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Application of CDPU sets in Data Structure Analysis

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Abstract

This paper discusses practical aspects of a new graph labeling technique, Complementary Distance Pattern Uniform sets in Computer Science. A graph $G = (V, E)$ is complementary distance pattern uniform (CDPU), if there exists $M \subset V(G)$ such that $f_M(u) = \{d(u, v) : v \in M\}$, for every $u \in V(G) - M$, is independent of the choice of $u \in V(G) - M$ and the set M is called the CDPU set. The least cardinality of CDPU set in G is called the CDPU number of G, denoted by $\sigma(G)$. Representation of CDPU number in Tree data structures and analog to digital conversion is discussed.

1. INTRODUCTION

Graph theory is considered to have begun in 1736 with the publication of Euler's solution of the Königsberg bridge problem. As a valuable tool for dealing with relations of events, graph theory has rapidly grown in theoretical results and its applications to real-life problems. Graph theory provides a vocabulary that can be used to label and denote many social structural properties, giving a set of primitive concepts that allow us to refer quite precisely to these properties. Graph theory gives mathematical operations and ideas with which many of these properties can be quantified and measured. Given this vocabulary and these mathematics, graph theory can prove theorems and, hence, represent social structure and networking.

A *graph* $G = (V, E)$ consists of a non-empty set $V(G)$ of objects called *vertices* and a (possibly empty) set $E(G)$ of two element subsets of $V(G)$, called *edges*. The set $V(G)$

is called the *vertex set* of G and $E(G)$, its *edge set*. In the mathematical discipline of graph theory, a *graph labeling* is the assignment of labels, traditionally represented by integers, to the edges or vertices, or both, of a graph. Graph labeling was introduced in late 1960 by A.Rosa [12]. Instead of a finite subset of the set of integers, Acharya [1] associated an arbitrary non-empty set to the graphs, thereby laying the foundations for the set-valuations of the graph. In the intervening years, dozens of graph labeling techniques have been studied in over 800 papers and surveyed by Gallian. Labeled graphs are becoming an increasingly useful family of mathematical models for various applications such as coding, X-ray, crystallography, radar tracking, remote control, radio-astronomy, communication networks, network flow etc. This paper discusses the practical aspects of a new graph labeling technique-Complementary Distance Pattern Uniform set(CDPU). For all terminology and notation in graph theory not defined specifically in this paper, we refer the reader to Harary [9]. Unless mentioned otherwise, all the graphs considered in this paper are simple, self-loop-free and finite.

Graph Labeling has wide applications in such as coding theory, radar, astronomy, circuit design, missile guidance, communication network addressing, x-ray crystallography, database management [14]. Graph labelLing can be used to solve issues in Mobile Adhoc Networks (MANETS) [6]. Connectivity, scalability, and routing can be analyzed using Graph models. Automatic routing of data in a network can also be achieved using graph labelling[6]. The work proposed in [11] explore the role of labeling in expanding the utility of the channel assignment process in communication networks.

In computer science, a data structure is a particular way of storing and organizing data in a computer to be used efficiently. Different kinds of data structures are suited to varying applications, and some are highly specialized to specific tasks. In the proposed paper, we propose some applications of graph labelling in data structures in computer science.

Sets are as fundamental to computer science as they are to Mathematics. Whereas mathematical sets are unchanging, the sets manipulated by algorithms can grow, shrink, or otherwise change over time. Here we extend the concept; Complementary Distance Pattern Uniform Sets (CDPU) proposed by authors [8] to data structures. This paper initiates a study on the application of CDPU sets in graph theory on data structures in Computer Science.

The rest of the paper is organized as follows. Section 2 describes Complimentary Distance Pattern Uniform Sets (CDPU) in detail. Applications of CDPU are discussed in Section 3. The paper is concluded in Section 4.

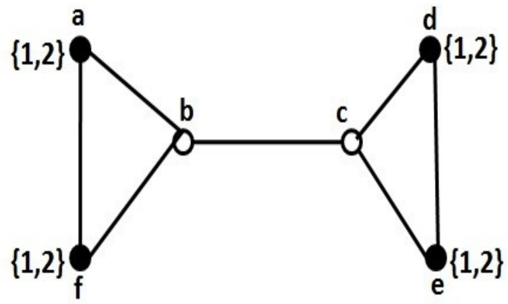

Figure 1: Graph with CDPU set $M = \{b, c\}$.

