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Bipolar Valued Fuzzy Subbisemigroups of a Bisemigroup

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Abstract

In this paper, bipolar valued fuzzy subbisemigroup is defined and introduced and using this concept, some important theorems are stated and proved. In bisemigroup, concept of bipolar valued fuzzy set is applied.
Keywords: Bipolar valued fuzzy set, bipolar valued fuzzy subsemigroup, bipolar valued fuzzy subbisemigroup, bisemigroup.

INTRODUCTION:

In 1965, Zadeh [7] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, soft sets etc. W.R.Zhang [8, 9] introduced an extension of fuzzy sets named bipolar valued fuzzy sets in 1994 and bipolar valued fuzzy set was developed by Lee [4, 5]. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [−1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [9]. Vasantha kandasamy.W.B[6] introduced the basic idea about the fuzzy group and fuzzy bigroup. M.S.Anithat et.al[1] introduced the bipolar valued fuzzy subgroup and A.Balasubramanian et.al[2] introduced the intuitionistic fuzzy subbigroup of a bigroup. Justin Prabu.T and K.Arjunan[3] introduced Q-fuzzy subbigroup of a bigroup. The above papers have motivated to work this area. Bisemigroup is one of the algebraic structures. The bipolar valued fuzzy concept is applied in bisemigroup and bipolar valued fuzzy subbisemigroup of a bisemigroup is introduced.

1. PRELIMINNRIES

1.1Definition.

[6] A set (G, + , $\bullet$) with two binary operations + and $\bullet$ is called a bisemigroup if there exist two proper subsets G_1 and G_2 of G such that (i) $G = G_1 \cup G_2$ (ii) (G_1 , +) is a semigroup (iii) (G_2 , $\bullet$) is a semigroup.

1.2 Definition.

[8] A bipolar valued fuzzy set N in X is defined as an object of the form $N = \{ < x, N^+(x), N^-(x) >/ x \in X\}$, where $N^+ : X \rightarrow [0, 1]$ and $N^- : X \rightarrow [-1, 0]$. The positive membership degree $N^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set N and the negative membership degree $N^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set N. If $N^+(x) \neq 0$ and $N^-(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for N and if $N^+(x) = 0$ and $N^-(x) \neq 0$, it is the situation that x does not satisfy the property of N, but somewhat satisfies the counter property of N. It is possible for an element x to be such that $N^+(x) \neq 0$ and $N^-(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X.

1.3 Example.

$N = \{ < a, 0.8, -0.4 >, < b, 0.6, -0.7 >, < c, 0.2, -0.4 >\}$ is a bipolar valued fuzzy subset of $X = \{ a, b, c \}$.

1.4 Definition.

[1] Let G be a semigroup. A bipolar valued fuzzy subset N of G is said to be a bipolar valued fuzzy subsemigroup of G if the following conditions are satisfied,

(i) $N^+(xy) \geq \min\{ N^+(x), N^+(y) \}$,

(ii) $N^-(xy) \leq \max\{ N^-(x), N^-(y) \}$, for all x and y in G.

1.5 Example.

Let $G = \{1, \omega, \omega^2 \}$ be a semigroup with respect to the ordinary multiplication. Then $N = \{ < 1, 0.9, -0.5 >, < \omega, 0.4, -0.3 >, < \omega^2, 0.4, -0.3 > \}$ is a bipolar valued fuzzy subsemigroup of G.

1.6 Definition.

Let $G = (G_1 \cup G_2, +, \bullet)$ be a bisemigroup. Then a bipolar valued fuzzy set A of G is said to be a BVFSBSG of G if there exist two bipolar valued fuzzy subsets A_1 of G_1 and A_2 of G_2 such that (i) $A = A_1 \cup A_2$ (ii) A_1 is a bipolar valued fuzzy subsemigroup of (G_1 , +) (iii) A_2 is a bipolar valued fuzzy subsemigroup of (G_2 , $\bullet$).

2. PROPERTIES

Theorem 2.1

Let $A = M \cup N = \langle (M\cup N)^+, (M\cup N)^- \rangle$ be a BVFSBSG of a bisemigroup $G = H \cup F$.

