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Bayesian Estimation of the parameter of Weibull Distribution with Hybrid Censored data

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Abstract

Survival analysis is used in different Engineering experiments for lifetime data analysis, reliability analysis or time to event analysis. One of the difficulties which arise in this area is the presence of censored data. The lifetime of an individual is censored when it cannot be exactly measured but partial information is available. Different circumstances can produce different types of censoring. The two most common censoring schemes used in life testing experiments are Type-I and Type-II censoring schemes. Hybrid censoring scheme is mixture of Type-I and Type-II censoring schemes. In this paper we have considered the hybrid censored lifetime data when the life time follows two parameter Weibull distribution. The parameters are estimated by the Maximum Likelihood and Bayesian Estimation methods. It is observed that the maximum likelihood estimates cannot be obtained in closed form. We obtain the maximum likelihood estimates of the unknown parameters using R Software. The Fisher information matrix has been obtained.

Keywords: Survival analysis, Hybrid censoring scheme, Weibull distribution, maximum likelihood estimates, Monte Carlo Simulation Technique.

1. INTRODUCION

Survival analysis is a branch of engineering experiments for lifetime data analysis, reliability analysis or time to event analysis. That focuses on analyzing the expected duration until one or more events happen, such as death in biological organisms or failure in mechanical systems. It is used in a variety of fields including medicine, biology, engineering, and social sciences. This refers to the time duration until an event of interest occurs. The event can be anything like death, relapse of a disease, or failure of a machine. In many cases, the event of interest may not be observed within the study period for all subjects. Censoring occurs when we have incomplete information about the event time. The two most common censoring schemes used in life testing experiments are Type-I and Type-II censoring schemes. Hybrid censoring scheme is mixture of Type-I and Type-II censoring schemes. They can be briefly described as follows. Suppose n units are put on a life test. In type-I censoring, experiment continues up to a pre-specified integer time say t_0 elapsed and the no. of item failed are say $m \leq n$. Therefore, in Type–I censoring scheme, the number of failures is random and in Type–II censoring scheme the experimental time is random and test is terminated when pre– specified number of items say $r \leq n$ failed. A mixture of Type–I and Type–II schemes is known as the hybrid censoring scheme.

The hybrid censoring scheme was first introduced by Epstein (1954, 1960). But recently it becomes quite popular in the reliability and life testing experiments. Suppose n identical units are put on a life test. The test is terminated when a pre-specified number r, out of n units has failed or a pre-determined time t_0, has been elapsed. Therefore, in hybrid censoring scheme, the experimental time and number of failures will not exceed t_0 and r respectively. It is clear that Type-I and Type-II censoring schemes can be obtained as special cases of hybrid censoring scheme by taking $r = n$ and t_0=∞ respectively.

Epstein (1954) first introduced the hybrid censoring scheme and analyzed the data under the assumption of exponential lifetime distribution of the experimental units. He also proposed a two-sided confidence interval of the unknown parameter, without any formal proof. Fairbanks et al. (1982) modified slightly the proposition of Epstein (1954) and suggested a simple set of confidence intervals. Chen and Bhattacharya (1988) obtained the exact one-sided confidence interval based on the distribution of the maximum likelihood estimator of the experimental parameter. Draper and Guttmann (1987) also considered the same problem but from the Bayesian point of view, and obtained two-sided credible interval of the mean lifetime using the invented gamma prior. Comparisons and criticisms of the different methods can be found in Gupta and Kundu (1998). For some of the relevant references on hybrid censoring and related topics the readers are referred to Ebrahimi (1986, 1992),Basu and Ebrahimi(1991), Jeong et al (1996) childs et.al. (2003) Kundu (2007). It should be mentioned that though hybrid censoring schemes seems to be an important censoring scheme, but not much work has done.