2. COMPLEMENTARY DISTANCE PATTERN UNIFORM SETS

One of the basic problems in graph theory is to select a minimum set S of vertices in such a way that each vertex in the graph is uniquely determined by the distances to the chosen vertices. This concept was first introduced by Slater [13] who called such a set, a *locating set*. The same concept was independently discovered by Harary and Melter [10] who used the term *resolving set*. Let $W = \{w_1, w_2, \ldots, w_k\}$ be an ordered subset of $V(G)$. We refer to the $k-$vector $r(v|w) = (d(v, w_1), d(v, w_2), \ldots, d(v, w_k))$ as a metric representation of v with respect to W. The set W is called a resolving set for G if $r(u|w) = r(v|w)$ implies that $u = v$ for all $u, v \in v(G)$.

Instead of considering the $k-$vector of distances, it was Acharya [15] who introduced the notion of taking the set of all distances from a vertex v to a subset of the vertex set and investigate under what condition this assignment of the set of all distances of each vertex is injective.

B.D.Acharya [15] define the M distance pattern of a vertex as follows: [15] Let $G = (V, E)$ be a (p, q) graph and M be any non-empty subset of $V(G)$. Then, the M - distance pattern of u is the set $f_M(u) = \{d(u, v) : v \in M\}$, where $d(u, v)$ denotes the usual distance between u and v in G.

Authors of [8] introduce complementary distance pattern uniform graphs as follows:

[8] If $f_M(u)$ is independent of the choice of $u \in V - M$, then G is called a Complementary Distance Pattern Uniform (CDPU) Graph and the set M is called the CDPU set. The least cardinality of CDPU set in G is called the CDPU number of G, denoted $\sigma(G)$.

In Figure 1, choose $M = \{b, c\}$. Then $f_M(a) = f_M(d) = f_M(e) = f_M(f) = \{1, 2\}$.

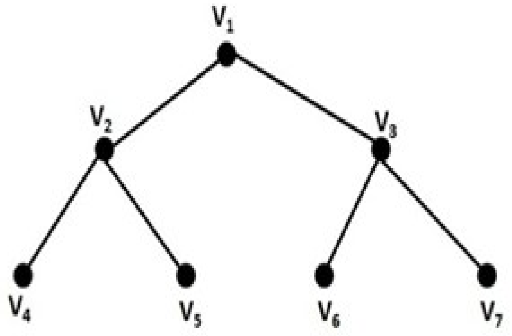

Figure 2: A Complete Binary Tree.

[8] Every connected graph has a CDPU set. [8] A graph G has $\sigma(G) = 1$ if and only if G has atleast one vertex of full degree.

[8] Let G be a non self-centered graph having no full degree vertex. Then, $\sigma(G) = 2$ if and only if the vertices of G have exactly two different eccentricities such that, the number of vertices corresponding to atleast one of the eccentricities should be two.

[8] Let G be a non self-centered graph with n vertices and CDPU set M. Then, the vertices in $V - M$ possess the same eccentricity.

[8] For any integer $n \geq 4, \sigma(P_n) = n - 2$. [8] $\sigma(K_{a_1,a_2}) = 2, a_1, a_2 \geq 2$.

[8] $\sigma(C_n) \leq \begin{cases} n-2, & \text{if n is odd;} \\ \frac{n}{2}, & \text{if } n \geq 8 \text{ is even.} \end{cases}$

[8] For a double star $B(m, n)$, $\sigma(B(m, n)) = 2, m, n \geq 1$. For a tree T, $\sigma(T) = 2$ if and only if $T \approx B(m, n)$.

3. DATA STRUCTURES AND CDPU APPLICATIONS

Data structures are used in almost every program or software system. Data structures provide a means to manage huge amounts of data efficiently, such as large databases and Internet indexing services In computer science, a tree is a widely-used data structure that emulates a hierarchical tree structure with a set of linked nodes. A tree is a connected acyclic graph; unless stated otherwise, trees and graphs are undirected. We can take an arbitrary undirected tree, arbitrarily pick one of its vertices as the root, and make all its edges directed by making them point away from the root node and assign an order to all the nodes results in tree data structure. The application of CDPU concepts on tree data structures gives valuable information that can be used in the analysis of memory operations in running time analysis

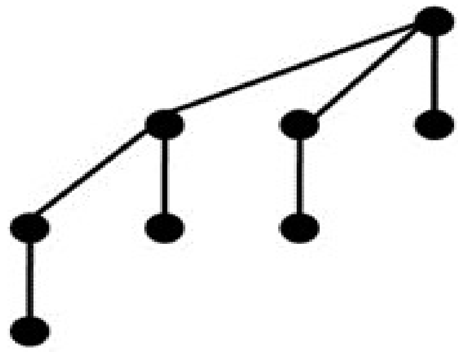

Figure 3: A Binomial Tree.