(i) If $M^+(x+y) = 0$, then either $M^+(x) = 0$ or $M^+(y) = 0$, for x and y in H.

(ii) If $M^-(x+y) = 0$, then either $M^-(x) = 0$ or $M^-(y) = 0$, for x and y in H.

(iii) If $N^+(xy) = 0$, then either $N^+(x) = 0$ or $N^+(y) = 0$, for x and y in F.

(iv) If $N^-(xy) = 0$, then either $N^-(x) = 0$ or $N^-(y) = 0$, for x and y in F.

Proof. (i) Let x and y in H. By the definition $M^+(x+y) \geq \min\{ M^+(x), M^+(y) \}$, which implies that $0 \geq \min\{ M^+(x), M^+(y) \}$. Therefore, either $M^+(x) = 0$ or $M^+(y) = 0$. (ii) By the definition $M^-(x+y) \leq \max\{ M^-(x), M^-(y) \}$, which implies that $0 \leq \max\{ M^-(x), M^-(y) \}$. Therefore, either $M^-(x) = 0$ or $M^-(y) = 0$. (iii) Let x and y in F. By the definition $N^+(xy) \geq \min\{ N^+(x), N^+(y) \}$, which implies that $0 \geq \min\{ N^+(x), N^+(y) \}$. Therefore, either $N^+(x) = 0$ or $N^+(y) = 0$. (iv) By the definition $N^-(xy) \leq \max\{ N^-(x), N^-(y) \}$, which implies that $0 \leq \max\{ N^-(x), N^-(y) \}$. Therefore, either $N^-(x) = 0$ or $N^-(y) = 0$.

Theorem2.2

If $A = M\cup N = \langle (M\cup N)^+, (M\cup N)^- \rangle$ is a BVFSBSG of a bisemigroup $G = H\cup F$, then

(i) $L = \{ x\in H \mid M^+(x) = 1, M^-(x) = -1 \}$ is either empty or is a subgroup of H.

(ii) $K = \{ x\in F \mid N^+(x) = 1, N^-(x) = -1 \}$ is either empty or is a subgroup of F.

(iii) $R = L \cup K$ is either empty or a subbisemigroup of G.

Proof. (i) If no element satisfies this condition, then L is empty. If x and y in L, then $M^+(x+y) \geq \min\{ M^+(x), M^+(y) \} = \min\{ 1, 1 \}= 1$. Therefore $M^+(x+y) = 1$. And $M^-(x+y) \leq \max\{ M^-(x), M^-(y) \} = \max\{ -1, -1\}= -1$. Therefore $M^-(x+y) = -1$. That is $x+y\in L$. Hence L is a subgroup of H. Hence L is either empty or is a subgroup of H.

(ii) If no element satisfies this condition, then K is empty. If x and y in K, then $N^+(xy) \geq \min\{ N^+(x), N^+(y) \} = \min\{ 1, 1 \}= 1$. Therefore $N^+(xy) = 1$. And $N^-(xy) \leq \max\{ N^-(x), N^-(y) \} = \max\{ -1, -1\}= -1$. Therefore $N^-(xy) = -1$. That is $xy^{-1}\in K$. Hence K is a subgroup of F. Hence K is either empty or is a subgroup of F.

(iii) Clearly $R = L \cup K$ is either empty or a subbisemigroup of G.

Theorem 2.3

If $A = M\cup N = \langle (M\cup N)^+, (M\cup N)^- \rangle$ and $B = R\cup S= \langle (R\cup S)^+, (R\cup S)^- \rangle$ are two BVFSBSGs of a bisemigroup $G = H\cup F$, then their intersection $A\cap B$ is a BVFSBSG of G.