The Weibull has been extensively used in life testing and reliability problems. The distribution has been named after the Swedish scientist, Weibull(1939) proposed it for the first time in connection with has studies on strength of material. Weibull(1951) showed that the distribution is also useful in describing the 'wear out' or fatigue failures.

The two parameters Weibull distribution widely used as a model for the failure time distribution is given by

$$f(x) = p\theta x^{(p-1)} e^{-\theta x^p}; \qquad x, \theta, \quad p > 0\,, \qquad \textbf{(1.1)}$$

Here θ is referred as a scale parameter and p as a shape parameter. Although we will follow this practice, we note that $\theta' = \theta^{\frac{1}{p}}$ should be regarded as the scale parameter.

The main aim of this paper is to provide different methods to first compute the point estimates of the unknown parameters. We have obtained the MLE's of the unknown parameters accordingly. It is observed that the MLEs can be obtained by solving a non-linear equation and we will use a simple iterative scheme to solve the non-linear equation. We also discuss the properties of the estimators by obtaining the variance covariance matrix.

The second aim of this paper is to provide the Bayes estimates under the assumptions of non-informative and conjugate prior on the parameters. It is observed that the Bayes estimates cannot be computed explicitly, so we have used R software to compute the Bayes estimates. We compared the performances of the different methods of estimation by Monte Carlo simulation technique.

2. Model Description.

Suppose x is distributed as Weibull distribution with probability density function and the cumulative distribution function defined respectively as

$$f(x) = P\theta x^{P-1} e^{-\theta x^P}; x, \theta, \quad p > 0 \qquad \textbf{(2.1)}$$

The reliability function is given by

$$R(t) = e^{-\theta t^p} \quad ; \qquad t > 0\,, \qquad \textbf{(2.2)}$$

The hazard rate function is given by

$$H(t) = P\theta t^{p-1} \quad ; \qquad t > 0\,, \qquad \textbf{(2.3)}$$

Where $'\theta'$the scale and 'p' is is the shape parameters and its cumulative density function is given as

$$F_x(x) = 1 - \exp(-\theta t^{\mathrm{P}}) \quad ; \qquad x > 0\,, \ \ \theta > 0\,, \qquad \textbf{(2.4)}$$

The characteristics of the two -parameter Weibull distribution can be exemplified by examining the two parameters, p and θ, and the effect they have on the p.d.f., reliability and failure rate functions. An extensive treatment on Weibull distribution is given by Johnson et al. (1994).

3. Maximum Likelihood Estimator

We will use maximum likelihood method to estimate the unknown parameters of Weibull distribution using hybrid censored sample. Suppose n identical units are put on life test. The test is terminated when a pre-chosen number 'r' out of 'n' items failed or when a pre-determined time t_0 on test has been reached. It is assumed the failed items are not replaced under the under the hybrid scheme, it is also assumed that 'r' and $'t_0'$ are known in advance.

The likelihood function of censored data may be written as

$$L(\theta, p|\underline{x}) = \frac{n!}{(n-D^*)!} p^{D^*} \theta^{D^*} \prod_{i=1}^{D^*} x_i^{(p-1)} e^{-\theta \sum_{i=1}^{D^*} x_i{}^p} [\exp(-\theta x^p)]^{(n-D^*)}; \qquad (\mathbf{3.1})$$

Taking logarithm on both sides as

$$\log L(\theta, p) = constt. + D^* \log p + D^* \log \theta + (p-1) \sum_{i=1}^{D^*} \log x_i - \theta \sum_{i=1}^{D^*} x_i{}^p - (\mathrm{n} - \mathrm{D}^*)\theta x^p$$

Now partially differentiating the above equation with respect to θ and equating to zero, we get

$$\frac{\delta}{\delta\theta} logL = \frac{D^*}{\theta} - \sum_{i=1}^{D^*} x_i{}^p + (\mathrm{n} - \mathrm{D}^*) x^p = 0,$$

