ by

$$f(x) = \begin{cases} \frac{1}{n^4} & \text{if } x = \frac{1}{n}, n \geq 2 \\ 0 & \text{otherwise.} \end{cases}$$

Then f has a unique fixed point $x^* \in X$.

Solution: We consider the following three cases.

Case-I. Let $x = 0$ (or $x = 1$), $y = \frac{1}{n}$ and $z = \frac{1}{n}$. Since, $fx = 0$ (or $x = 1$), $fy = \frac{1}{n^4}$ and $fz = \frac{1}{n^4}$ for all $n \in \mathbb{N}$ then,

$$H(0, \frac{1}{n}, z, t) = \min \left\{ \begin{array}{l} G(0, \frac{1}{n}, \frac{1}{n}, t), G(0, 0, 0, t), G(\frac{1}{n}, \frac{1}{n^4}, \frac{1}{n^4}, t), \\ G(\frac{1}{n}, \frac{1}{n^4}, \frac{1}{n^4}, t), \frac{1}{2}[G(0, \frac{1}{n^4}, \frac{1}{n^4}, t) + G(\frac{1}{n}, \frac{1}{n^4}, \frac{1}{n^4}, t)], \\ \frac{1}{3}[G(0, \frac{1}{n^4}, \frac{1}{n^4}, t) + G(\frac{1}{n}, \frac{1}{n^4}, \frac{1}{n^4}, t) + G(\frac{1}{n}, 0, 0, t)] \end{array} \right\} = 1. \quad (4.1)$$

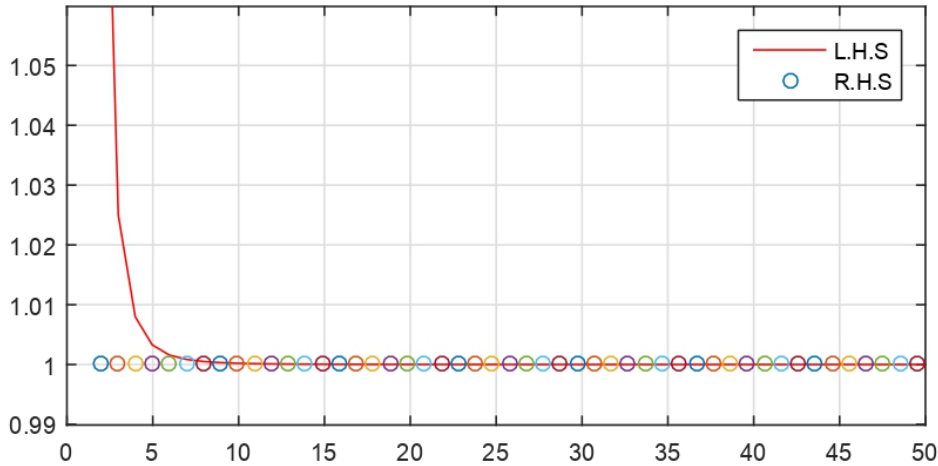
Hence the L.H.S of (3.1), $\phi(G(fx, fy, fz)) = 1 + \frac{2}{n^4}$ and the R.H.S of (3.1), $\psi[\phi(H(x, y, z))]$ = 1. Therefore, $\phi(G(fx, fy, fz)) \geq \psi[\phi(H(x, y, z))]$.

Case-II. Let $x = \frac{1}{n}, y = \frac{1}{m}$ and $z = \frac{1}{m}$, when $m \geq n \geq 2$. Since, $fx = \frac{1}{n^4}, fy = \frac{1}{m^4}$ and $fz = \frac{1}{m^4}$, then,

$$H(\frac{1}{n}, \frac{1}{m}, \frac{1}{m}, t) = \min \left\{ \begin{array}{l} G(\frac{1}{n}, \frac{1}{m}, \frac{1}{m}, t), G(\frac{1}{n}, \frac{1}{n^4}, \frac{1}{n^4}, t), G(\frac{1}{m}, \frac{1}{m^4}, \frac{1}{m^4}, t), \\ G(\frac{1}{m}, \frac{1}{n^4}, \frac{1}{n^4}, t), \frac{1}{2}[G(\frac{1}{n}, \frac{1}{m^4}, \frac{1}{m^4}, t) + G(\frac{1}{m}, \frac{1}{m^4}, \frac{1}{m^4}, t)], \\ \frac{1}{3}[G(\frac{1}{n}, \frac{1}{m^4}, \frac{1}{m^4}, t) + G(\frac{1}{m}, \frac{1}{m^4}, \frac{1}{m^4}, t) + G(\frac{1}{m}, \frac{1}{n^4}, \frac{1}{n^4}, t)] \end{array} \right\} = 1. \quad (4.2)$$

Hence the L.H.S of (3.1), $\phi(G(fx, fy, fz)) = 1$ and the R.H.S of (3.1), $\psi[\phi(H(x, y, z))]$ = 1. Therefore, $\phi(G(fx, fy, fz)) \geq \psi[\phi(H(x, y, z))]$.

Figure1.Comparison of L.H.S and R.H.S of Case I and Case II of (3.1)in 2D view



Case-III. Let $x = \frac{1}{n}$, when $n \geq 2, y = 0$ (or $y = 1$) and $z = 0$ (or $z = 1$). Since, $fx = \frac{1}{n^4}, fy = fz = 0$ then,

$$H(\frac{1}{n}, 0, 0, t) = \min \left\{ \begin{array}{l} G(\frac{1}{n}, 0, 0, t), G(\frac{1}{n}, \frac{1}{n^4}, \frac{1}{n^4}, t), G(0, 0, 0, t), \\ G(0, 0, 0, t), \frac{1}{2}[G(\frac{1}{n}, 0, 0, t) + G(0, 0, 0, t)], \\ \frac{1}{3}[G(\frac{1}{n}, 0, 0, t) + G(0, 0, 0, t) + G(0, \frac{1}{n^4}, \frac{1}{n^4}, t)] \end{array} \right\} = 1. \quad (4.3)$$

Hence the L.H.S of (3.1), $\phi(G(fx, fy, fz)) = 1 + \frac{2}{n^4}$ and the R.H.S of (3.1), $\psi[\phi(H(x, y, z))] = 1$. Therefore, $\phi(G(fx, fy, fz)) \geq \psi[\phi(H(x, y, z))]$. Thus all the conditions of Theorem 3.2 are satisfied and $x = 1$ is fixed point of f .

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