Proof. Let $C = A\cap B$ and $C = U\cup V = \langle (U\cup V)^+, (U\cup V)^- \rangle$. Now $U^+(x+y) = \min\{ M^+(x+y), R^+(x+y) \}\geq \min\{ \min\{M^+(x), M^+(y) \}, \min\{R^+(x), R^+(y) \} \}\geq \min\{ \min\{M^+(x), R^+(x) \}, \min\{M^+(y), R^+(y) \} \}= \min\{ U^+(x), U^+(y) \}$. Therefore $U^+(x+y) \geq \min\{U^+(x), U^+(y) \}$. Also $U^-(x+y) = \max\{ M^-(x+y), R^-(x+y) \}\leq \max\{ \max\{M^-(x), M^-(y) \}, \max\{R^-(x), R^-(y) \}\}\leq \max\{ \max\{ M^-(x), R^-(x) \}, \max\{M^-(y), R^-(y)\}\} = \max\{ U^-(x), U^-(y) \}$. Therefore $U^-(x+y) \leq \max\{ U^-(x), U^-(y) \}$. Thus U is a bipolar valued fuzzy subgroup of H. Now $V^+(xy) = \min\{ N^+(xy), S^+(xy) \}\geq \min\{\min\{N^+(x), N^+(y) \}, \min\{S^+(x), S^+(y) \} \}\geq \min\{ \min\{N^+(x), S^+(x) \}, \min\{N^+(y), S^+(y) \} \}= \min\{ V^+(x), V^+(y) \}$. Therefore $V^+(xy) \geq \min\{V^+(x), V^+(y) \}$. Also, $V^-(xy) = \max\{ N^-(xy), S^-(xy) \}\leq \max\{ \max\{N^-(x), N^-(y)\}, \max\{S^-(x), S^-(y) \}\} \leq \max\{ \max\{N^-(x), S^-(x)\}, \max\{N^-(y), S^-(y)\}\} = \max\{ V^-(x), V^-(y) \}$. Therefore $V^-(xy) \leq \max\{ V^-(x), V^-(y) \}$. Thus V is a bipolar valued fuzzy subgroup of F. Hence $C = U\cup V$ is a BVFSBSG of G.

Theorem 2.4

The intersection of a family of BVFSBSGs of a bisemigroup $G = H\cup F$ is a BVFSBSG of G.

Proof. The proof follows from the Theorem 2.3.

Theorem 2.5.
If $A = M\cup N = \langle (M\cup N)^+, (M\cup N)^- \rangle$ and $B = R\cup S = \langle (R\cup S)^+, (R\cup S)^- \rangle$ are any two BVFSBSGs of the bigroups $G_1 = H_1\cup F_1$ and $G_2 = H_2\cup F_2$ respectively, then $A\times B = (M\cup N)\times(R\cup S) = \langle (M\times R)^+\cup(M\times S)^+\cup(N\times R)^+ \cup(N\times S)^+, (M\times R)^-\cup(M\times S)^-\cup(N\times R)^-\cup(N\times S)^- \rangle$ is a bipolar valued fuzzy sub-four-semigroup of $G_1\times G_2$.