Which provides

$$\hat{\theta} = \frac{D^*}{\sum_{i=1}^{D^*} x_i{}^p - (\mathrm{n} - \mathrm{D}^*) x^p}, \qquad (\mathbf{3.2})$$

Again partially differentiating with respect to p and equating to zero, we get

$$\frac{\delta}{\delta p} \log L = \frac{D^*}{p} + \sum_{i=1}^{D^*} \log x_i - \theta \Sigma \mathrm{x}_\mathrm{i}^p \log \mathrm{x}_\mathrm{i} - (\mathrm{n} - \mathrm{D}^*)\theta p x^{(p-1)} = 0$$

The MLE's of $\hat{\theta}$ and $\hat{P}$ are the solutions of the equations given below as

$$\sum_{i=1}^{D^*} \log x_i = \hat{\theta} \Sigma \mathrm{x}_\mathrm{i}^{\hat{p}} \log \mathrm{x}_\mathrm{i} + (\mathrm{n} - \mathrm{D}^*)\hat{\theta}\hat{p} x^{(\hat{p}-1)} - \frac{D^*}{\hat{p}}$$

$$\hat{p} = \frac{D^*}{\hat{\theta} \Sigma \mathrm{x}_\mathrm{i}^{\hat{p}} \log \mathrm{x}_\mathrm{i} + (\mathrm{n} - \mathrm{D}^*)\hat{\theta}\hat{p} x^{(\hat{p}-1)} - \sum_{i=1}^{D^*} \log x_i}; \qquad (\mathbf{3.3})$$

$$\text{And } \hat{\theta} = \frac{D^*}{\sum_{i=1}^{D^*} x_i{}^{\hat{p}} - (\mathrm{n} - \mathrm{D}^*) x^{\hat{p}}}; \qquad (\mathbf{3.4})$$

Now to obtain the MLE's of $\hat{\theta}$ and $\hat{p}$ the computer simulation is required.

The asymptotic variance- covariance matrix of (θ, p) is obtained by inverting the information matrix with elements that are negative of expected values of the second order derivatives of logarithms of the likelihood function. In the present situation it seems appropriate to approximate the expected values by their maximum likelihood estimate. Accordingly, we have the following appropriate variance- covariance matrix

$$I(\theta, p) = -\begin{vmatrix} \frac{\delta^2 \log L}{\delta\theta^2} & \frac{\delta^2 \log L}{\delta\theta\delta p} \\ \frac{\delta^2 \log L}{\delta\theta\delta p} & \frac{\delta^2 \log L}{\delta p^2} \end{vmatrix} at \ \theta = \hat{\theta}, p = \hat{p}, \qquad (\mathbf{3.5})$$

The elements of fisher information matrix are given as follows

$$-\frac{\delta^2 l}{\delta\theta^2} |_{\hat{\theta},\hat{p}} = \frac{D^*}{\theta^2} \text{ and } \frac{\delta^2 \log L}{\delta p^2} |_{\hat{\theta},\hat{p}} = \frac{D^*}{p^2} - (n - D^*)\theta x^{(p-1)}$$

$$\frac{\delta^2 l}{\delta\theta\delta p} |_{\hat{\theta},\hat{p}} = \Sigma x_i log x_i - (n - D^*) p x^{(p-1)} \text{ and } \frac{\delta^2 l}{\delta\theta\delta p} |_{\hat{\theta},\hat{p}} = \Sigma x_i log x_i - (n - D^*) p x^{(p-1)}$$

4. Bayesian Inference

To obtain the Bayes estimates of the unknown parameters for two parameter Weibull distribution eqn.(1.1), assume that the two-parameter θ and p are independent and let the non-informative prior (NIP), the function for p and θ are respectively given by

$$\pi(p)\alpha\delta^{k-1}\ ; \qquad p > 0, k > 0 \qquad \textbf{(4.1)}$$

$$\pi(\theta)\alpha\theta^{-1}\ ; \qquad \theta > 0, \qquad \textbf{(4.2)}$$

Hence the joint prior density of θ and p will be

$$\pi(\theta, p) = \theta^{-1}p^{k-1} \quad ; \quad p > 0, k > 0, \theta > 0 \qquad \textbf{(4.3)}$$

