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An Extension of Method of Proportion for Solving Nonlinear Equations

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Abstract

In this paper an iterative method is presented for solving nonlinear equations based on the proportion of two real parameters, associated with the given equations. In the proposed method, a single iterative formula is generated by considering the additional terms in an expression given in the method of proportion developed earlier. It is also shown that the number of iterations decreases with the increases of the number terms in the right hand side of a key expression. The methods are supported by various numerical examples and shown that the new proposed methods are effective and comparable to the well-known Newton's method.

Key words: Nonlinear equation, Proportion, Iterative method, Newton's method.

INTRODUCTION

One of the most basic and fundamental problems in numerical analysis is that of finding values of the variable x, which satisfies

$$f(x) = 0 \qquad (1)$$

for a given function f. There are so many existing standard methods are available for solving such equation (1). Of course out of them Newton's method is very popular and useful. In past few years so many authors have considered the nonlinear equations and gave us a new idea towards the solution and compare their result with the well known Newton's methods [1-4]. Basto et al.[1] and Chen at al.[2] were presented an iterative formula in which the order of convergence is fixed and in particular, in [1] when the other iterative method is considered by taking more terms in the series solution obtained by Adomian modified method [5, 6] or by considering Taylor's expansion of higher order, the order of convergence was not increased. A new method of proportion [7] was presented in which, the order of convergence was increased by taking more terms in an expression taking the vital role for the increment of order of convergence. In the method [7], two iterative formula were presented by considering the first one and first two terms of an expression to find out the solution of a non-linear equation (1). In this method an iterative formula is presented by considering the first three terms of an expression described in [7].

THE METHOD

In the method [7], after finding the sufficient small interval [a_0,b_0], the expression for the root α of the equation f(x) = 0 is given by

$$\alpha = b_0 - \frac{b_0 - a_0}{1 - \frac{f(a_0)}{f(b_0)}} - D, \quad \text{if } \left|\frac{f(a_0)}{f(b_0)}\right| < 1$$

$$= a_0 + \frac{b_0 - a_0}{1 - \frac{f(b_0)}{f(a_0)}} - D, \quad \text{if } \left|\frac{f(b_0)}{f(a_0)}\right| < 1 \qquad (2)$$

$$= \frac{a_0 + b_0}{2} - D, \qquad \text{if } \left|\frac{f(a_0)}{f(b_0)}\right| = 1$$

where D is given by, $D = \dfrac{f(w_0)}{f^{/}(\xi_0)}$, where w_0 is a point in [a_0,b_0] given by

$$w_0 = a_0 - \frac{b_0 - a_0}{f(b_0) - f(a_0)} f(a_0) \qquad (3)$$

which is our well-known Regula-falsi formula and $\min\{\alpha, w_0\} < \xi_0 < \max\{\alpha, w_0\}$ (4) or, D satisfies the expression,

$$D\left[f^{/}(w_0) + \frac{D}{2} f^{//}(w_0) + \frac{D^2}{3!} f^{///}(w_0) + \frac{D^3}{4!} f^{iv}(w_0) + \ldots\ldots + \frac{D^{n-2}}{(n-1)!} f^{n-1}(w_0) + \frac{D^{n-1}}{n!} f^{n}(w_0 + \theta D)\right]$$
$$= f(w_0), \text{ where } 0 < \theta < 1 \qquad (5)$$

Here ξ_0 is a point where the tangent to the curve y = f(x) at (ξ_0 , f (ξ_0)) is parallel to the chord joining the points (α, 0) and (w_0, f (w_0)). Since α is unknown we cannot locate exactly the point ξ_0 and consequently, we cannot exactly determine the value of θ. So, we approximate the value of D by taking the first few terms of (5).

Particular Method.