Proof. Let A and B be two BVFSBSGs of the bigroups G_1 and G_2 respectively. Let a_1 and a_2 be in G_1, b_1 and b_2 be in G_2. Then (a_1, b_1) and (a_2, b_2) are in $G_1\times G_2$. If (a_1, b_1) and (a_2, b_2) are in $H_1\times H_2$, then $(M\times R)^+[(a_1, b_1)+ (a_2, b_2)] = (M\times R)^+(a_1+a_2, b_1+b_2) = \min \{ M^+(a_1+a_2), R^+(b_1+b_2)\} \geq \min \{ \min\{M^+(a_1), M^+(a_2)\}, \min\{R^+(b_1), R^+(b_2)\}\} = \min\{ \min \{M^+(a_1), R^+(b_1) \}, \min\{ M^+(a_2), R^+(b_2)\}\} = \min\{(M\times R)^+(a_1, b_1), (M\times R)^+(a_2, b_2) \}$. Therefore, $(M\times R)^+[(a_1, b_1)+ (a_2, b_2)] \geq \min\{ (M\times R)^+(a_1, b_1), (M\times R)^+(a_2, b_2) \}$. If (a_1, b_1) and (a_2, b_2) are in $H_1\times F_2$, then $(M\times S)^+[(a_1, b_1)+(a_2, b_2)] = (M\times S)^+(a_1+a_2, b_1b_2) = \min \{ M^+(a_1+a_2), S^+(b_1b_2)\} \geq \min \{ \min\{M^+(a_1), M^+(a_2)\}, \min\{S^+(b_1), S^+(b_2)\}\} = \min\{ \min \{M^+(a_1), S^+(b_1) \}, \min\{ M^+(a_2), S^+(b_2) \}\} = \min\{(M\times S)^+(a_1, b_1), (M\times S)^+(a_2, b_2) \}$. Therefore, $(M\times S)^+[(a_1, b_1)+(a_2, b_2)] \geq \min \{(M\times S)^+(a_1, b_1), (M\times S)^+(a_2, b_2) \}$. If (a_1, b_1) and (a_2, b_2) are in $F_1\times H_2$, then $(N\times R)^+[(a_1, b_1)(a_2, b_2)] = (N\times R)^+(a_1a_2, b_1+b_2) = \min \{N^+(a_1a_2), R^+(b_1+b_2)\} \geq \min \{\min\{N^+(a_1), N^+(a_2)\}, \min\{R^+(b_1), R^+(b_2)\}\} = \min \{\min \{N^+(a_1), R^+(b_1) \}, \min \{N^+(a_2), R^+(b_2)\}\}= \min\{(N\times R)^+(a_1, b_1), (N\times R)^+(a_2, b_2) \}$. Therefore, $(N\times R)^+[(a_1, b_1)(a_2, b_2)] \geq \min\{ (N\times R)^+(a_1, b_1), (N\times R)^+(a_2, b_2) \}$. If (a_1, b_1) and (a_2, b_2) are in $F_1\times F_2$, then $(N\times S)^+[(a_1, b_1)(a_2, b_2)] = (N\times S)^+(a_1a_2, b_1b_2) = \min \{N^+(a_1a_2), S^+(b_1b_2) \}\geq \min \{ \min\{N^+(a_1), N^+(a_2)\}, \min\{ S^+(b_1), S^+(b_2) \}\} = \min\{ \min \{N^+(a_1), S^+(b_1) \}, \min\{ N^+(a_2), S^+(b_2) \}\}= \min\{(N\times S)^+ (a_1, b_1), (N\times S)^+(a_2, b_2) \}$. Therefore, $(N\times S)^+[(a_1, b_1)(a_2, b_2)] \geq \min\{ (N\times S)^+(a_1, b_1), (N\times S)^+(a_2, b_2) \}$. If (a_1, b_1) and (a_2, b_2) are in $H_1\times H_2$, then $(M\times R)^-[(a_1, b_1)+ (a_2, b_2)] = (M\times R)^-(a_1+a_2, b_1+b_2) = \max \{M^-(a_1+a_2), R^-(b_1+b_2)\} \leq \max \{ \max\{M^-(a_1), M^-(a_2)\}, \max\{R^-(b_1), R^-(b_2)\}\} = \max \{\max\{M^-(a_1), R^-(b_1)\}, \max\{M^-(a_2), R^-(b_2)\}\}= \max\{(M\times R)^-(a_1, b_1), (M\times R)^-(a_2, b_2)\}$. Therefore $(M\times R)^-[(a_1, b_1)+ (a_2, b_2)] \leq \max \{ (M\times R)^-(a_1, b_1), (M\times R)^-(a_2, b_2) \}$. If (a_1, b_1) and (a_2, b_2) are in $H_1\times F_2$, then $(M\times S)^-[(a_1, b_1)+ (a_2, b_2)] = (M\times S)^-(a_1+a_2, b_1b_2) = \max\{ M^-(a_1+a_2), S^-(b_1b_2)\} \leq \max\{ \max\{M^-(a_1), M^-(a_2)\}, \max\{S^-(b_1), S^-(b_2)\}\} = \max\{ \max\{M^-(a_1), S^-(b_1) \}, \max\{ M^-(a_2), S^-(b_2) \}\}= \max\{(M\times S)^-(a_1, b_1), (M\times S)^-(a_2, b_2) \}$. Therefore, $(M\times S)^-[(a_1, b_1)+(a_2, b_2)] \leq \max\{ (M\times S)^-(a_1, b_1), (M\times S)^-(a_2, b_2) \}$. If (a_1, b_1) and (a_2, b_2) are in $F_1\times H_2$, then $(N\times R)^-[(a_1, b_1)(a_2, b_2)] = (N\times R)^-(a_1a_2, b_1+b_2) = \max \{ N^-(a_1a_2), R^-(b_1+b_2)\} \leq \max\{ \max\{N^-(a_1), N^-(a_2)\}, \max\{R^-(b_1), R^-(b_2)\}\} = \max\{\max\{N^-(a_1), R^-(b_1)\}, \max\{N^-(a_2), R^-(b_2)\}\}= \max\{(N\times R)^-(a_1, b_1), (N\times R)^-(a_2, b_2) \}$. Therefore, $(N\times R)^-[(a_1, b_1)(a_2, b_2)] \leq \max \{(N\times R)^-(a_1, b_1), (N\times R)^-(a_2, b_2) \}$. If (a_1, b_1) and (a_2, b_2) are in $F_1\times F_2$, then $(N\times S)^-[(a_1, b_1)(a_2, b_2)] = (N\times S)^-(a_1a_2, b_1b_2) = \max \{N^-(a_1a_2), S^-(b_1b_2) \}\leq \max \{\max\{N^-(a_1), N^-(a_2)\}, \max\{ S^-(b_1), S^-(b_2) \}\} = \max\{ \max \{N^-(a_1), S^-(b_1) \}, \max \{N^-(a_2), S^-(b_2) \}\}= \max\{(N\times S)^-(a_1, b_1), (N\times S)^-(a_2, b_2) \}$. Therefore