Combining the eqns. (3.1) and (4.3), the joint posterior density function of θ and p under hybrid censored sampling scheme will be

$$\rho(\theta, p|\underline{x}) = \frac{\theta^{(D^*-1)}p^{(D^*+k-1)}\{\prod_{i=1}^{D^*} x_i^{(p-1)} e^{-\theta\sum_{i=1}^{D^*} x_i{}^p}\}\ [\exp(-\theta x^p)]^{(n-D^*)}}{\phi}\ ;\ \theta > 0, p > 0 \qquad \textbf{(4.4)}$$

Where φ is the normalizing constant equals to

$$\phi = \int_0^\infty \int_0^\infty \theta^{(D^*-1)}p^{(D^*+k-1)}\{\prod_{i=1}^{D^*} x_i^{(p-1)} e^{-\theta\sum_{i=1}^{D^*} x_i{}^p}\}[\exp(-\theta x^p)]^{(n-D^*)}d\theta dp\ ; \qquad \textbf{(4.5)}$$

The joint mode of the posterior distribution eqn.(4.4) may be considered as Bayes estimates clearly if k=1 and $\theta = 1$ maximum likelihood estimates will be the same as the joint posterior mode.

Now the marginal posteriors of any parameter is obtained by integrating the joint posterior distribution with respect to the other parameters, so the marginal posterior probability density functions of θ is given as

$$\rho(\theta|p, \underline{x}) = \frac{p^{(D^*+k-1)} \int_0^\infty \theta^{(D^*-1)}\{\prod_{i=1}^{D^*} x_i^{(p-1)} e^{-\theta\sum_{i=1}^{D^*} x_i{}^p}\}\ [\exp(-\theta x^p)]^{(n-D^*)}d\theta}{\phi}; \qquad \textbf{(4.6)}$$

Similarly integrating the joint posterior distribution with respect to the p, we get the marginal posterior probability density functions of p is given as

$$\rho(\theta|p, \underline{x}) = \frac{\theta^{(D^*-1)} \int_0^\infty p^{(D^*+k-1)}\{\prod_{i=1}^{D^*} x_i^{(p-1)} e^{-\theta\sum_{i=1}^{D^*} x_i{}^p}\}\ [\exp(-\theta x^p)]^{(n-D^*)}dp}{\phi}; \qquad \textbf{(4.7)}$$

It is well known that under a squared error function the Bayes estimator of a parameter will be its posterior expectation and posterior variance will be the Bayes risk. To obtain the posterior mean and variance a numerical integration is required.

Then the posterior mean and variance of the scale and shape parameter (θ, p) are expressed as follows

$$\hat{\theta} = E(\theta|p, \underline{x});$$

$$\hat{\theta} = \int_0^\infty \int_0^\infty \theta^{D^*}p^{(D^*+k-1)}\{\prod_{i=1}^{D^*} x_i^{(p-1)} e^{-\theta\sum_{i=1}^{D^*} x_i{}^p}\}\ [\exp(-\theta x^p)]^{(n-D^*)}\, dpd\theta; \qquad \textbf{(4.8)}$$

Now the variance of $\hat{\theta}$ of the scale parameter θ is given by

$$v(\theta|p, \underline{x}) = \int_0^\infty \int_0^\infty (\hat{\theta}^2 - \theta)^2\theta^{(D^*-1)}p^{(D^*+k-1)}\{\prod_{i=1}^{D^*} x_i^{(p-1)} e^{-\theta\sum_{i=1}^{D^*} x_i{}^p}\}\ [\exp(-\theta x^p)]^{(n-D^*)}\, dpd\theta \qquad \textbf{(4.9)}$$