3.1. Complete k-ary tree

A complete k-ary tree is a k-ary tree in which all leaves have the same depth and all internal nodes have degree k. A complete k-ary tree has $\frac{k^h-1}{k-1}$ internal nodes, where h is the height of the tree. These internal nodes form a CDPU set for the tree and hence, $\sigma = \frac{k^h-1}{k-1}$. Thus, the number of nodes in the complement of the CDPU set = the number of leaves, for a complete k-ary tree. In Figure 2, $M = \{v_1, v_2, v_3\}$ form a CDPU set for that graph with $\sigma = 3$. For a complete k –ary tree G, relationship between CDPU number (σ) and the number of leaves (L) can be obtained as, $L = (k-1)\sigma(G) + 1$,

ie. when $k = 2$, $L = 1 \times \sigma(G) + 1$. When $k = 3$, $L = 2 \times \sigma(G) + 1$.

3.2. Binomial tree

The binomial tree B_k is an ordered tree defined recursively with height k. B_0 consists of a single node. B_k consists of two binomial trees B_{k-1} that are linked together. Figure 3 shows B_3. For a binomial tree, we get $\sigma(B_1) = 1, \sigma(B_2) = 2, \sigma(B_3) = 4, \sigma(B_4) = 6$ and $\sigma(B_5) = 8$ and in general, $\sigma(B_k) = 2 \times (k-1)$ and $\sigma(B_{k+1}) = \sigma(B_k) + 2$.

3.3. Vertex cover and Clique number of a tree

Vertex cover for a graph G is the set of all vertices which covers all the edges of G. The size of a vertex cover is the number of vertices in it. Minimum vertex cover is an important classical optimization problem in computer science. Let G be any graph. Then $\sigma(G) = 1$ if and only if the size of the vertex cover $= 1$. Also if $\sigma(G) = 2$, then clearly, the size of the vertex cover $= 2$.

But the converse need not be true. For example, for a path P_5, the size of the vertex

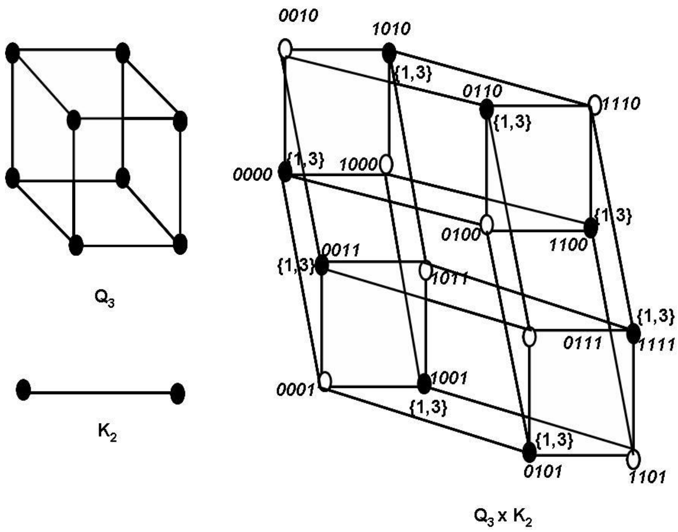

Figure 4: Quantization levels

cover $= 2$, but $\sigma(P_5) = 3$.

Hence for all graphs, the size of the vertex cover of the graph $\leq$ the CDPU number of that graph.

A clique is a complete subgraph of G. The number of vertices in the largest clique of G is called the clique number of G, denoted by, $cl(G)$. For a tree T with at least two vertices, $cl(T) = 2$. Except for a star, $\sigma(T) \geq 2$. Also for a tree, $cl(T) = \sigma(T)$ if and only if $T \approx B(m, n)$, where $B(m, n)$ represents a double star. Hence, for a tree except star, $cl(T) \leq (T)$.