$(N\times S)^{-}[\ (a_1, b_1)(a_2, b_2)\ ] \le \max\{\ (N\times S)^{-}(a_1, b_1), (N\times S)^{-}(a_2, b_2)\ \}$. Hence A×B is a bipolar valued fuzzy sub-four-semigroup of $G_1\times G_2$.

Theorem 2.6.

If $A_1 = M_1\cup N_1 = \langle (M_1\cup N_1)^{+}, (M_1\cup N_1)^{-} \rangle$, $A_2 = M_2\cup N_2 = \langle (M_2\cup N_2)^{+}, (M_2\cup N_2)^{-} \rangle, \ldots$, $A_n = M_n\cup N_n = \langle (M_n\cup N_n)^{+}, (M_n\cup N_n)^{-} \rangle$ are BVFSBSGs of the bisemigroups $G_1 = H_1\cup F_1$, $G_2 = H_2\cup F_2$, …, $G_n = H_n\cup F_n$ respectively, then $A_1\times A_2\times\ldots A_n$ is a bipolar valued fuzzy sub-n-semigroup of $G_1\times G_2\times\ldots\times G_n$.

Proof. The proof follows from the theorem 2.5.

Theorem 2.7

If $A_1 = M_1\cup N_1 = \langle (M_1\cup N_1)^{+}, (M_1\cup N_1)^{-} \rangle$, $A_2 = M_2\cup N_2 = \langle (M_2\cup N_2)^{+}, (M_2\cup N_2)^{-} \rangle, \ldots$ are BVFSBSGs of the bisemigroups $G_1 = H_1\cup F_1$, $G_2 = H_2\cup F_2$, …, respectively, then $A_1\times A_2\times\ldots$is a bipolar valued fuzzy sub ∞-semigroup of $G_1\times G_2\times\ldots$.

Proof. The proof follows from the theorem 2.6.

Theorem 2.8

Let $A = M\cup N = \langle\ (M\cup N)^{+}, (M\cup N)^{-}\ \rangle$ be a bipolar valued fuzzy subset of a bisemigroup $G = H\cup F$ and $V = (M\cup N)\times(M\cup N) = \langle\ (M\times M)^{+} \cup (M\times N)^{+} \cup (N\times M)^{+} \cup (N\times N)^{+}, (M\times M)^{-} \cup (M\times N)^{-} \cup (N\times M)^{-} \cup (N\times N)^{-}\ \rangle$ be the strongest bipolar valued fuzzy relation of G. If A is a BVFSBSG of G, then V is a bipolar valued fuzzy sub-four-semigroup of G×G.