Similarly we can find the posterior mean and variance of the shape parameter p as

$$\hat{p} = \int_0^\infty \int_0^\infty \theta^{(D^*-1)} p^{(D^*+k)} \{\prod_{i=1}^{D^*} x_i^{(p-1)} e^{-\theta \sum_{i=1}^{D^*} x_i{}^p}\} \ [\exp(-\theta x^p)]^{(n-D^*)} \, d\theta dp; \quad \textbf{(4.10)}$$

$$Var\left(p|\theta,\underline{x}\right) = \int_0^\infty \int_0^\infty (\hat{p}-p)^2 \ \theta^{(D^*-1)} p^{(D^*+k-1)} \{\prod_{i=1}^{D^*} x_i^{(p-1)} e^{-\theta \sum_{i=1}^{D^*} x_i{}^p}\} \ [\exp(-\theta x^p)]^{(n-D^*)} \, d\theta dp; \quad \textbf{(4.11)}$$

Equations (4.9) to (4.11) are very difficult to evaluate theoretically. A numerical procedure is needed to solve these equations numerically. R Software is used for the estimation of the scale and shape parameter(θ, p).

5. Numerical Results

The numerical simulations have been carried out by comparing the performances of the MLE's and the Bayes estimators. The different choices of n, r and T values have been taken for simulation. It is clear that, there are no complete solutions for obtaining new estimators either in both non-Bayesian and Bayesian approaches. So we have taken random data, and applied numerical integration for the solution through computer programming. We have illustrated the results numerically which are obtained in the previous sections like the maximum likelihood estimators and their variance-covariance matrix for the unknown parameters of Weibull distribution under hybrid censored sample obtained with following procedures

1. The random sample of size 10, 20, 30, 40and 50 are generated from Weibull distribution by Monte Carlo random number generation method. The selected values of the true parameters are θ= (3, 5) and p = (2.5, 3.5).
2. For hybrid censored sample, we have chosen the censoring time T and the number of failed items r.
3. The equations (3.4) and (3.5) are solved in order to obtain the maximum likelihood estimators $\hat{\theta}$ and $\hat{p}$ of θ and p respectively.
4. The variance covariance matrix of ($\hat{\theta}$,$\hat{p}$) is obtained by using eqn. (3.5).

The results are presented in Table (1) .From Table (1), we note that as we increase the sample size the standard deviation starts decreasing, similarly the maximum likelihood estimator of the parameters has the same behaviors when the sample size becomes large and the properties of two parameters $\theta$and p at (2.5 and 3.5) respectively is better than the other values. To obtain the posterior mean and the posterior variance of θ and p, the numerical procedures is described as follow:

Step 1 and 2 is repeated as above, then

2: For hybrid censored sample, we have chosen the censoring time *'T'* and the number of failed *items 'r*'.

3: The equations (4.8) and (4.9) are solved to obtain non linear Bayes estimator and its posterior variance of the scale and shape parameter (θ, p).

5: The posterior mean and the posterior variance of the estimators for the shape and scale parameter (θ, p) for all sample size and for sets of parameters were obtained.

Numerical results are summarized in Table (2), it is noted that the posterior mean decreases when n is increasing. Similarly, the posterior variance of the parameters has the same behaviors when the sample size becomes large.

ssss For the value of D*, we have to take decision as

H terminate the experiment at min $(T, X_{(r)})$.

$$H=\begin{cases} 1\,, if\ H = X(r) \\ 0\,, if\ H = T \end{cases} \quad ; \quad \text{and} \quad k\text{: The prior}$$

Table 1: The maximum likelihood Estimator, the standard deviation and covariance of the Weibull distribution with two parameter under hybrid censored sample when