3.4. Analog to digital conversion(ADC)

Analog to digital conversion(ADC) is an inevitable component in digital data transmission. ADC process utilizes sampling and quantization of the continuous analog signal. Consider a continuous-time signal digitized using $2n$ quantization levels, where n is the number of bits. Then 4 bit binary is needed to represent 16 quantization levels. Let's try to make an analogy using CDPU concepts of graph labeling. Consider n cube Q_n with vertices as binary representations of quantization levels. Figure 4 shows Q_4 with 16 quantization levels.

We have $Q_n = K_2 \times Q_{n1}$ and has $2n$ vertices which may be labeled $a_1, a_2, ...a_n$,where each a_i is either 0 or 1 . Also, two vertices in Q_n are adjacent if their binary representations differ at exactly one place. Let $V(Qn) = \{v_1, v_2, v_3, ..., v_{2n}\}$. Choose M as the set of all vertices whose binary representation differ at two places. Clearly,the vertices in M are non adjacent and also maximal with respect to the property of independence. Now, to check whether M is CDPU, choose $M = \{v_1, v_3, ..., v_{2n1}\}$ .Consider a vertex v_i which does not belong to M . Clearly, v_i is adjacent to a vertex v_j in M . Hence, $1 \in f_M(v_i)$. Then, since v_j is in M , v_j is adjacent to a vertex v_k not in M . Hence, 2 does not belong to $f_M(v_i)$. Since v_k is not an element of M and v_k is adjacent to a vertex v_l in M , $3 \in f_M(v_i)$. Proceeding in the same manner, we get $f_M(v_i) = \{1, 3, ..., n-1\}$, for every $v_i \in VM$. Hence, Q_n is a CDPU graph with $|M| = \frac{2n}{2}$

4. CONCLUSION

Here we tried to relate the concepts of CDPU in graph theory to tree data structures in computer science. CDPU number can be used for the asymptotic analysis of running time for the memory operations. Further study is needed to develop the exact relation of CDPU to running time analysis.

REFERENCES

[1] B.D. Acharya, *Set-valuations And Their Applications*, MRI Lecture notes in Applied Mathematics, **2** (1983).

[2] C. Berge, *Graphs and Hypergraphs*, North-Holland, Amsterdam, (1973).

[3] J.A.Bondy and U.S.R.Murthy, *Graph Theory with Applications*, The Macmillan Press Ltd., (1976).

[4] F. Buckley, F. Harary, *Distance in Graphs*, Addison-Wesley Publishing Company, (1990).

[5] G. Chartrand and P. Zhang, *Introduction to Graph Theory*, McGraw-Hill International Edition, Singapore, (2005).

[6] A. Elumalai *Graph coloring and labeling applications in computer science*, Malaya Journal of Matematik, 5, 4039-4041, 2020.

[7] Germina K.A. and R.Ananthakumar, *Distance pattern distinguishing sets in graphs*, Advanced studies in Contemporary Mathematics, 21 (2011), No.1, 107-114.

[8] Germina K.A., Beena Koshy, *Complementary Distance Pattern Uniform Graphs*, Int. J. of Contemporary Mathematical Sciences,**5**no.55(2010),2745 – 2751.

[9] F. Harary, *Graph Theory*, MAddison-Wesley Publishing Company, Massachusetts, (1969).

[10] F. Harary and R.A. Melter, *On metric dimension of graph*, Ars Combin., 2 (1976), 191-195.

[11] Prasanna, Nissankara Lakshmi et al. *Applications of Graph Labeling in Communication Networks*, (2014).

[12] A. Rosa, *On Certain Valuation of the Vertices of a Graph*, In Theory of Graphs, Gordon and Breach, New York and Dunod, Proceedings of the International Symposium in Rome Paris, (1967).

[13] P.J.Slater, *Leaves of trees*, Congr. Numer., 14 (1975), 549-559.

[14] Sravanthi, Kuchipudi et al. Applications of Graph Labeling in Major areas of Computer Science." (2014).

[15] Technical Report, *Set-valuations of Graph*, DST grant-in-aid project No.SR/S4/MS:277/05, funded by the Department of Science Technology (DST)(2011).

[16] Thomas H. Cormen. Charles E. Leiserson. Ronald L. Rivest. Clifford Stein, *Introduction to algorithms*, cGraw-Hill Higher Education, (2001).

[17] D. B. West, *Introduction to Graph theory with Applications*, Prentice Hall of India Private Limited, New Delhi, Eastern Economy Edition, (1996).