Proof. Suppose that A is a BVFSBSG of G. Then for any $a = (a_1, a_2)$ and $b = (b_1, b_2)$ are in G×G. If $a = (a_1, a_2)$ and $b = (b_1, b_2)$ are in H×H, then $(M\times M)^{+}(a+b) = (M\times M)^{+}[\ (a_1, a_2)+(b_1, b_2)] = (M\times M)^{+}(a_1+b_1, a_2+b_2\ ) = \min\{\ M^{+}(a_1+b_1), M^{+}(a_2+b_2)\ \} \ge \min\{\ \min\{\ M^{+}(a_1), M^{+}(b_1)\ \}, \min\{\ M^{+}(a_2), M^{+}(b_2)\ \}\} = \min\{\min\{M^{+}(a_1), M^{+}(a_2)\}, \min\{M^{+}(b_1), M^{+}(b_2)\}\} = \min\{(M\times M)^{+}(a_1, a_2), (M\times M)^{+}(b_1, b_2)\} = \min\{(M\times M)^{+}(a), (M\times M)^{+}(b)\ \}$. Therefore $(M\times M)^{+}(a+b) \ge \min\{(M\times M)^{+}(a), (M\times M)^{+}(b)\}$, for all a and b in H×H. Also $(M\times M)^{-}(a+b) = (M\times M)^{-}[(a_1, a_2)+(b_1, b_2)] = (M\times M)^{-}(a_1+b_1, a_2+b_2) = \max\{\ M^{-}(a_1+b_1), M^{-}(a_2+b_2)\} \le \max\{\max\{M^{-}(a_1), M^{-}(b_1)\}, \max\{\ M^{-}(a_2), M^{-}(b_2)\}\} = \max\{\ \max\{\ M^{-}(a_1), M^{-}(a_2)\ \}, \max\{\ M^{-}(b_1), M^{-}(b_2)\}\} = \max\{\ (M\times M)^{-}(a_1, a_2), (M\times M)^{-}(b_1, b_2)\} = \max\{(M\times M)^{-}(a), (M\times M)^{-}(b)\}$. Therefore $(M\times M)^{-}(a+b) \le \max\{(M\times M)^{-}(a), (M\times M)^{-}(b)\}$, for all a and b in H×H. Thus M×M is a BVFSSG of H×H. If $a = (a_1, a_2)$ and $b =(b_1, b_2)$ are in H×F, then $(M\times N)^{+}(a+b) = (M\times N)^{+}[(a_1, a_2)+(b_1, b_2)] = (M\times N)^{+}(a_1+b_1, a_2b_2\ ) = \min\{\ M^{+}(a_1+b_1), N^{+}(a_2b_2)\} \ge \min\{\min\{\ M^{+}(a_1), M^{+}(b_1)\ \}, \min\{\ N^{+}(a_2), N^{+}(b_2)\}\ \} = \min\{\ \min\{\ M^{+}(a_1), N^{+}(a_2)\ \}, \min\{\ M^{+}(b_1), N^{+}(b_2)\ \}\} = \min\{(M\times N)^{+}(a_1, a_2), (M\times N)^{+}(b_1, b_2)\ \} = \min\{(M\times N)^{+}(a), (M\times N)^{+}(b)\ \}$. Therefore $(M\times N)^{+}(a+b) \ge \min\{(M\times N)^{+}(a), (M\times N)^{+}(b)\ \}$, for all a and b in H×F. Also $(M\times N)^{-}(a+b) = (M\times N)^{-}[\ (a_1, a_2)+(b_1, b_2)] = (M\times N)^{-}(a_1+b_1, a_2b_2) = \max\{M^{-}(a_1+b_1), N^{-}(a_2b_2)\} \le \max\{\max\{M^{-}(a_1), M^{-}(b_1)\ \}, \max\{N^{-}(a_2), N^{-}(b_2)\ \}\} = \max\{\max\{M^{-}(a_1), N^{-}(a_2)\ \}, \max\{M^{-}(b_1), N^{-}(b_2)\}\} = \max\{(M\times N)^{-}(a_1, a_2), (M\times N)^{-}(b_1, b_2)\} = \max\{(M\times N)^{-}(a), (M\times N)^{-}(b)\ \}$. Therefore $(M\times N)^{-}(a+b) \le \max\{(M\times N)^{-}(a), (M\times N)^{-}(b)\ \}$, for all a and b in H×F. Thus M×N is a BVFSSG of H×F. If $a = (a_1, a_2)$ and $b= (b_1, b_2)$ are in F×H, then $(N\times M)^{+}(ab) = (N\times M)^{+}[(a_1, a_2)(b_1, b_2)] = (N\times M)^{+}(a_1b_1, a_2+b_2\ ) = \min\{\ N^{+}(a_1b_1), M^{+}(a_2+b_2)\ \} \ge \min\{\ \min\{\ N^{+}(a_1), N^{+}(b_1)\ \},$