(θ = 2.5, p = 3.5)

n	**r**	**T**	**H**	$\hat{\boldsymbol{\theta}}$ MLE	$\hat{\boldsymbol{\theta}}$ S.D.	$\hat{\boldsymbol{p}}$ MLE	$\hat{\boldsymbol{p}}$ S.D.	**Cov($\hat{\boldsymbol{\theta}}$,$\hat{\boldsymbol{p}}$)**
10	5	0.5	0	3.4824	1.3045	3.5145	1.3409	30.3834
			1	3.9811	1.9705	4.4062	1.2235	
		1.0	1	3.2515	1.3682	3.7270	1.3092	9.3277
			0	2.5197	1.1177	2.6503	1.9687	
20	10	0.5	0	2.1638	1.0369	2.2837	1.8507	27.9243
			0	2.4211	1.1438	2.6012	1.9098	
		1.0	1	1.9782	0.9496	2.1778	0.9999	25.7656
			1	1.6297	1.9988	1.7024	1.0689	
30	20	0.5	0	1.4290	2.5402	1.4785	2.3713	22.0097
			1	1.3102	2.6347	1.5573	2.2051	
		1.0	1	1.1966	3.0196	1.2907	1.7087	20.0079
			0	1.1199	3.0312	1.3328	1.4849	
40	30	0.5	1	1.0715	3.7478	1.1500	1.3203	20.0079
			1	1.0708	3.7199	1.4505	1.2196	
		1.0	1	1.0551	3.3374	1.4040	1.2196	20.0079
			0	1.0412	3.1931	1.3357	1.1286	

Table 1(continued): The maximum likelihood Estimator, the standard deviation and covariance of the Weibull distribution with two parameter under hybrid censored sample when (θ = 3, p = 4)

n	**r**	**T**	**H**	$\hat{\boldsymbol{\theta}}$ MLE	$\hat{\boldsymbol{\theta}}$ S.D.	$\hat{\boldsymbol{p}}$ MLE	$\hat{\boldsymbol{p}}$ S.D.	**Cov($\hat{\boldsymbol{\theta}}$,$\hat{\boldsymbol{p}}$)**
10	5	0.5	0	1.7221	2.9368	3.1010	2.2290	9.1721
			1	1.6301	2.9159	3.1311	2.3897	
		1.0	1	1.6150	2.9014	2.6216	1.9925	16.0421
			1	1.5452	2.9009	2.7201	1.9434	
		0.5	0	1.5157	2.8632	2.2884	1.8363	24.7854
			0	1.4638	2.8415	2.3656	1.1358	

20	10	1.0	1 1	1.4239 1.3860	2.7233 2.7017	2.0050 2.0633	1.0660 1.0822	30.7062
30	20	0.5	0 1	1.3389 1.3121	2.6927 2.6466	1.7645 1.8069	1.0874 1.0705	32.8731
		1.0	1 0	1.2603 1.2421	2.6424 2.5820	1.5602 1.5897	1.1058 1.1573	22.2539
40	30	0.5	1 1	1.1875 1.1760	2.5530 2.5052	1.3862 1.4054	1.5252 1.6696	19.7964
		1.0	1 0	1.1202 1.1137	2.4650 2.4346	1.2376 1.2487	1.7484 1.0086	11.4555

Table 1(continued): The maximum likelihood Estimator, the standard deviation and covariance of Weibull distribution with two parameter under hybrid censored sample when (θ = 3.5, p = 4.5)