If we consider the first three terms of the expression (5) then the approximate value of D is given by

$$D = \frac{f(w_0)}{f^{/}(w_0) + \frac{D}{2} f^{//}(w_0) + \frac{D^2}{6} f^{///}(w_0)} \qquad (6)$$

i.e. D satisfies the cubic equation given by,

$$D^3 f^{///}(w_0) + 3D^2 f^{//}(w_0) + 6D f^{/}(w_0) - 6 f(w_0) = 0 \qquad (7)$$

Now again we approximated the value of D by putting $D = \dfrac{f(w_0)}{f^{/}(w_0)}$ in the right hand side of (6) and we get $D = \dfrac{f(w_0)}{f^{/}(w_0) + \dfrac{f(w_0)}{2f^{/}(w_0)} f^{//}(w_0) + \dfrac{f^2(w_0)}{6f^{/2}(w_0)} f^{///}(w_0)}$

$$= \frac{6f(w_0)f^{/2}(w_0)}{6\{f^{/}(w_0)\}^3 + 3f(w_0)f^{/}(w_0)f^{//}(w_0) + f^2(w_0)f^{///}(w_0)} \qquad (8)$$

Hence the n-th approximation of the root α is given by

$$x_{n+1} = b_n - \frac{b_n - a_n}{1 - \dfrac{f(a_n)}{f(b_n)}} - D, \quad \text{if } \left|\frac{f(a_n)}{f(b_n)}\right| < 1$$

$$= a_n + \frac{b_n - a_n}{1 - \dfrac{f(b_n)}{f(a_n)}} - D, \quad \text{if } \left|\frac{f(b_n)}{f(a_n)}\right| < 1 \qquad (9)$$

$$= \frac{a_n + b_n}{2} - D, \quad \text{if } \left|\frac{f(a_n)}{f(b_n)}\right| = 1$$

where D is given by $D = \dfrac{6f(w_n)f^{/2}(w_n)}{6\{f^{/}(w_n)\}^3 + 3f(w_n)f^{/}(w_n)f^{//}(w_n) + f^2(w_n)f^{///}(w_n)}$

and w_n is given by $w_n = a_n - \dfrac{b_n - a_n}{f(b_n) - f(a_n)} f(a_n), \quad a_n < \alpha < b_n$

Numerical Experiments and Comparison

We now compare our new methods with the Newton's method and in Table-1. All the examples are taken from the references at the end of this paper

The test functions f(x) is as follows:

$f_1(x) = x - 2 - e^{-x}$, α = 2.1200282574,

$f_2(x) = x^3 + x + 1$, α = - 0.6823511720,

$f_3(x) = \ln x$, α = 0.9999467134,

$f_4(x) = xe^{-x} - 0.1$, α = 0.1118325517,

$f_5(x) = \dfrac{1}{x} - \sin x + 1 = 0$, α = - 0.6294364929.

Table-1:-(Comparative Statement)

f(x)	Method	Initial Approx./ intervalx_0/[a_0,b_0]	Tolerance error(ε)	No. of iteration	Obtained Solution
$f_1(x)$	Newton's Method	2	10^{-7}	3	2.1200282574
	New Method	[2, 3]		3	2.1200282574
$f_2(x)$	Newton's Method	0	10^{-4}	6	-0.6823278069
	New Method	[-1, 0]		4	- 0.6823511720
$f_3(x)$	Newton's Method	0.5	10^{-4}	5	1.0000000000
	New Method	[0.5, 5]		9	0.9999467134
$f_4(x)$	Newton's Method	0	10^{-7}	4	0.1118325591
	New Method	[0, 1]		5	0.1118325517
$f_5(x)$	Newton's Method	-3	10^{-4}	5	-3.9885725975
	New Method	[-1.3, 0.5]		3	- 0.6294364929

All computations are done by the 'C' programming language. Here we take the approximate solution, depending upon the precision (ε) of the computer. For the computer program the stopping criteria $\left|x_{n+1} - x_n\right| < \varepsilon$ is used, and when the stopping criteria is satisfied α is taken as the approximate value of the root. For all the numerical examples given in Table 1 the fixed stopping criteria (ε), as shown in the Table-1, is used.

CONCLUSION

In this work, a single general iterative formula (9) is proposed in which linear convergence is assured, if we remove the term D from the expression (9) and further if we take only the first three term of (5) then the algorithm gives better than the Newton's method in some cases with respect to the number of iterations.. The performance of our methods has been compared with the well known Newton's methods and it gives equal or better result. Of course we can give another iterative formula by considering more than first three terms of (5) and it is an open question whether this another new iterative formula.

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