min { $M^+(a_2)$, $M^+(b_2)$ }} = min { min{ $N^+(a_1)$, $M^+(a_2)$}, min { $N^+(b_1)$, $M^+(b_2)$ }} = min {$(N\times M)^+(a_1, a_2)$, $(N\times M)^+(b_1, b_2)$}= min {$(N\times M)^+(a)$, $(N\times M)^+(b)$}. Therefore $(N\times M)^+(ab) \geq \min\{(N\times M)^+(a), (N\times M)^+(b)\}$, for all a and b in F×H. Also $(N\times M)^-(ab)$ = $(N\times M)^-[(a_1, a_2)(b_1, b_2)]$ = $(N\times M)^-(a_1b_1, a_2+b_2)$ = max{$N^-(a_1b_1)$, $M^-(a_2+b_2)$} ≤ max { max{ $N^-(a_1)$, $N^-(b_1)$ }, max{$M^-(a_2)$, $M^-(b_2)$}} = max { max { $N^-(a_1)$, $M^-(a_2)$ }, max { $N^-(b_1)$, $M^-(b_2)$ }} = max{ $(N\times M)^-(a_1, a_2)$, $(N\times M)^-(b_1, b_2)$ }= max { $(N\times M)^-(a)$, $(N\times M)^-(b)$ }. Therefore $(N\times M)^-(ab) \leq \max\{(N\times M)^-(a), (N\times M)^-(b)\}$, for all a and b in F×H. Thus N×M is a BVFSSG of F×H. If a = (a_1, a_2) and b = (b_1, b_2) are in F×F, then $(N\times N)^+(ab)$ = $(N\times N)^+[(a_1, a_2)(b_1, b_2)]$ = $(N\times N)^+(a_1b_1, a_2b_2)$ = min { $N^+(a_1b_1)$, $N^+(a_2b_2)$ }≥ min{min {$N^+(a_1)$, $N^+(b_1)$}, min {$N^+(a_2)$, $N^+(b_2)$}} = min {min{$N^+(a_1)$, $N^+(a_2)$}, min{$N^+(b_1)$, $N^+(b_2)$}} = min{$(N\times N)^+(a_1, a_2)$, $(N\times N)^+(b_1, b_2)$} = min {$(N\times N)^+(a)$, $(N\times N)^+(b)$}. Therefore $(N\times N)^+(ab) \geq \min\{(N\times N)^+(a), (N\times N)^+(b)\}$, for all a and b in F×F. Also $(N\times N)^-(ab)$ = $(N\times N)^-[(a_1, a_2)(b_1, b_2)]$ = $(N\times N)^-(a_1b_1, a_2b_2)$ = max{$N^-(a_1b_1)$, $N^-(a_2b_2)$} ≤ max{ max {$N^-(a_1)$, $N^-(b_1)$}, max{$N^-(a_2)$, $N^-(b_2)$}} = max {max {$N^-(a_1)$, $N^-(a_2)$}, max {$N^-(b_1)$, $N^-(b_2)$}}= max{$(N\times N)^-(a_1, a_2)$, $(N\times N)^-(b_1, b_2)$}= max{$(N\times N)^-(a)$, $(N\times N)^-(b)$}. Therefore $(N\times N)^-(ab) \leq \max\{(N\times N)^-(a), (N\times N)^-(b)\}$, for all a and b in F×F.Thus N×N is a BVFSSG of F×F. Hence V is a bipolar valued fuzzy sub-four-semigroup of G×G.

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