n	r	T	H	$\hat{\theta}$		$\hat{p}$		Cov($\hat{\theta}$,$\hat{p}$)
				MLE	**S.D.**	**MLE**	**S.D.**	
10	5	0.5	0 0	2.0625 1.9852	2.3290 2.2897	7.4247 6.2610	3.2258 3.0956	13.2442
		1.0	1 0	1.9922 1.9667	2.2925 2.2434	7.1729 5.1460	3.3960 2.8817	24.5450
20	10	0.5	0 0	1.9272 1.9018	2.1363 2.0358	5.0463 4.1047	2.5227 1.9619	25.7779
		1.0	1 1	1.8843 1.7993	1.0660 1.0002	3.6978 3.2509	1.5699 1.5781	35.9740
30	20	0.5	0 1	1.6912 1.6713	0.9999 0.9999	2.8028 2.5888	1.1017 1.0169	34.9265
		1.0	1 0	1.6282 1.5303	0.9998 0.9973	2.1870 2.0849	0.9946 0.8633	23.5734
40	30	0.5	1 1	1.5186 1.3866	0.9952 0.7696	1.7489 1.7015	0.7239 0.6966	12.5127
		1.0	1 0	1.4134 1.2476	0.9484 0.7086	1.4277 1.4076	0.5451 0.5206	11.7491

Table 1(continued): The maximum likelihood Estimator, the standard deviation and covariance of Weibull distribution with two parameter under hybrid censored sample when (θ = 3.5, p = 4.5)

n	r	T	H	$\hat{\theta}$		$\hat{p}$		Cov($\hat{\theta}$,$\hat{p}$)
				MLE	**S.D.**	**MLE**	**S.D.**	
10	5	0.5	0 1	6.3230 5.5561	1.5380 1.3152	5.7167 5.6993	2.1952 2.1863	13.0083
		1.0	1 0	5.5173 4.8450	1.4325 1.2747	4.8784 3.7624	1.9663 1.8905	22.7741
		0.5	0 0	4.4352 3.9359	1.3221 1.2211	4.3947 4.9769	1.7594 2.0930	22.9455

20	10	1.0	1	3.8882	1.2121	3.9922	1.9770	32.1153
			1	2.5487	1.5880	3.0878	1.8938	
30	20	0.5	0	2.8643	1.6602	3.3050	1.5287	21.7955
			1	2.3707	1.2887	4.9214	2.0928	
		1.0	1	2.2212	1.1067	2.6056	1.1363	19.5155
			0	2.1638	1.0166	3.3589	1.7901	
40	30	0.5	1	2.0163	0.9148	2.0237	1.0070	16.2796
			1	1.9909	0.9442	2.3548	1.2057	
		1.0	1	1.5523	1.8311	1.5782	1.6967	11.0848
			0	1.5416	1.8735	1.7123	1.8916	

Table 2: The posterior mean and posterior variance of the Weibull distribution under hybrid censored sample when (θ = 2.5, p = 3.5, k=2)

n	r	T	H	$\boldsymbol{\theta}$		$\boldsymbol{p}$	
				posterior mean	**posterior variance**	**posterior mean**	**posterior variance**
10	5	0.5	0	1.0190	2.0064	2.0341	1.3151
			0	1.4956	2.5633	2.0322	1.8718
		1.0	1	1.2199	2.1550	1.8594	1.5932
			0	2.0642	0.9024	1.8588	2.5400
20	10	0.5	0	1.4305	2.2570	1.7047	1.8614
			0	2.8129	1.3091	1.7049	3.3141
		1.0	1	1.6148	2.8610	1.5675	2.0445
			1	3.5482	1.9576	1.5681	3.8708
30	20	0.5	0	1.7229	1.7437	1.4454	2.0692
			1	3.7965	2.0467	1.4461	3.7949
		1.0	1	1.7168	2.0088	1.3363	1.9240
			0	3.3309	1.6636	1.3370	13.1500
40	30	0.5	1	1.5988	1.6251	1.2385	1.6708
			1	2.5491	1.6657	1.2391	2.3834
		1.0	1	1.4097	1.3126	1.1507	1.3897
			0	1.8532	1.5572	1.1511	1.7530

Table 2(continued): The posterior mean and posterior variance of the Weibull distribution under hybrid censored sample when (θ = 3, p = 4, k=2)

n	r	T	H	$\hat{\boldsymbol{\theta}}$		$\hat{\boldsymbol{p}}$	
				posterior mean	**posterior variance**	**posterior mean**	**posterior variance**
10	5	0.5	0	4.1338	1.2290	10.6634	1.2290
			0	4.2152	2.3897	15.7899	2.3897
	7	1.0	1	3.3754	1.1925	8.2480	1.1925
			0	3.4296	2.2434	10.9033	2.2434
20	10	0.5	0	2.7926	1.1363	5.9877	1.1363
			0	2.8267	2.0358	7.1931	2.0358

	15	1.0	1	2.3398	1.0660	4.3274	1.0660
			1	2.3637	1.8022	4.8721	1.8022
	20	0.5	0	1.9835	0.9874	3.1901	0.9874
30			1	1.9991	1.5705	3.4435	1.5705
	25		1	1.6996	0.9058	2.4145	0.9058
		1.0	0	1.7095	1.3573	2.5350	1.3573
			1	1.4704	0.8252	1.8756	0.8252
	30	0.5	1	1.4764	1.1696	1.9325	1.1696
40			1	1.2833	0.7484	1.4915	0.7484
	35	1.0	0	1.2865	1.0086	1.5165	1.0086

Table 2(continued): The posterior mean and posterior variance of the Weibull distribution under hybrid censored sample when (θ = 3.5, p = 4.5,k=2)

n	**r**	**T**	**H**	$\boldsymbol{\theta}$		$\boldsymbol{p}$	
				posterior mean	**posterior variance**	**posterior mean**	**posterior variance**
	5		0	0.8117	1.6881	1.4346	0.8679
		0.5	1	0.8425	1.8349	1.3322	1.3917
10	7		1	1.1671	0.9693	1.3797	0.8368
		1.0	0	1.2624	0.8121	1.3000	1.3159
	10	0.5	0	1.7577	0.7491	1.3262	0.8014
20			0	2.0374	1.0066	1.2656	1.2309
	15	1.0	1	2.7245	1.3280	1.2743	0.7632
			1	3.5333	1.8588	1.2295	1.1415
	20	0.5	0	3.9178	2.0061	1.2241	0.7233
30			1	5.9265	2.7294	1.1922	1.0517
	25	1.0	1	4.3580	2.1839	1.1757	0.6827
			0	6.7585	3.0991	1.541	0.9466
		0.5	1	3.4359	1.9915	1.1290	0.6424
40	30		1	4.5310	2.2683	1.1155	0.8820
		1.0	1	2.2727	1.0091	1.0842	0.6030
	35		0	2.5843	1.0974	1.0768	0.8052

Table 2(continued): The posterior mean and posterior variance of the Weibull distribution under hybrid censored sample when (θ = 4.5, p = 5)

n	**r**	**T**	**H**	$\boldsymbol{\theta}$		$\boldsymbol{p}$	
				posterior mean	**posterior variance**	**posterior mean**	**posterior variance**
	6	0.5	0	5.2147	1.2687	8.2480	4.2354
10			1	4.9517	1.2312	6.2354	3.1002
	8	1.0	1	4.2370	1.0121	5.2781	2.0090
			0	3.9604	0.9991	4.6697	2.0074
	14	0.5	0	2.5215	1.3280	3.5077	1.9125
20			0	2.3719	1.3988	2.5975	1.4998

	18	1.0	1	1.9683	2.1294	1.5217	1.8210
			1	1.5486	2.5839	1.4573	1.7989
30	22	0.5	0	1.4001	2.5615	1.2672	1.6658
			1	1.3368	2.7683	1.1956	1.4412
	26	1.0	1	1.2927	2.9347	1.8756	1.8991
			0	1.2350	2.9170	1.9626	1.9995
40	30	0.5	1	1.1817	3.0527	1.4915	1.6541
			1	1.1257	3.1966	1.5165	1.5976
	36	1.0	1	1.1500	3.0070	1.2105	1.7421
			0	1.1325	3.5650	1.2192	1.8897

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